**Activity 4.5.1 Rational Equations**

**Section I**

You have used the Multiplication Property of Equality at least since Algebra 1 when you solved a linear equation such as 2x + 4 = 12. Here is the statement of the Multiplication Property of Equality:

If a, b, and c are real numbers with c ≠ 0 , then a = b is equivalent to ac = bc. This means that if each side of an equation is multiplied by the same non zero number, the new equation is equivalent to the old one. For example:

(1) 8x = 35

(2) 8x (1/8) = 35(1/8). Since 1/8 ≠ 0 the solution 35/8 is not only the solution to equation (2) but it is also the solution of equation (1).

Note if:

(3) 8x = 35 and we multiply both sides by 0, we get

(4) 0 = 0 an equation for which all Real numbers are solutions, not just 35/8.

We introduced extra solutions known as extraneous solutions. An extraneous solution is a solution of a transformed version of an equation that does not satisfy the original equation because it was excluded from the domain of the original equation.

When a variable expression is in the denominator of a fraction, we need to multiply by the variable expression. We have no way of knowing whether the expression will equal zero or not. Thus the original equation and the new equation may not be equivalent.

Thus the check is not just a good thing to do, it is essential because a solution to the new equation may not be a solution of the original equation. Consider:

(5) Multiply both sides by x2 – 4 in its factored form (x-2)(x+2)

(6)

Work out the solution and you should get

(7) (x + 2) + (x – 2) = 4

(8) 2x = 4

(9) Hence x = 2.

But 2 does not satisfy the original equation. So the original has no solution even though (x + 2) + (x – 2) = 4 and 2x = 4 are satisfied by 2 and thus have a solution. Two is not on the domain of the original equation and 2 is an extraneous solution.

We introduced extra solutions known as extraneous solutions. An extraneous solution is a solution of a transformed version of an equation that does not satisfy the original equation because it was excluded from the domain of the original equation. For our example, 2 is not in the domain of the original expressions making up the equation.

Try the next four problems with your neighbor:

2. = 9



Your teacher now has an activity for you.

Teacher Notes:

**Section II.**

Each student or pair of students should be given one of the following 8 problems. After solving it they will look around the room for the solution for a card posted on a classroom wall. Teacher Round Robin full page sheets is provided. On the card with the solution will be another equation or problem from the ones below. If a student or two or three is having problems with the same problem they can work in a group for a bit till they get the solution and can move on. They continue to move around the room until they have solved all 8 problems. If students are not done a sheet with all 8 problems can be distributed for homework and students can x out the ones they have done in class or each of the 8 problems can be on a separate colored sheet. Each student gets one colored sheet and as they do a problem they turn it in and pick up the next colored sheet at the station that had the answer posted. Then if not done, students just pick up the sheets with the colors they need to complete.

1. 5/(x + 6) + 7/(x + 6) = 1
2. 4/(a + 2) + 2/(a – 6) = 8/(a + 2)
3. 8/(x2 – 16) = 1/(x – 4) - 9/(x + 4)
4. Jon and James like to shovel the driveway for their grandparents when it snows. When only Jon shovels alone it takes him 80 minutes for a moderate snowfall. When James shovels alone it takes him 90 minutes. Nana makes cookies and cocoa for them. How long does it take them when they can come together?
5. 1/5 + 2/3 + 5/(2x + 4) = 3/5
6. Johanna wants to know if a rational function ever crosses its horizontal asymptote. She does not have her grapher with her. Find the equation of the horizontal asymptote for the following function. \_\_\_\_\_\_ Use the equation of the horizontal asymptote and the equation of the function to algebraically determine if the graph of the function does cross the horizontal asymptote. If it does, what are the coordinates of the crossing point? \_\_\_\_\_\_\_\_\_\_\_\_\_