**Unit 1: Investigation 6 (2 Days)**

**Symmetry**

***Common Core State Standards***

* G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**Overview**

Having studied transformations, students will now apply transformations to the concept of symmetry. Students will discover that isosceles triangles and isosceles trapezoids have a line of symmetry through the midpoint of a side, that kites have a line of symmetry through two vertices, and that parallelograms have 180° rotational symmetry. There will also be a discussion about the symmetries of other geometric shapes such as rectangles, rhombi and squares.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Find the line(s) of symmetry in a given polygon
* Identify figures with rotational symmetry.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 1.6.1** gives students an isosceles triangle on a coordinate axis and asks them to draw in the line of symmetry and identify the coordinates of the point in which the base and the line of symmetry intersect.
* **Exit slip 1.6.2** asks students to identify the types of symmetry found in a few different polygons.
* **Journal Entry** asks students to explain which types of trapezoids will exhibit symmetry.

**Launch Notes**

You can begin this investigation with a video on how symmetry is vital to the field of architecture. When the video is done, prompt the class to think of where they have seen symmetry in architecture—whether in local structures or places they have visited. The video can be accessed at: <https://www.youtube.com/watch?v=B9W7IABkFuU>.

**Teaching Strategies**

In an attempt to make the discussion of symmetry a concrete one, **Activity 1.6.1** **Creating Tangrams** has the teacher leading the class through the construction of a set of tangrams. Students may have explored tangrams in an earlier grade but most likely, they never created their own set. The directions are relatively simple but the key is discussing the geometry during and in-between the steps.

**Differentiated Instruction (for students needing more help)**

Some students may benefit from working at their own pace to create the set of tangrams. They can use the video found at: <http://www.linkslearning.k12.wa.us/kids/1_math/2_illustrated_lessons/7_Tangrams/index.html>. This is a nice resource because it goes at a student’s own pace. They advance to the next step when they are done.

Additionally, if you have some students that you feel will not be able to even follow directions, you can give them the following template to cut out: <http://www.tangram-channel.com/tangrams-to-cut-out/one-set-of-tangram-pieces-free-template/>

Step 1

Give every student a pair of scissors and a rectangular piece of construction paper. Don’t give them the whole sheet. Usually, half a sheet is a good rectangle to start with. Now ask if anyone knows how we can make a square out of the sheet. Some students will suggest simply folding the rectangle in half but that won’t assure a square. If need be, have them use a ruler to prove it isn’t a square. If no student suggests it, inform them to fold one vertex of the rectangle until the width of the rectangle is lined up with the length of the rectangle creating a trapezoid. They should then cut the rectangle that is formed off leaving just the triangle. Be sure to have students throw out the rectangle they cut off. It is not part of the tangram set.

Step 2

Take a moment to review triangle vocabulary. Ask what kind of triangle is formed. Some may recognize it as a right triangle and some as isosceles. So the most specific name is “right isosceles triangle.” Also ask what the “long” side is called—the side across from the right angle. Again, some may be familiar with the term hypotenuse. Now have the students unfold the triangle and ask what shape we have. They should recognize it is now a square. In Unit 3, we will examine formal definitions for polygons but you may want to informally ask what the required elements of a square are. Also be sure to ask what the term is that represents the fold (diagonal). Now have them cut along the fold and ask them what kind of triangles we get. Some should recognize that we now have two isosceles right triangles.

Step 3

Now have them put one of the triangles off to the side and turn the other so that the hypoentuse is on the bottom. Have them fold the triangle in half—from bottom vertex to bottom vertex. At this point, discuss the fact that the fold is a line of symmetry. You may want to also ask where the line of symmetry is in relation to the base. What you are getting at here is the notion that the line of symmetry occurs at the midpoint of the base and is a line of reflection. One half of the triangle is the mirror image of the other half.

Ask students what else the fold represents (altitude or median). Now have students cut along the fold and again ask what kind of triangles are created.

**Differentiated Instruction (for enrichment)**

You may want to use this opportunity to talk about how the two smaller triangles are similar to the larger triangle that is off to the side. The concept of similarity comes up repeatedly in the steps that follow, but is not emphasized in this course until Unit 4.

These triangles are the first two pieces of our set of tangrams. As an option, you may want to have students mark them with a ‘1’ and a ‘2’.

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Step 4

Now have students take the other large triangle and again place it so the hypotenuse is on the bottom. This time they are going to fold from the top vertex down to the midpoint of the base and again cut along the fold. This will give us our third piece—a medium-sized right isosceles triangle and an isosceles trapezoid. Most students will recognize the shape is a trapezoid but very few of them would identify the term “isosceles “used with one. They are much more familiar with the notion of isosceles triangles. An isosceles trapezoid is a trapezoid with a line of symmetry: consequently, the legs (the non-parallel sides) have the same length. You may want to also informally ask for the definition of a trapezoid. This is one of the polygons that will be explored in more detail in Unit 3. For now, you may want to informally define it as a four-sided figure with at least one pair of parallel sides.

Discuss the symmetry in the trapezoid. Note that after this many cuts, it is doubtful that students’ pieces are going to be perfect. Inform them that this trapezoid should look symmetrical.

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Step 5

Have students take the trapezoid and fold in half—from bottom vertex to bottom vertex. Take a moment to identify the fold as the line of symmetry. Also ask if it, as in the isosceles triangle, crosses the midpoint of the base. Some students may also recognize that it crosses the midpoint of the “smaller” base on top of the trapezoid.

Now have students cut along the fold. Ask them what shape we get. Most should recognize that we get non-isosceles trapezoids. Ask if they can find a line of symmetry at all in the shapes.





Step 6

It is helpful to think of the two trapezoids as sneakers or shoes. The easier fold is to take one of them and fold from the “heel” to the “toes”. After cutting along the fold, students will get the fourth and fifth pieces—a sqaure and a small isosceles right triangle. At this point, you may ask about the four lines of symmetry found in a square.



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Step 7

The final step is to take the other trapezoid and fold it from the “heel” to the “laces”. This is the most difficult fold of all and students are often confused by it. In fact, you may want to have students show it to you before they cut it. After cutting they should get a second small isosceles right triangle and a parallelogram. Ask students if they can find a line of symmetry in the parallelogram. Students often incorrectly think a parallelogram will have a line of symmetry because it “looks” symmetric.



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As a wrap-up, the activity sheet asked students to identify the symmetry, if any, in the shapes they encountered.

The activity sheet also has half the class working in pairs to arrange the seven tangram shapes into a square and the other half arranging the shapes into a non-square rectangle. When they have completed the task, ask them to find all the lines of symmetry in their objects and share with the class. Below are the solutions to the two puzzles.

**Differentiated Instruction (for students needing more help)**

The creation of the square and the rectangle are puzzles that help build spatial awareness. The Common Core Mathematical Practice Standard 1 states that students should “make sense of problems and persevere in solving them”. However, there is a fine line between perseverance and non-helpful frustration. So, if you feel as if students are really becoming frustrated, you can share the solution with them—or give them hints.



Note: You may want to have a discussion before students begin looking for symmetry that they should only consider the entire shape, not the pieces.

At the end of Day 1 you may give students **Exit Slip 1.6.1** which gives them an isosceles triangle on a coordinate axis and asks them to draw in the line of symmetry and identify the coordinates of the point in which the base and the line of symmetry intersect.

To begin Day 2 and **Activity 1.6.2**, show students a “perfectly cut” parallelogram from a set of tangrams. Remind them that in the previous activity they determined that the parallelogram did not have a line of symmetry. This activity will show, however, that the parallelogram does have rotational symmetry. They will see that if they rotate the parallelogram 180° they obtain the same image. Students will also explore the concept of rotational symmetry in a rectangle and a rhombus.

 They will also discuss what, if any, symmetry exists in a rhombus and in a rectangle.

**Technology Tip**

If you choose to use Geogebra to explore rotational symmetry in the rhombus, the following steps walk you through how to create a rhombus:

(1) Draw segment $\overbar{AB}$ as the diagonal.

(2) Use the **Perpendicular Bisector** tool to make the perpendicular bisector of $\overbar{AB}$

(3) Draw point *C* on this line

(4) Draw a circle with center *A* through *C*

(5) Label point *D* as the second intersection point of this circle and the perpendicular bisector.

(6) Draw segments from *A* to *C* to *B* to *D* to *A*.

(7) Hide the circle and the perpendicular bisector.

<http://archive.geogebra.org/en/upload/files/english/steve_phelps/constructions/rhom_diag.html>

Additionally, students will see that a kite has one line of symmetry through two of its vertices.

At the end of Day 2 you may give students **Exit Slip 2**, which asks them to identify the types of symmetry found in a few different polygons.

**Journal Entry:** What kind of trapezoid has a line of symmetry? How would you describe its location? Look for students to recognize that the line of symmetry passes through the midpoints of the parallel sides even if they do not identify name “isosceles.”

**Closure Notes**

In this investigation, students go through the steps to create a set of tangrams. This construction is used as a way to discuss the symmetry found in various geometric shapes. Students should come away from this investigation with the understanding that many shapes, including kites, isosceles triangles, and isosceles trapezoids have a single line of symmetry; some shapes such as rectangles and squares have more than one line; and several shapes do not have any lines of symmetry. Furthermore, some shapes have rotational symmetry.

**Vocabulary**

altitude
base
isosceles triangle

isosceles trapezoid

line of symmetry

rotational symmetry

symmetry

**Resources and Materials**

Video for creating tangrams:

<http://www.linkslearning.k12.wa.us/kids/1_math/2_illustrated_lessons/7_Tangrams/index.html>

Resources for creating tangrams:

<http://mathforum.org/trscavo/tangrams/construct.html>

Tangram Template:

<http://www.tangram-channel.com/tangrams-to-cut-out/one-set-of-tangram-pieces-free-template/>

Steps for Creating a rhombus in Geogebra:

<http://archive.geogebra.org/en/upload/files/english/steve_phelps/constructions/rhom_diag.html>

Activities:

 Activity 1.6.1 Creating Tangrams

 Activity 1.6.2 Symmetry in Other Shapes

Construction paper and scissors for Activity 1.6.1