**Activity 3.5.3 Can You Count It?**

Launch Problem: The football coach has to choose two captains for the final game of the season out of the five seniors that he has on the team. How many ways can he choose the two captains for the game?

 The class should already have a good idea how to solve this problem based on Pascal’s Triangle. The answer can be determined from the line of Pascal’s triangle that gives all the subsets of a set of five elements: 1 5 10 10 5 1. There would be 10 ways to choose the two captains.

This problem can also be solved by examining the coefficients of the Binomial Expansion of (x + 1)5 = x5 + 5 x4 + 10 x3 + 10 x2 + 5 x + 1. The coefficient of the term x2 is 10, which represents the number of ways the captains are chosen. This occurs because the term x2 appears in the product by choosing the factor of x from 2 factors of (x+1) in the expansion. To help visualize this, the images below show how a term of x2 can appear in the product.

$\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)$

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The images indicate each way to get a term of x2 in the product (x+1)•(x+1)•(x+1)•(x+1)•(x+1). There would be 10 ways for that to happen.

When using the product of the polynomials this way, we call the result a generating function. In essence, it generates the coefficients that represent the number of ways an outcome can happen.

**Making Change!**

Your teacher has challenged you to make exactly $1.50 in change, but you can only use 10 coins and you must use only pennies, dimes, or quarters. Can you do it?

While that problem could take a long time trying to come up with all the possible ways to combine 10 coins made up of pennies, dimes, and quarters, we could use generating functions to predict all the possible combinations that could occur. The generating function representing this problem is the 10th power of the polynomial (x1 + x10 + x25) where x1 represents the choice of a penny, x10 represents the choice of a dime, and x25 represents the choice of a quarter for each coin.

1. Examine what (x1 + x10 + x25)2 would give as an answer and what the terms of the product would represent.

2. Use a CAS to expand (x1 + x10 + x25)10 and see if you can determine if it’s possible to get a total of $1.50 using 10 coins.

3. Find how many coins between 6 and 15, would enable you to make exactly $1.50.

**Making a Fair Committee**

The Governor of Connecticut wants to create a committee of 6 members made up of at least one member of from each of four groups of women: Caucasian, African American, Hispanic American, and Asian American. If there are 6 Caucasian women, 7 African American women, 4 Hispanic American women, and 3 Asian American Women, how many ways can the committee be chosen if it does not matter which women from a group are chosen. In mathematics we say that the members of the group are indistinguishable?

1. To solve this problem using generating functions we need to create the polynomials that represent the possible choices. Since there were 6 Caucasian women and at least one must be chosen, the factor representing those choices is *(x1 + x2 + x3 + x4 + x5 + x6).* Find the factors that represent the choices for African American, Hispanic American, and Asian American women.

2. The product of the different factors would give the possible combinations of forming committees of 4 to 20 members. Use a CAS to create the generating function.

3. Note that the coefficients of x4 and x20 are each 1. That’s because there’s only one way to get a committee of 4 people, that is 1 representative from each group, and there’s only one way to get a committee of 20 people, that is taking every member of every group. Interpret what each coefficient represents and determine how many ways can you make a committee of 6 people.

4. Verify your answer in the previous question by finding all the committee configurations that you could have.

5. Write a word problem for which the polynomial (1+x+x2)3 would be a generating function. Interpret what the meaning of the coefficient of the x4 term would be in the context of your word problem.