**Unit 1: Investigation 1 (5 Days)**

**Systems of Linear Inequalities and Linear Programming**

**Common Core State Standards**

**A.REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**A.REI.D.11** Explain why the x-coordinate of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include where f(x) and/or g(x) are linear, rational, absolute value, exponential and logarithmic functions.

**A.REI.D.12** Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality) and graph the solution set of a system of linear inequalities as the intersection of the corresponding half-planes.

**Overview**

This investigation builds on the mathematics learned in algebra one and extends students’ understanding of algebra and graphing techniques to the solution of a system of linear inequalities. It then extends their understanding to modeling and solving LP optimization problems. Students will graph the solution set of a linear inequality in two variables and solve systems of inequalities graphically by hand, and with a graphing calculator. They will identify the boundary lines, half-planes, feasible region and vertices of a feasible region and determine the objective function for a real-world problem. Students will apply the Fundamental Principle of Linear Programming (the maximum/minimum solution occurs at a vertex of the feasible region when certain conditions are met such as the feasible region is bounded) and determine the optimal solutions to real-world problems. The students’ experience solving optimization problems will result in an understanding of the historical applications and practical efficacy of linear programming and its importance in present day decision-making.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Students will graph the solution set of a linear system of inequalities in two variables as the intersection of the corresponding system of half-planes, and interpret the result.
* Students will be able to define an objective function for a given context.
* Students will be able to use the seven step linear programming algorithm to identify optimal solutions to practical problems.
* Students will be able to use linear programming to identify optimal solutions to practical problems.

**Assessment Strategies: How Will They Show What They Know?**

* **Activity 1.1.1** **Group Activity Defining Variables and Writing Constraints** students will carefully read a complex problem and identify the variables and write constraints
* **Activity 1.1.2 A** **Homework Activity Defining Variables and Writing Constraints** provides more practice and sets conditions for finding the solution of a linear inequality and a system of linear inequalities.
* **Activity 1.1.2 B Homework Activity Defining Variables and Writing Constraints** provides more practice and sets conditions for finding the solution of a linear inequality and a system of linear inequalities.
* **Activity 1.1.3** **Graphing Constraints and Determining the Objective Function**

Is both an inclass group and individual in determining an objective function and graphing a system of linear inequalities.

* **Activity 1.1.4** **Homework for Role of the Corner Points and Some Practice** examines the importance of the corner points of the feasible region.
* **Activity1.1.5** **The Rational Behind Only Checking Corner Points** provides students with a graphical rational for being able to test just a finite number of points to optimize the problem.
* **Activity1.1.6** **Meera’s Jobs** has students solve another linear programming problem
* **Activity1.1.7** **The Farmer** is another LP problem that is a bit more challenging
* **Activity 1.1.8 Farm Subsidies** provides additional LP problems
* **Activity 1.1.9 Natasha’s Cat** is another LP problem with a slight twist.
* **Activity 1.1.10 Supplemental Problems** has more LP problems.
* **Exit slip 1.1.1** Student will graph the solution of one linear inequality and of a system of inequalities in an applied setting and interpret their result.
* **Journal Entry 1** If a new student comes to our class and has missed the last 4 days, explain in your own words to this new student the process needed to solve a Linear Programming problem.
* Student presentations (optional)
* A differentiated Linear Programming Problem to be included in the midunit assessment

**Launch Notes :**

**Real Life Context** –

Linear programming provides a mathematical way to identify optimal conditions. Linear programming was developed out of necessity during World War II, and with the invention of computers, made advances into the 21st century. Linear programming was used, beginning with World War II and then other conflicts, to optimize the use of resources for the military: food, vehicles, ships, personnel, etc. (View Video news Clip <https://screen.yahoo.com/u-troops-head-ebola-hot-163128163.html>? Or use an equivalent clip that will make students wonder how do we move so much equipment or other goods effectively and efficiently.)

In the 1960’s Linear programming was used in the launch of the first rocket that carried an astronaut into space. In industry, linear programming is used to determine optimal solutions to many real life situations. Linear programming is a process or algorithm that determines a maximum or minimum value (optimal solution) for a linear function of more than one variable where the independent variables are subject to linear constraints. The process may be represented graphically or algebraically. In this investigation we will examine the graphing approach to finding the optimal solution.

**Closure Notes**

This investigation provides several opportunities for end-of-investigation assessment: a student presentation (optional)– both collaborative (optional) or individual as well as a differentiated culminating problem that can be done individually or in pairs or groups. Either of these would be due by the end of the unit affording about three weeks for students to work on them while continuing to work through the remainder of the unit. There will also be a question on the midunit assessment. Two questions are provided. One is more challenging. The end of unit assessment will include problems that require students to solve a linear inequality in two variables and a system of linear inequalities in two variables. The CCSS only require the later.

In the problem assessment, students identify a practical application and create a corresponding problem. Or, to differentiate, the teacher may select an application and give the student a problem to solve. Students can work in small groups or alone to apply the seven-step linear programming algorithm. They could present the solution to the class orally. The students should submit a detailed written solution. This task would be completed by the end of the unit, not the investigation. It should be assigned as students complete investigation one and due by the end of the unit.

For the presentation assessment (optional), students will investigate the role of linear programming. They may research the history of linear programming, identify mathematicians who have had or may continue to have a major role in the area, or identify linear programming’s role in contemporary decision-making (**See Resources: Suggested Research Topics** at the endof this overview**).** Students will write a report about their research so that all members of the class learn more about linear programming’s development and benefits. Skits, videos, power point presentations, blogs and other creative presentation modes may be used. This task can be used for enrichment. Interdisciplinary planning and sharing are encouraged. This task would be completed by the end of the unit, not the investigation. It should be assigned as students complete investigation one and due by the end of the unit.

Students will be assessed individually through the solution of an application problem. Form A is more challenging than Form B and is included in the midunit assessment. Note students are first given Form A or B part 1 and they turn that in to get Part 2. Part 2 has the correct constraints so that students can demonstrate their ability to apply steps 3 – 7 even if they are still having trouble writing constraints. The end of unit assessment will include solving an inequality on 2 variables and solving linear system of inequalities.

**Teaching Strategies**

During the first two days, provide an overview of linear programming and within that context the need to graph the solution set of a linear inequality in two variables and then of a system of linear inequalities in two variables. Then be prepared to have students hone their abilities to apply components of the seven-step algorithm using practical linear programming contexts. To launch the project, students may watch the two- minute video clip about army planning and the movement of goods and troops to assist in the war against Ebola. A clip of your choice can be substituted. Students need to consider how we move so many goods and people efficiently, effectively and in a timely fashion.

The teacher can give students a real world problem involving transportation of military vehicles. Distribute just page one of **Activity Sheet 1.1.1 Defining** **Variables and Writing Constraints** to investigate in small groups, possibly with teacher guidance. Challenge the groups to work together for 20 minutes and find a “best” solution in the allotted time. You may need to remind students that they have heard the terms “is greater than” and “is less than” when they solved linear inequalities in one variable in algebra one and they have found solutions to equations in two variables. If they need prompting, ask them what problem solving techniques they possess—guess, check , revise and maybe organize in a table, for example. Students then share and explore all the different solutions each group proposes and agree on the one that appears to be the “best”. Then pose the question “How do we know if we have the very best solution?”

**Group Activity Sheet 1.1.1 page one only, Defining** **Variables and Writing Constraints**. Challenge the groups to work together for 20 minutes and find a “best” solution in the allotted time. Students then share and explore all the different solutions and agree on the one that appears to be the “best”.

Now distribute the remainder of **Activity Sheet 1.1.1 Defining** **Variables and Writing Constraints.** Introduce the idea that there is a mathematical way to judge the solution. Have students try the first two steps of the linear programming algorithm: defining the variables and writing constraints as well as determining solutions and non-solutions for the weight constraint. Have each group graph these on a transparency. Transparencies when used, should all have the same scale throughout this problem. Collect each group’s transparency.

 (**Teacher Note**: Prior to the next class, take the groups’ transparencies with the ordered pairs from the weight constraint and plot all points on one master transparency for use the next day. **Activity 1.1.1** has students graph the solution set of one inequality in two variables—the weight constraint. **Activities 1.1.2a** and **1.1.2b** will do the same, but for the area constraint. However, then the two solution sets will be superimposed to obtain the solution of the system on day 2 of this investigation.)

**Differentiated Instruction (For Learners Needing More Help)** Many students are overwhelmed with the amount of reading in a linear programming problem. You may prompt students needing assistance by verbally (or with highlighter) highlighting the sentences that contain the variable definitions or constraints. For example, “The first constraint uses the statement ‘No more than 100 shirts can be made’.”

 **Activity 1.1.2a** and **Activity** **1.1.2b** **Defining** **Variables and Writing Constraints** need to be assigned for homework so they can be used for the opening of the next day’s lesson. (**Teacher Note**: **Activity Sheets 1.1.2a and1.1.2b Homework** are provided so that students come to class on the second day with solutions and non-solutions and a graph (that has the same scaling as the one used in class for the weight constraint) of the ordered pairs for the area constraint. Two forms of the homework sheet are provided. Each one has different ordered pairs so that there will be sufficient points to draw a reasonable conclusion regarding how to quickly graph the solutions of an inequality with respect to the boundary line. The teacher may also want to provide some homework examples for students who need to review x- and y-intercepts of a linear function and solve some linear systems. Students will need to find x- and y-intercepts and solve systems on the next day to obtain the corner points of a feasible region.)

**Differentiated Instruction (For Learners Needing More Help)** The extensive

new vocabulary may be supported by diagrams, graphic organizers, and a student-constructed glossary.

**Differentiated Instruction (For Learners Needing More Help)** Student may need to review x- and y-intercepts and how to use them to graph a linear function

Students should be reminded that the intercepts are easy to find because the boundary equations are generally in standard form.

On the second day, display the master transparency ( teacher made from the group transparencies) showing all ordered pair solutions and non-solutions to the weight constraint that the groups tested the day before. Guide students to draw the conclusion that all the solutions are on one side of the boundary line. Then have students form their groups again and using a transparency you provide with the same scaling have group members plot their homework points on the one group transparency.

**Group Activity Sheet 1.1.2a and Sheet 1.1.2a second page** Using the points students found for homework have each group have its members transfer their points to a “master group transparency.” Then using one group’s master transparency as the class master have the other groups add their points. Hopefully they will notice like yesterday all solutions are on one side and all non solutions are on the other

 Now emphasize the use of the “test point”; i.e. graph the boundary line and then use one “test” point. If the inequality is satisfied, that is the side that contains the entire solution set. If the inequality is not satisfied then the other side of the line contains the solutions. The boundary line is included since we are graphing ax + by ≤ c or ax + by ≥ c. Often the origin can be used as the test point. (Teacher note: Keep class master transparencies for future reference.) Thus we do not have to test a lot of points and quickly can determine which side of the boundary line should be shaded.

Using **Activity Sheet 1.1.3 Graphing the** **Constraints and Determining the Objective** **Function** and student homework from Activity Sheet 1.1.2, each group will make a transparency for the area constraint.

**Group Activity Sheet 1.1.3 Graphing Constraints and Determining the Objective Function**. Have each group make a transparency for the area constraint.

You may then overlay all the area constraint transparencies. Again pose a question, “Where are all the solutions to the inequality?” After students agree on the answer to that question, overlay the master weight constraint transparency onto the area constraint transparencies so students can “see” the feasible region. All ordered pairs in the feasible region are solutions of the system but which one is the best? There are still too many solutions to test.

Then continue with **activity 1.1.3** to complete the 7-step process. This will provide the optimal solution to the military transport problem. Have students compare it to the earlier suggested class “best” solution.

Teacher Note:

Students may need a few examples of functions defined in terms of two variables. Maybe a function like an area function for rectangles. We give it a length and a width (an ordered pair)and it computes the area. The objective function is called linear when the collection of ordered pairs that satisfy f(x,y) = c from a line.

Emphasize the need to check the final answer with the actual problem to be sure it is indeed the solution. The remainder of this sheet can be done independently and completed for homework if necessary. But do also assign homework **Activity Sheet 1.1.4 The Objective Function**. Work from it will be used to launch the next lesson. **Activity** **1.1.4** should be used for homework and in addition to the critical LP task on it needed for the next lesson it contains some practice problems graphing inequalities if your class needs some practice. If you have extra time, it can be used to review solving linear systems and graphing a linear inequality in 2 variables using the test point method or the shading method.

**Differentiated Instruction (Enrichment)-** Activity 1.1.3 sections 2 – 7 can be completed by students working in pairs or alone. They can then share their observations at the close of class.

(**Teacher Notes**: Keep the transparencies produced today so that future work for the feasible region can be demonstrated.) Homework **Activity Sheet 1.1.4** where students graph the objective function for several force values is provided so students can come to class the next day and see that for changing values of c in ax + by = c, parallel lines are generated. Thus, we only need to look at edges of the feasible region and ultimately just the corners of the feasible region. For students needing more practice finding x- or y-intercepts or solving systems of equations, additional homework practice can be provided.

Using **Activity Sheets 1.1.5 The Rationale Behind** **Only Checking Corner Points**, students will gain an understanding of the relationship of the objective function to the corner points of the feasible region; that is, that the optimal solution to a linear programming problem will be found at a corner point (vertex). Students can discuss the first 2 pages of section one and then in group use the prior night’s homework , **Activity 1.1.4** where they graphed force lines to see why they need only consider the other edges of the feasible reason. Then they can continue to follow the work in **Activity 1.1.5** to reduce the number of points that need to be examined to just the corner points. Emphasize the need to check the final answer with the actual problem to be sure it is indeed the solution.

Students can then apply the 7-step algorithm to a new problem, the Stop World Hunger Fundraiser. Depending upon time and your class you may need to walk the students through the process up to the graph of the feasible region (steps 1 – 4). Then, you can have the students carry out steps 5, 6 and 7 to gain practice with the final steps of the linear programming algorithm.

**Exit Slip 1.1.1** should be distributed on either day 3 or 4 of this investigation

The URL below dynamically demonstrates objective function lines moving in parallel fashion. It may be used here or with the next lesson. <http://people.richland.edu/james/ictcm/2006/slope.html>

The URL above reinforces the relationship between the objective function and the corner points of the feasible region and it also opens up the discussion of what happens when the slope of the graph of an objective function is equal to the slope of an edge. Except for STEM intending, it is not necessary to spend a large amount of time on this last situation.

**Differentiated Instruction (Enrichment)-** Students can investigate the role of linear programming. See **Suggested Research List in** the Resources at the end of this unit overview.

Students may watch a video about the manufacture of Belgian chocolates. You may use the video at [www.metalproject.co.uk/METAL/Resources/Films/linear\_programming/index.html](http://www.metalproject.co.uk/METAL/Resources/Films/linear_programming/index.html). (produced by Mathematics to Enhance Economics: Enhancing Teaching and Learning). You may pause the video and have students do the associated linear programming problem and then restart the clip to "see" the solution. Or you can just let the teacher in the video talk students through the problem-- it is done slowly and with clarity. The problem has a fractional solution, which is fine since the company can produce a fraction of a batch of chocolate. Students can then work on **Activity Sheet 1.1.6 Meera’s Jobs or it can be assigned for homework.** Or for students needing a bit more challenge **Activity sheet 1.1.7 The Farmer** could be used instead. This feasible region does not have corners at the origin or on the x or y axis. Only one of the constraints is in the form . The second constraint isor *x*  ≤ *y* . Furthermore, instead of the non-negative constraints , The Farmer requires the constraints . This is a good problem too to use say *a* and *c* rather than *x* and *y.*

**Differentiated Instruction (For Learners Needing More Help)** When students are creating their graphs, you may need to remind students to label their lines clearly so that they know which line matches each constraint. This will help in formulating the equations that are used to find the intercepts.

**Differentiated Instruction (Enrichment)-** **Activity Sheet 1.1.7, The Farmer**, is a challenging problem in which only one of the constraints is in the form . The second constraint is. Furthermore, instead of the non-negative constraints , The Farmer requires the constraints .

You may emphasize that linear programming problems (now often called operations research) are seen in many real world contexts where a maximum or minimum solution is sought: transportation, manufacturing, agriculture, business, health care, advertising, military, telecommunications, financial services, energy and utilities, marketing and sales. See [www.hsor.org](http://www.hsor.org) for a very rich site: the Mathematics for Decision Making in Industry and Government, the High School Operations Research site. It has modules from Bethlehem Steel, the Meat Industry, LL Bean, and Special Education School Buses. Videos are available too. Most materials are for free. There are some that can be purchased. This site was a link from the NCTM Illuminations site.

Students should begin **Activity Sheet 1.1.8 Farm Subsidies** in class. The teacher may want to check the students’ constraints before they delve too deeply into the problem. The same set of constraints applies to Parts A1, A2 and B. A2 may be assigned for homework and B may be assigned for homework for students ready for a challenge. Or part B might be done in class or omitted for nonSTEM intending students. Problem B illustrates that when two adjacent corners both produce the optimum result, then all points along the entire edge do also. If the video <http://people.richland.edu/james/ictcm/2006/slope.html>

listed above **after Activity 1.1.5** was shown in class, students will have been forewarned of this possibility. Students who do not need scaffolding could do **Activity 1.1.9 Natasha’s Cat**. It requires students to *minimize* an objective function. The coordinates of one of the corner points are fractions, but make sense since one can feed a cat a half serving of cat food.

It is not necessary to do both **Activity 1.1.7** and **1.1.8** nor is it necessary to do all parts of **Activity 1.1.8.** These two activity sheets and their parts could be assigned by group, each group would do just one of them and then discuss the solution with the whole class. Lastly **Activity 1.1.10** contains a list of a few supplemental problems that can be used if more problems are needed, that can be used for group work/presentations, used as a substitute for some of the earlier activities or not used at all. In the Resources under the heading, **Supplemental LP Problem Resources** at the end of this documents are additional resources to obtain LP problems. The list also contains references some TI Activities and these include instructions for using the TI Graphing Inequalities APP.

**Journal Entry 1** If a new student comes to our class and has missed the last 4 days, explain in your own words to this new student the process needed to solve a Linear Programming problem. Students should be able to relate the 7 steps in their own words. Once they define their variables of course whether they tell the new student to define the objective function or to write the constraints first is immaterial.

**Vocabulary**

Algorithm

Boundary line

Constraint

Equivalent equation and inequality

Feasible region

Half-plane

Inequality

Maximum/minimum

Objective function

Optimization

Shading

Solution of a system of linear equations

Solution of a system of linear inequalities

System of equations

System of inequalities

Test point

**Resources and Materials**

* <https://screen.yahoo.com/u-troops-head-ebola-hot-163128163.html>? Activity 1.1.1
* Green and red pencils for plotting the solutions and non solutions in Activity 1.1.1, and Activity 1.1.2A and 1.1.2B
* <http://people.richland.edu/james/ictcm/2006/slope.html> (1.1.5)
* Belgian Chocolates and Tomato Farmers videos (produced by Mathematics to Enhance Economics: Enhancing Teaching and Learning – at [www.metalproject.co.uk/METAL/Resources/Films/linear\_programming/index.html](http://www.metalproject.co.uk/METAL/Resources/Films/linear_programming/index.html) (1.6)
* The following website may be used to give insight into who is using linear programming and why: http://www.ilog.com/products/optimization/. (anytime after 1.1.5)
* See [www.hsor.org](http://www.hsor.org) for many high school linear programming problems and information on linear programming (1.5)
* Contemporary College Algebra: Data, Functions, Modeling, 6th ed., McGraw – Hill Primis Custom Publishing . Don Small author (Farm Subsidy problem, 1.1.9)
* Farm subsidy data base at<http://farm.ewg>.

org/farm/region.php?fips=09003 (1.6)

* video on Operational Research use in 1.6 or later- [http://www.bnet.com/2422-13950\_23-178846.html](https://mail.wcsu.edu/owa/redir.aspx?C=5deda06d8de846579753f0ed1e80a694&URL=http%3a%2f%2fwww.bnet.com%2f2422-13950_23-178846.html)
* <http://illuminations.nctm.org> (supplemental problem #3)
* <http://www.wikihow.com/Make-a-Duct-Tape-Wallet>

<http://www.ducttapefashion.com/products/prod01.htm> (supplemental problem #4)

* TI Numb3rs Linear Programming Activity at <http://education.ti.com/educationportal/activityexchange/Activity.do?cid=US&aId=7508>

(supplemental problem #6)

* Bakers Choice published by Key Curriculum Press (supplemental problem #7)
* Information relevant to George Dantzig: <http://www.lionhrtpub.com/orms/orms-8-05/dantzig.html> and <http://www.lionhrtpub.com/orms/orms-6-05/dantzig.html> (Suggested research question #4)

**Activities 1.1.1 – 1.1.6 should be done with all students**. Activity 1.1.7 is designed for students needing a bit more challenge. Not all of 1.1.8 must be done with all students and Activity 1.1.9 and Activity 1.1.10 are additional problems and can be skipped unless there is time or different groups can do just one of them and then present to the class.

* **Activity 1.1.1** Group Activity Defining Variables and Writing Constraints
* **Activity 1.1.2 A** Homework Activity Defining Variables and Writing Constraints
* **Activity 1.1.2 B** Homework Activity Defining Variables and Writing Constraints
* **Activity 1.1.3** Graphing Constraints and Determining the Objective Function

both an inclass group and individual

* **Activity 1.1.4** Homework for Role of the Corner Points and Some Practice
* **Activity1.1.5** The Rational Behind Only Checking Corner Points
* **Activity1.1.6** Meera’s Jobs
* **Activity1.1.7** The Farmer
* **Activity 1.1.8** Farm Subsidies
* **Activity 1.1.9** Natasha’s Cat
* **Activity 1.1.10** Supplemental Problems

**Suggested Research Question List**

1. Research how linear programming came to being. How did it get its name? What role did World War II and computers play in its development?
2. Have there been any recent developments in the field of linear programming, sometimes called Operations Research (OR)? How, if at all, does linear programming or OR affect our daily lives and how we live?
3. Joseph Fourier proved the Linear Programming Theorem in 1826. It states: If a feasible region in a linear programming problem is convex and bounded, then the maximum or minimum quantity for the linear objective function is determined at one (or more) of the vertices of the region. Like many mathematical discoveries and proofs, it waited many years before being widely used, in this case more than 100 years. Who is Fourier? What is his role in the field of mathematics?
4. In 1947 George Danzig, Leonld Hurwitz and T. C Koopmans invented the simplex algorithm or method used for solving “big” linear programming problems. President Gerald Ford in 1975 gave George Danzig a Presidential Award for his contribution. Koopmans received a Nobel Prize in 1982. Research the lives of these men and their contribution to the simplex method.
5. In 1947 [John von Neumann](http://en.wikipedia.org/wiki/John_von_Neumann) developed the theory of the duality. Who is von Neumann and what is meant by the duality? Demonstrate its usefulness. (For students needing challenge)
6. Study the simplex method and solve a small linear programming problem using it. (For students needing challenge)
7. In 1945, George Stiegler worked on the problem of determining a least expensive, yet healthy diet. He received a Nobel Prize for his work. Find out more about what he did and his diet.
8. In 1979 the Soviet mathematician L. G. Kachian developed the ellipsoid method, which has not proved as useful as the simplex method, but it is noteworthy in this developing field of Operations Research. Research Kachian and his contributions.
9. In 1984, 29-year old Neredra Karmarkar of ATT Bell Laboratory announced his algorithm. Is it better than the simplex method? Research Karmarkar and the state of his linear programming algorithm.

Students can research large linear programming problems. See below for some sources

#### Large Scale Linear Programming Application Areas

Problem areas where large linear programming problems arise are:

* Pacific Basin facility planning for AT&T

The problem is to determine where undersea cables and satellite circuits should be installed, when they will be needed, the number of circuits needed, cable technology, call routing, etc. over a 19 year planning horizon (a linear programming problem with 28,000 constraints, 77,000 variables).

* Military officer personnel planning

The problem is to plan US Army officer promotions (to Lieutenant, Captain, Major, Lieutenant Colonel and Colonel), taking into account the people entering and leaving the Army and training requirements by skill categories to meet the overall Army force structure requirements (a linear programming with 21,000 constraints and 43,000 variables).

* Military patient evacuation

The US Air Force Military Airlift Command (MAC) has a patient evacuation problem that can be modeled as a linear programming problem. They use this model to determine the flow of patients moved by air from an area of conflict to bases and hospitals in the continental United States. The objective is to minimize the time that patients are in the air transport system. The constraints are:

1. All patients that need transporting must be transported; and
2. Limits on the size and composition of hospitals, staging areas and air fleet must be observed.

MAC have generated a series of problems based on the number of time periods (days). A 50 day problem consists of a linear programming problem with 79,000 constraints and 267,000 variables (solved in 10 hours).

* Military logistics planning

The US Department of Defense Joint Chiefs of Staff have a logistics planning problem that models the feasibility of supporting military operations during a crisis.

The problem is to determine if different materials (called movement requirements) can be transported overseas within strict time windows.

The linear programming problem includes capacities at embarkation and debarkation ports, capacities of the various aircraft and ships that carry the movement requirements and penalties for missing delivery dates.

One problem (using simulated data) that has been solved had 15 time periods, 12 ports of embarkation, 7 ports of debarkation and 9 different types of vehicle for 20,000 movement

**Supplemental LP Problem Resources**

1. At <http://illuminations.nctm.org> under grade 9 – 12 algebra select “Dirt Bike Dilemma”. You may just use the problem or parts of the lesson and activity sheets. (Type linear programming in the search box)
2. Numb3rs Season 3 “The Mole - Branch and Bound” provided on the Activities Exchange at <http://education.ti.com/en-GB/UK/teachers/numb3rs/series-three-archive>. Then select episode 5 the Mole. This problem is designed to answer how linear programming can be used to find the most probable outcome of a situation. It will study the branching and bounding algorithm that can be used when integer solutions are desired but not obtained. It is written for Algebra 1 students.(As an aside can search Numb3rs activitiesti to find other interesting tidbits to use throughout the year. Also if the URL is giving you a problem use this search too to get to The Mole.)
3. How many of Each kind? [www.pct.edu?K12/govmath/2004/docs/Linear-Programming-appendix.pdf](http://www.pct.edu?K12/govmath/2004/docs/Linear-Programming-appendix.pdf) . Presents a problem about a small bakery and a solution in all its glory.
4. Bakers Choice published by Key Curriculum Press has a real life problem involving a Bakery that wants to maximize their profits. It is a supplemental module for Interactive Mathematics.
5. At the TI Activities Exchange TI.com/en/us/activities and search by TI 84 or the Inspire and LP Or systems of inequalities and you will get a number of activities most of which use the Inequality Graphing App too if you want to introduce some technology. One title is Maximizing Your Efforts. These activities change with time so it is worth checking the site periodically. Some activities will disappear and new ones appear.