**Activity 4.3.3 – Graphing Rational Functions III**

In **Activity 4.3.2** you worked with vertical and horizontal asymptotes where the degree of the numerator and the degree of the denominator were equal. You found that when the degrees are equal the equation of the horizontal asymptote is defined by the quotient of the leading coefficients of the numerator and denominator. You also found that, if the degree of the numerator is less than the degree of the denominator, the equation of the horizontal asymptote is *y* = 0.

1. Sketch the graph of $g\left(x\right)=\frac{2(x+3)(x-2)}{(3x-2)(x+4)}$ and identify the important features.



VA:

HA:

Zeros:

*y*-intercept:

Domain:

1. Sketch the graph of $f\left(x\right)=\frac{3x(x+4)}{(2x-1)(x+3)}$ and identify the important features.



VA:

HA:

Zeros:

*y*-intercept:

Domain:

By now your class definition of a vertical asymptote might be something like: a **vertical asymptote** is a vertical line that a graph approaches but **never** crosses or touches. But as you saw in Questions 1 and 2 of this activity, perhaps to your surprise, the graph of a function can (and often does) touch, and even cross, a horizontal asymptote. You may need to adjust your class definition of a horizontal asymptote now.

1. Sketch the graph of $g\left(x\right)=\frac{x}{(x - 3)(x+2)}$ and identify the important features.



VA:

HA:

Zeros:

*x*-intercepts:

*y*-intercept:

Domain:

To determine if a function’s graph crosses its horizontal asymptote, we need to see if there is an input *a* that has the output *b*, that is, *g*(*a*) = *b*, where *y* = *b* is the equation of the horizontal asymptote.

In above example the HA has the equation *y* = 0. Let $\frac{x}{(x - 3)(x+2)}$ = 0 and solve.

 The numerator equals 0 when *x* = 0.

The graph crosses the horizontal asymptote at (0, 0).

We will study techniques for solving rational equations in Investigation 5 of this unit. Being able to determine if a graph will cross its horizontal asymptote provides one reason to study rational equations and their solutions. For now we will informally state that we multiplied both sides of the above equation by the product (*x* – 3)(*x* + 2). We know it is okay to multiply both sides of an equation by a nonzero number. In Investigation 5 we will explore how it is possible to multiply both sides of an equation by an expression that is not a number.

1. Find the point, if one exists, where the graph of the function crosses its horizontal asymptote.

$r\left(x\right)=\frac{3(x+3)(x-2)}{x^{2}}$

1. Sketch the graph of $k\left(x\right)=\frac{x^{2 }+ 3x }{x + 5}$ and identify the important features.

VA:

HA:

*x*-intercepts:

Zeros:

*y*-intercept:

Domain:

1. Sketch the graph of $r\left(x\right)=\frac{x^{2}-1}{x}$ and identify the important features.

VA:

HA:

*x*-intercepts:

Zeros:

*y*-intercept:

Domain:

1. Does every rational function have a horizontal asymptote?

Let’s summarize what we discovered so far about the HA and VA for rational functions.

1. Fill in the blanks below:

* When the degree of the numerator and denominator are the same, we find the value of the

HA by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* When the degree of the denominator is larger than the degree of the numerator, the HA is

always the line *y* = \_\_\_\_\_\_\_

* When the degree of the denominator is smaller than the degree of the numerator, a HA

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* The VA is related to the restrictions of the domain and can be found by setting the

denominator of the function equal to \_\_\_\_\_\_\_\_\_ and solving for the variable. (This will be refined later in future activities.)

* If the denominator is a quadratic or higher degree, then you will need to \_\_\_\_\_\_\_\_\_\_\_\_\_\_

in order to find the vertical asymptote(s).

1. Sketch the graph of $r\left(x\right)=\frac{1}{1+x^{2}}$ and identify the important features.



VA:

HA:

*x*-intercepts:

Zeros:

*y*-intercept:

Domain:

1. Do all rational functions have graphs with a vertical asymptote? Explain.
2. Sketch the graph of $f\left(x\right)=\frac{3}{(x + 3)^{2}}$ and identify the important features.



VA:

HA:

Zeros:

*y*-intercept:

Domain:

Consider the function $f\left(x\right)=\frac{2x-8}{x^{2}-16}$ = $\frac{2(x-4)}{(x-4)(x+4)}$. Notice that when *x* = 4 both the numerator and denominator equal 0.

* Put the above equation in Y1 and graph. You see no holes in the graph.
* Evaluate the function at *x* = 4 using the value command. Select 2nd TRACE, select 1:value, enter 4, press ENTER.
* You will see y =
* This means that when *x* = 4, there is no value for y.

We mark this on the graph with a hole, *an open circle*.

When a value of *x* sets both the denominator and the numerator of a rational function equal to 0, there is a hole\* in the graph; this is a point at which the function has no output value.

To find the coordinates of the hole, cancel the common factors in the numerator and denominator. This will create a "new" function since reducing the expression changes the domain of the function.  Evaluate the new function at the excluded value to obtain the output for the new function. ***Note*: The functions are no longer equal if they have different domains.**

\*If you evaluate the new function and the denominator is still zero but the numerator is not zero you have a vertical asymptote again and not a hole! We will not in this course work with functions that are this bizarre.

1. Graph $f\left(x\right)=\frac{2x-8}{x^{2}-16}$ = $\frac{2(x-4)}{(x-4)(x+4)}$ and identify important features.

VA:

HA:

Zeros:

*y*-intercept:

Domain:

Hole: (4, $\frac{1}{4})$ – Remember to put an open circle

at the point (4, $\frac{1}{4})$.

1. Graph $f\left(x\right)=\frac{4(x-2)}{x^{2}+x-6}$ and identify important features.



VA:

HA:

Hole:

Zeros:

*y*-intercept:

Domain:

1. Graph $f\left(x\right)=\frac{(x+2)(x-4)^{2}}{(x+2)(x+1)}$ and identify important features.



VA:

HA:

Hole:

Zeros:

*y*-intercept:

Domain: