**(+) Activity 4.3.7 – Queueing Theory Application**

Queueing theory is the mathematical study of waiting lines. Understanding queues and managing them is one of the most important areas in operations management. Waiting in line is part of our everyday life. We wait in line at the movie theaters, amusement parks, deli counter, Department of Motor Vehicles, and many other places. It is estimated that Americans spend about 37 billion hours each year waiting in line.

**Your Mission:**

The vice president in charge of operations for an amusement park is concerned about complaints regarding long waits at the ticket window for a particular ride. He hires you (since you are an expert on queueing theory) to reduce the number of complaints.

You identify that this is a *single-server* model and you make the following assumptions:

a. Individual riders arrive at random.

 b. The time to complete the purchase of the ticket is random. This can be due to the customer paying by check, cash or credit, the number of tickets being purchased, or the customer asking for information about the ride.

You collect data on customers and find that the average number of customers arriving per hour, *a*, is 36, and the average number of customers the single ticket agent can help, *h,* is 38 per hour.

1. If 36 customers arrive per hour, on average, how much time occurs between arrivals?
2. If *a* customers arrive per hour, on average, how much time occurs between arrivals?
3. If 38 customers are helped per hour, on average, how long does it take to help one customer?
4. If *h* customers are helped per hour, on average, how long does it take to help one customer?

Next you need to find the *traffic intensity*, *t*.

$$ t=\frac{average rate of customer arrivals}{average rate of customers being helped}=\frac{a}{h}$$

1. Find the traffic intensity *t*.
2. The average number of customers in the system, *C*, (which includes people in line and people at the ticket window) can be represented by the function:

$$C=\frac{t}{1 - t} where 0<t<1.$$

1. Calculate the average number of customers in the system:
2. What does this value of *C* tell you about this queueing system?

Customer’s satisfaction depends upon the length of time it takes to purchase a ticket rather than the length of the line. Let *W* equal the average time a customer waits in a system (this includes the time in line and the time to make a purchase).

The function *C* = *aW* expresses the relationship between *C* and *W*.

1. Given that *C* = *aW,* solve for *W* in terms of *a* and *C.*
2. Use your values of *a* and *C* to calculate the value for *W.*
3. What does this value represent?
4. Graph the function *C* = $\frac{x}{1 - x}$ on your graphing calculator, sketch the graph, and identify important features.

V.A.:

H.A.:

Hole:

Zero(s):

*x*-intercept(s):

*y*-intercept:

1. Using the graph what happens when *x* = 1?
2. How is this represented on the graph?
3. What happens to the function $C\left(x\right)$ when *x* > 1?
4. What is the domain of the function $C\left(x\right)$?
5. For the queueing problem, what was the domain?
6. Solve the function for $C\left(x\right)=-1$.
7. How is this represented on the graph?
8. What is the range of the function $C\left(x\right)$?
9. Which of the variables can the vice president control, *a* or *h*?
10. What recommendations can you make to improve customer satisfaction?