**Unit 4: Investigation 1 (4 Days)**

**Indirect Variation Functions**

**Common Core State Standards**

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.

F.IF.5Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes.

F.IF.7Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.9Compare properties of two functions each represented in a different way (algebraically, graphically, by table, or verbally)

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable in a modeling context.

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

**Overview**

This investigation will use an experiment with light to determine the Inverse Square Law of Light and to introduce the family f(x)= kx –n for natural numbers n. Students will complete a table, plot the points on graph paper, and draw a smooth curve. Students should try to obtain a mathematical model. The graph will be nonlinear so the graph can promote a review of the discussion of direct variation from algebra 1 and now a discussion of indirect (inverse) variation and functions of the form f(x) = kx –n for natural numbers n. Graphs of other nonlinear functions that students studied earlier this year and at the end of algebra 1 can be used for comparison and contrast. Students will examine the domains and ranges for f(x) = k/x and g(x) = k/x2, the behavior of this new family as x gets larger and larger, and the behavior near 0. Lastly, they will examine the graphs f(x) = kx-p  for *p* a rational number > 0. A review of integer and rational exponents will be provided. If students have not completed investigation 2 of unit 7 in Algebra 1, activities 7.2.2 – 7.2.4 should be completed before the review problems in 4.1.2 and 4.1.6 are assigned. Irrational values of *p* will not be studied in this course.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Distinguish, given a graph, linear vs nonlinear behavior.
* Distinguish, given a graph, differing nonlinear behaviors, in particular between polynomial degree ≥ 3 and quadratic, exponential, and now the new families--the power and selected rational.
* Explain how indirect variation differs from direct variation by equation and graph.
* Model situations with a direct or indirect variation function as appropriate.
* Demonstrate that the brightness of a source of light is a function of the inverse square of its distance.
* Make a quick sketch of f(x) = kx-n for n = 1, 2, 3, 4.
* Describe in words the end behavior of f(x) = kx-*n*, *n* a natural number, and the behavior near x = 0. Later, for *n* a rational number.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit slip 4.1.1** asks students to compare and contrast a decaying exponential function with an inverse square function with regard to y – intercepts, domain, range, end behavior, behavior near 0, concavity (+). It provides both a graph and equation for the two functions.
* **Exit 4.1.2 Slip** asks students to compare and contrast direct (y = kx) and indirect variation (y = kx-1) by graph, table, and equation.
* **Journal Prompt 1** asks students to explain why it gets so dark so fast outside the circle of a campfire.
* **Journal Prompt 2** asks students to describe how you would match the function f(x) = x-3 with its graph and how you would match f(x) = x-3/4 with its graph. Students may use symmetry or lack of it, asymptotes, end behavior, or any other characteristics of a function and its graph.
* **Activity Lab Sheet** **4.1.1** will require students to model a light experiment with an equation of the form f(x) = kx-2 and will require them to study the graph and make observations about the behavior of this function.
* **Activity 4.1.2 Evaluating and Graphing f(x) = kx-n for n a natural number** asks students to evaluate and graph members of the indirect variation family and has some review of exponents.
* **Activity 4.1.3 Direct Variation and Indirect Variation** This activity sheet will focus on applications of direct and inverse variation by table, verbal description, graph and equation.
* **Activity 4.1.4 Other Applications of an Inverse Square Law** will encourage students to explore other applications of the inverse square law. It is optional.
* **Activity 4.1.5 Evaluating and Graphing f(x) = kx-p for *p* a rational number > 0.** This activity asks students to evaluate and extend their graphing members of the power family where f(x) = kx-p, *p* a rational number > 0. Focus is on quadrant 1 behavior.
* **Activity 4.1.6** **A Review of Exponents and Radical Notation** provides some skill review with integral and rational exponents. It can be used any time after Activity 4.1.3.
* **Activity 4.1.7 Collecting Bird Data (Weights and Wingspans)** has students gather data for homework that will be needed for the launch activity of investigation 2.

**Launch Notes**

How do astronomers measure the distance to far away objects? Why does a street light look dimmer the farther we walk away from it? Terms such as distance, brightness, illuminated squares, area, amount of light called power or luminosity need to be discussed. Students can be placed in groups of three to four. Once in groups, the teacher may want to do a large group demonstration of the lab set up. **Activity 4.1.1 The Inverse Square Law of Light**, a lab activity,provides the instructions for the set up. But here are some helpful hints. It was easy to make the box using a k-cup box from Costco which contained 100 K-cups. Also, if you use scissors, there is a problem making the hole in the index card. The hole is either too small or too big.  But when an Exacto knife is used the hole comes out just fine. You get a more exact cut. That also means the teacher must make the boxes ahead of time. Science teachers may already have boxes that can be used. Teacher notes are provided and contain additional support for the lab.

Two sets of graph paper can be available for students, ¼ cm and 1 cm. For students needing support, the cm graph paper can be used. For others, it is a good activity to have them count ¼ cm squares and convert to area in cm2. Groups should also be encouraged to obtain a defining equation for the relationship that has been graphed. Obtaining this equation can be facilitated by having students have the area illuminated be a perfect square number of squares 1, 4, 9, 16, and then measure the distances. Although students are using range (output) values to obtain domain (input) values which may be a new experience for them, it affords a new strategy they can employ in future circumstances.

**Teaching Strategies**

Students will need to discuss the vocabulary associated with the light lab and a demonstration of the use of the modest equipment will assist students when they get into groups. After a whole class demonstration students can be grouped and can work on the lab.

**Activity 4.1.1 The Inverse Square Law of Light**

**Group Lab Activity – Activity 4.1.1. Students will discover the Inverse Square Law and meet a function that has a vertical asymptote.**

**Differentiated Instruction (For Learners Needing More Help)**

In the experiment Activity 4.1.1, teachers can have students use the cm square graph paper and have them illuminate say 25 cm2, then 16 cm2, 9, 4 and measure the distance instead of the other way around, making it easier to see the reciprocal of the squaring relationship.

Activity 4.1.2 can be started in class with teacher assistance and completed for homework on day one of this investigation.

**Differentiated Instruction (Enrichment)**

Students can extend this lab and answer the question: How do astronomers measure distances to far away objects and present to the class.

Once all groups have their graphs (and equation if possible) of relative brightness as a function of distance, and they are placed on the board, the teacher can havethestudents report out their results to the entire class. Students should be encouraged to explain how the domain of their graph differs from other nonlinear functions they have met. End behavior should be discussed in detail and related to earlier functions studied. The behavior near zero should be explored. The table feature of a grapher can be very helpful here. Students should be asked to consider if a graph can ever cross a vertical asymptote. The teacher can ask what is happening to the numerator and denominator as values of x get close to zero. Making a table might help explain this behavior better.The teacher can add graphs of a linear function that describes a direct proportion, a quadratic, an exponential and perhaps a cubic and students can be asked to compare and contrast the graphs with regard to domain and range, end behavior depending upon the time that remains. A class equation should be determined for the light experiment. Both from the graph and the equation, students should be able to describe that with increasing distance from the light source, the relative brightness decreases by the square of the distance.

**Differentiated Instruction (For Learners Needing More Help)**

In the class discussion that follows activity 4.1.1, when students are examining selected graphs of the functions to date, copies of the parent function sheet used in the algebra 1 course, or unit 1 of this course or an equivalent reference sheet can be made available. Time permitting; you might be able to have students generate a class parent function reference sheet promoting discussion of the functions studied in Algebra 1 and units 1 – 3 in Algebra 2.

**Journal** **Prompt** **1** Why does it get dark so fast when you walk outside the campfire circle of friends**?** Students might say that since brightness or illumination is inversely related to the distance from the campfire that as one walks from one foot away to two feet away, the illumination will be ¼ of what is was at one foot away.

**Activity 4.1.2 Evaluating and Graphing f(x) = kx-n for n a natural number** asks students to evaluate and graph members of the indirect variation family. It can be used in part or whole depending upon the class as a homework assignment on the first day of this investigation. This activity includes some linear functions and polynomial including quadratic functions so comparisons and contrasts can be made. Some work with integer exponents, as a review is also included.

Students should examine the graphs they made in activity 4.1.2, noting that for f(x) = kx-n, n a natural number, 0 must be excluded from the domain of each function. Students should be encouraged to use the table feature of their graphers to explore the behavior of the graphs near 0. Vertical asymptotes should be discussed more formally but it is too soon for a textbook definition. A class definition of direct and indirect (inverse) variation should be developed. The class should be prompted to note that one quantity has to be a constant multiple of a second quantity for direct variation so y = kx for k not 0 or y/x =k. They should note that k is the slope of the line and that the line passes through the origin since the y-intercept is 0. They should be prompted to note that for k > 0 as x increases, y will increase; and as x decreases the values of y will decrease. The teacher can then extend this concept to any power function whose p > 0 and say that if y = kxp, then y is directly proportional to the pth power of x, k not 0. For indirect also called inverse variation, where k > 0 students should note that as x increases, y will decrease so that y = k/x or y = k x-1 or xy = k. Again this can be extended to y = k/xn, n a natural number, for now and k not 0 and x > 0. The teacher can also note that generally for applications of inverse and direct variation, x > 0**. Exit slip 4.1.**1 can be used any time after Activity 4.1.2 has been discussed in class.

**Activity 4.1.3 Direct Variation and Indirect Variation** can be used in class or for homework.

**Pair Activity – Activity 4.1.3- Direct Variation and Indirect Variation**

This sheet will focus on applications of direct and inverse variation by table, verbal, graph and equation.

**Activity 4.1.6, A Review of Exponents and Radical Notation** can be used any time after **Activity** **4.1.3**.

**Activity 4.1.4 Other Applications of an Inverse Square Law** will encourage students to explore other applications of the inverse square law for it applies to gravitational forces, electrical forces, to name a few. It can be skipped or used as enrichment or challenge.

**Activity 4.1.4** Teacher Notes: Before giving out the activity it is suggested to show the following PowerPoint presentation -The Inverse Square Law – as it applies to radiation (or some of it) or you can go to an equivalent site for the inverse square law for radiation.

<http://medphys.mednet.ucla.edu/CSUN/Physics376/Week1/Lecture%2001%20-%20Intro/PDF%20Handouts/4%20-%20inverse%20square%20law.pdf>

After showing the presentation assign **Activity 4.1.4** with the problems. The last question, in the activity, asks the students to go to suggested websites or they are allowed to find their own web sites (If they do, they must bring their addresses to class) and give a brief talk in class on what they have learned about the inverse square law.

**Journal Prompt 2** Describe how you would match the function f(x) = x-3 with its graph and how you would match g(x) = x-3/4 with its graph. Use symmetry or lack of it, asymptotes, end behavior, or any other characteristics of a function and its graph. Students might say for f(x) = x-3 that the graph will exhibit odd symmetry, will be in the first and third quadrants, have a vertical asymptote with equation x = 0, horizontal asymptote with equation y = 0, the points (1,1) and (-1,-1) will be on the graph. For g(x) = x-3/4 they might say the graph is only in the first quadrant, it has a vertical asymptote with equation x = 0, horizontal asymptote with equation y = 0, no symmetry and the point with coordinates (1,1) is on the graph. They need not include all characteristics.

**Differentiated Instruction (Enrichment)**

Students can explain why the moon had a greater effect on the earth’s tides than on the sun. The force that creates the ocean tides on the surface of the Earth varies inversely (indirectly) with the cube of the distance from the earth to any other large body in space and varies directly with the mass of the other body. First translate the statement into an equation and use it to show the moon does indeed have more effect on the Earth’s tides than the sun.

A review of rational exponents may be needed. More work with rational exponents will occur later in the unit. Some work with rational functions of the form f(x) = d + x –p  with *p* mainly a natural number and f(x) = (a + x)/ (b + x) is provided. The intent is to demonstrate some more real word applications of a rational function and to obtain an informal class definition of a rational function. Class discussion could include problems like a student who has a 75% but wants a B (85) for the term, and the teacher asking how many 100s the student would need to get that 85 (modeled by f(x) = (100 x+ 75)/(x + 1), x the number of 100s or a basketball team with 36 wins and 34 losses is still hoping for a 60% winning record and asking how many consecutive games they must win to have a season end with that record, modeled by f(x) = (36 + x)/(70 + x), x the number of additional wins. Students should be encouraged to notice that power functions with negative integer exponents are simple examples of rational functions. Students will see that vertical lines other than x = 0 can serve as a vertical asymptote. Investigation 3 of this unit will develop graphing rational functions whose defining equations are a bit more complex, will provide more intense work with vertical asymptotes and will work with horizontal asymptotes besides y = 0. Some work with oblique asymptotes for STEM intending students will be included in a later investigation.

**Activity 4.1.5 Evaluating and Graphing f(x) = kx-p for *p* a rational number > 0** can be used in class and/or for homework. This activity asks students to evaluate and graph members of the rational family but especially for f(x) = kx-p, p a rational number. This activity is intended to strength the students’ comfort with the power family. It can be used as a homework assignment. This activity will also include some review of rational exponents.

**Closure Notes**

Students should in large group give a brief summary of the major properties of linear, quadratic, polynomial of degree three or higher, exponential, power and rational functions. Finding the domain, range, y-intercept and end behavior should be stressed. In addition, the impact of an application problem on the domain should be stressed. Students should especially compare and contrast direct and indirect variation by graph and equation and in instances where one is presented by graph and the other by equation or table. **Exit 4.1.2 Slip** can be distributed any time after Activity 4.1.3 has been discussed**.**

**Activity 4.1.7** can be distributed for homework. Two forms of the table are provided for some teachers may prefer the weight of the bird to be the independent variable, others may want to use the wingspan. In some classes the teacher may want half the class to use the weight as the independent and the other half of the class the wingspan and then compare the graphs. The data will be needed for the launch of the next class period activity in **Investigation 4.2.1.** (Teacher note: A list for selected birds has been provided in the Activity 2.1.1Answer Key so assignments of the birds from this list can be made and hopefully students may want to add a few birds from their backyards. The teacher should assign at least 20 birds in total but just a few per student so that there are at least 20 data points when the data is pooled.) Web sites are included in the investigation 2 resources for looking up birds not included in the answer key for **Activity 4.2.1**. Students should be encouraged to look up birds not on the teacher answer list.

**Investigation 4.2.1** will encourage using technology to assist with modeling a power function.

**Vocabulary**

Amount of light called power or luminosity

Area

Asymptote-- vertical and horizontal

Brightness

Direct variation or varies directly or directly proportional

Distance

Domain

Factors

Indirect (Inverse) variation or inversely proportional

Inverse Square Law

Power function

Proportion

Range

Ratio

Rational function

Scaling factor or stretch /compression factor/or constant of proportionality

Squares illuminated

Values excluded from the domain

*y* varies directly as the *n*th power of *x*

*y* varies inversely as the *n*th power of *x*

**Resources and Materials**

**Activities 4.1.1, 4.1.2, 4.1.3, 4.1.5, 4.1.7 should be completed in this investigation. Activity 4.1.7 is needed for the launch in Investigation Two. Activity 4.1.4** can be omitted**.** It is enrichment**. Activity 4.1.6** is optional. It provides practice for students needing additional review with rational exponents or radical notation.

Activity 4.1.1 The Inverse Square Law of Light

Activity 4.1.2 Evaluating and Graphing f(x) = kx-n for n a natural number

Activity 4.1.3 Direct Variation and Inverse Variation

Activity 4.1.4 Other Applications of an Inverse Square Law

Activity 4.1.5 Evaluating and Graphing f(x) = kx-p for p a rational number > 0

Activity 4.1.6 A Review of Rational Exponents and Radical Notation

Activity 4.1.7 Collecting Bird Data (Weights and Wingspans)

Graphing calculator/computer software with a graphing utility for all activities

Ruler with centimeter markings 4.1.1

Graph paper for activity 4.1.1

Flash lights mini Maglite recommended for activity 4.1.1

Cardboard for activity 4.1.1

Heavy construction paper or poster board for activity 4.1.1

Scissors or an Exacto knife for activity 4.1.1

Transparent tape for activity 4.1.1

Graph paper for all activities