**Unit 1: Investigation 7 (2-3 Days)**

**ROOT FUNCTIONS**

Common Core State Standards addressed in this Investigation:

• F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

• F.BF.4 Find inverse functions

**Overview**

This Investigation builds on the concepts of inverse functions from Investigation 6 to study root functions. Students will see that not all power functions have an inverse over their natural domain; for example, f(x) = x2 does not have an inverse unless we restrict the domain of the function to either positive or negative values. By restricting the domain for even functions, power functions do have an inverse: f-1(x) = $x^{\frac{1}{n}}$ = $\sqrt[n]{x}$. Such functions have applications in unexpected domains, such as music, that students will explore.

**Assessment Activities**

**Evidence of success: What will students be able to do?**

* Review the properties of exponents.
* Find the inverse of a power function f(x) = xn.
* Be able to graph the functions f(x) =. $\sqrt{x}$ and g(x) = $\sqrt[3]{x}$.
* Apply the knowledge of root functions to contexts in which they occur.

**Assessment tools: How will they show what they know?**

* **Activity 1.7.1 Review of Exponents** provides a review of the important properties of exponents, such as am × an = am+n**,** $\left(a^{m}\right)^{n}=a^{mn}$, etc.
* **Activity 1.7.2 Inverses of Power Functions** asks students to use the process they learned in Investigation 6 to determine the inverses of the functions f(x) = x2 and g(x) = x3. They will see that g(x) does have an inverse, and that g-1(x) = $\sqrt[3]{x}$. The function f(x) does not have an inverse unless we restrict the domain to x ≥ 0, in which case f-1(x) =. $\sqrt{x}$.
* **Exit Slip 1.7** asks students to determine whether a given power function has an inverse. If it does not, students will find a suitable restriction of the domain of the function and find a formula for the inverse of the function.
* **Activity 1.7.3 Tuning Up** guides students through the mathematics of tuning an instrument such as a piano. Tuning an instrument properly requires the use of root functions.

**Launch Notes**

Ask the class if anyone has heard of something called the Quadrivium. Can anyone at least answer how many parts the Quadrivium must have? (Remember: a “quad” is a four-wheeled vehicle.) The Quadrivium was the division of education in ancient Greece into four parts, and mastery of these was considered the pinnacle of an educated person. The four parts were Arithmetic (Number), Geometry (Number in Space), Astronomy (Number in Space and Time), and … Music (Number in Time). Are your students surprised that Music was considered the equivalent of Arithmetic, Geometry, and Astronomy? We generally do not consider them equivalent today.

Ask the class if anyone plays an instrument, either in the school band or orchestra or otherwise. How do you tune your instrument? How does a professional musician know how to tune their instrument? If two orchestras were playing the same piece at the same time, but one was in Vienna and one was in Boston, how could we be sure both are playing at the same pitch? One way is to set a standard for tuning instruments. The standard in music is to tune Concert A (the A above middle C on a piano keyboard) to a frequency of 440 Hertz (Hz). Ask the class, if this is the tuning for Concert A, what is the frequency for Upper A, the A above Concert A? How does one octave vary from the other? If no student knows the answer, tell the class that one octave doubles the frequency, so Upper A is at a frequency of 880 Hz. Now ask, how many half-steps (from A to A sharp/B flat, from A sharp/B flat to B) are there on a piano keyboard? There are 12. It turns out that the keys are set not by dividing the number 440 by 12, as that would not sound right to our ears. Instead, they are set by making the ratio between Hz the same for each.

If time permits (or as an alternative way to launch this Investigation), you might play the same music (or an excerpt of the same music) set at the standard 440 Hz, then at a different setting such as 432 Hz. For example, there are versions of Pachalbel’s Canon available on YouTube at 440 Hz (<https://www.youtube.com/watch?v=PkSp8wc8lKw>) and at 432 Hz (<https://www.youtube.com/watch?v=AJsPmOMaiVc>). If you play an excerpt from each, can your students tell the difference? Alternatively, you might ask them a “quiz” in which students listen to brief clips of music played at both settings. The video <https://www.youtube.com/watch?v=LVoVr9UwOQM> plays a clip (starting at about 1:06 of the video) of the same music tuned two different ways. Later in the video (starting at about 3:09) the video reveals which is which.

**Teaching Strategies**

**Activity 1.7.1 Review of Exponents** is a review of the properties of exponents. Consider introducing this Activity by asking individual students to both simplify expressions such as am × an, $\left(a^{m}\right)^{n}$, etc., and to explain why these expressions simplify that way. Point out that these operations with exponents are not a mystery; they are based on the meaning of exponential expressions such as am. Depending on students’ responses, this Activity might be used as a quick review of the properties of exponents or as a homework assignment.

**Activity 1.7.2 Inverses of Power Functions** asks students to start with the functions x2 and x3 and to develop the inverse functions $\sqrt{x}$ and $\sqrt[3]{x}$, respectively. In order to complete the tables in the Activity, students will need to remember how to find the table of an inverse from the table of the function. If students have difficulty with this, remind them that “an inverse function undoes what a function does”; that is, the roles of the input and output are reversed. Students will also need to remember how to find a formula for the inverse function given a formula for the function itself. Refer students back to Activities 1.6.1 and 1.6.2 as needed.

**Exit Slip 1.7** can be used upon completing Activity 1.7.2.

Musical scales make a very interesting application of root functions. These are explored in **Activity 1.7.3 Tuning Up**. While it is obviously important to emphasize the mathematics of root functions in this Activity, it is also important to discuss the musical applications of this Activity, as these provide an excellent opportunity to make mathematics relevant to a group of students who do not often see its relevance to their interests.

**Differentiated Instruction (Enrichment)**

Activity 1.7.3 lends itself to several kinds of enrichment for musically inclined students. For example, you might ask students to research the history of music to find when and where “Concert A is 440 Hz” became standard. You might ask students to research the differences between 440 Hz and other settings, such as 432 Hz (as you might have played in the Launch to Investigation 7). What is the relationship between the setting 440 Hz and the length of the strings on a piano?
As mentioned above, music formed part of the Quadrivium of education in ancient Greece. Students could research the Quadrivium, or research other ways in which mathematics is connected to music.

**Closure Notes**

This Investigation represents the end of Unit 1 in Algebra 2. Be sure to remind students about all they have learned about functions, paying special attention both to families of functions and to transformations of functions. Throughout much of the rest of Algebra 2, students will study particular families of functions in more detail, often through the lens of functions and transformations. Also be sure to remind students of all the various applications of functions included in Unit 1, from such different areas as climate change, tax tables, and music. Mathematics is a useful subject precisely because it finds application to so many aspects of our everyday lives.

**Vocabulary**

Hertz

Inverse function

Quadrivium

Root function

**Resources and Materials**

Graphing calculator/computer software with a graphing utility for all activities

Graph paper for all activities

**Activities 1.7.2 and 1.7.3 should be completed in this Investigation by all students. Activity 1.7.1 is a review of the properties of exponents; the material in this Activity can be reviewed and used as needed by the class.**

Activity 1.7.1 Review of Exponents

Activity 1.7.2 Inverses of Power Functions

Activity 1.7.3 Tuning Up