**Unit 1: Investigation 4 (6-7 Days)**

**BUILDING NEW FUNCTIONS FROM OLD**

Common Core State Standards addressed in this Investigation:

• F.BF.1b Combine standard function types using arithmetic operations.

• F.BF.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Overview**

Given a small number of “base” functions, we can create a vast number of useful functions through combinations of those functions and through transformations of those functions. This unit starts with combinations of functions: starting with two functions f(x) and g(x), students will find the functions f + g, f – g, fg, and f ÷ g symbolically, and the functions f + g and f – g graphically. Transformations are introduced through the language of “inside changes,” changes to the independent variable, and “outside changes,” changes to the dependent variable. It is expected that students will be able to describe these changes verbally in contexts. For example, if A(r) represents the area A of a circle as a function of its radius r, then A(r) + 5 represents an area 5 units larger than the area of the circle, and A(r + 5) represents the area of a circle with a radius 5 larger than 4.

Activities 1.4.3, 1.4.4, and 1.4.5 all involve transformations of functions. The main ideas are split between Activity 1.4.3, which involves transformations of the forms f(x) + k and f(x + k), and Activity 1.4.4, which involves transformations of the forms kf(x), and f(kx). Students then “put transformations together” in Activity 1.4.5, where they are asked to find the graphs of transformed functions by considering several points on the original graph and to find the effect of the given transformation on those points. The final Activity in this Investigation asks students to study transformations of functions in the context of greenhouse gas emissions.

**Assessment Activities**

**Evidence of success: What will students be able to do?**

* Given two functions f(x) and g(x), be able to find f + g, f – g, fg, and f ÷ g symbolically and (for f + g and f – g) graphically.
* Represent a verbal description of a function transformation symbolically.
* Understand the difference between a transformation of an independent variable and a dependent variable.
* Given a function f(x), be able to describe the effects of the transformations f(x) + k, f(x + k), kf(x), and f(kx) for a constant k.
* Given a graph of a function and a transformation of that function, be able to determine the transformation that is represented in the graph.
* Given a graph of a function, be able to graph (by hand) a transformation of that function.

**Assessment tools: How will they show what they know?**

* **Activity 1.4.1 Putting Functions Together** asks students to take a collection of functions and to find the sum, difference, product, and quotient of the functions.
* **Activity 1.4.2 Inside Change, Outside Change** provides verbal descriptions of several contexts and asks students to describe the meaning of a transformation of the independent variable (an “inside change”) or the dependent variable (an “outside change”).
* **Activity 1.4.3 Move It! Part One** asks students to describe the effect of the transformations f(x) + k and f(x + k).
* **Activity 1.4.4 Stretch It! Part One** asks students to describe the effect of the transformations kf(x) and f(kx).
* **Exit Slip 1.4** asks students to describe functions in terms of a “base function” and a set of transformations.
* **Activity 1.4.5 Transformations Made Easy (Seriously)** provides students with a template for describing a transformation by asking them to analyze separately the effects of addition and multiplication on the independent and dependent variable in a transformation. Students start with several points, find these points under the given transformation, and has them sketch a graph of the transformed function by plotting the points.
* **Activity 1.4.6 Greenhouse Gas Emissions** asks students to analyze transformations of functions in the context of greenhouse gas emissions.
* **Journal Entry 1.4** asks if any function they have encountered thus far in Algebra could be described in terms of a “base function” and a set of transformations.

**Launch Notes**

Go to the JASON Digital Lab Coaster Creator at the website <http://content3.jason.org/resource_content/content/digitallab/4859/misc_content/public/coaster.html>. This website, sponsored by National Geographic, allows anyone (without login or password) to create their own roller coaster and test it online. (The website does require Adobe Flash Player to be installed; this is a free download from [www.adobe.com](http://www.adobe.com)). Ideally, you should allow the students to test the software on their own for 5-10 minutes, but if this is not possible, you could do a whole-class demonstration of the software using an overhead projector. The software will draw a profile of a roller coaster in a window and allow the user to test it when complete. Once students have had a chance to test the software, ask students as a whole group to look at the profile of two roller coasters you have created on the overhead, one with at least one loop and one without. Ask them to think of these profiles as graphs. Are they the graphs of functions? They should recognize that the graph from the roller coaster with a loop is not a function, but the other one is.

Now imagine that we want to manipulate the roller coasters as was done with the software. How can we make the roller coaster taller, that is, how do we stretch the function up or down? Suppose we want to stretch the roller coaster to the right, so that there is a longer distance to slow it down. How would we stretch the function in that direction? How might we shift the roller coaster (the graph of the function) to the right or left? These movements might be done inside the program by shifting and stretching the graph of the function that represents the side profile of the roller coaster. This Investigation will show how to start with any function and to shift and stretch it in any direction desired.

**Teaching Strategies**

Students have done algebraic manipulations in Algebra 1. Now we are ready to do the same kinds of algebra, but on functions rather than variables. **Activity 1.4.1 Putting Functions Together** is about how to combine two functions through addition, subtraction, multiplication, and division.

**Differentiated Instruction (For learners needing more help)**

If you are working with students who might experience difficulty in combining like terms (needed to complete Activity 1.4.1), you might want to review combining like terms from Algebra 1. Consider using Activities such as 2.3.1 or 2.3.2 from Algebra 1 for this purpose.

Activity 1.4.1 shows how we can take two functions and combine them in various ways. It doesn’t really apply to the roller coaster in the Launch, because there we were not “combining two roller coasters,” we were using one roller coaster and considering how to shift and stretch it. The next four Activities will demonstrate how to move functions in this way. **Activity 1.4.2 Inside Change, Outside Change** is preliminary to the next three Activities, and concerns how students can distinguish between transformations to the independent variable and transformations to the dependent variable. It will be important to introduce to the whole class the language used in this Activity. An “inside change” is a change where you alter the input into a function, a transformation of the independent variable. The transformations considered later are when we transform f(x) to either f(x + k) or f(kx). An “outside change” is a change where you alter the output from a function, a transformation of the dependent variable. The transformations considered later are when we transform f(x) to either f(x) + k or kf(x). In order to make this distinction clearer, you might want to write y = f(x) and then write the transformations as y + k and ky. You might want to use one of the problems from Activity 1.4.2 as an example for the whole class to follow.

**Pair Activity – Activity 1.4.3 – Move It! Part One**

**and Activity 1.4.4 – Stretch It! Part One**

These Activities are designed to be completed by students in pairs.

Activities 1.4.3 and 1.4.4 are very important for the remainder of Algebra 2, because the language and ideas of transformations will be used when students investigate families of functions throughout the course. It is therefore very important not to rush through these Activities. By investigating the basic transformations first with a table of values and then with the graphs of functions, students should be able to discover on their own the effects of the various transformations.

In Activity 1.4.3, the effect of the transformation f(x) + k is fairly straightforward, and students should be able to describe it satisfactorily. However, the effect of the transformation f(x + k) is more challenging, because students will often conclude that the graph of the function f(x + k) is shifted by k units to the right, rather than k units to the left. One way to help students see this distinction is to have them think of the value of f(0). For example, suppose that in a particular function, f(0) = 3 and g(x) = f(x + 2). Then g(2) = f(2 + 2) = f(4), but g(-2) = f(-2 + 2) = f(0) = 3. By graphing the points (0, f(0)) and (-2, g(-2)) we can see that the function has shifted 2 units to the left, not to the right.

While students should see the effects of the transformations kf(x) and f(kx), they may have a difficult time thinking of language to describe them. You might call on students to share their conclusions from Activity 1.4.4 with the whole class, and suggest (if no student has expressed the transformation this way) that we describe kf(x) as a “vertical stretch by a factor of k,” and f(kx) as a “horizontal stretch by a factor of k.” However, we can consider the transformations of the form f(kx) (the horizontal stretches) as being a different form of the vertical stretches (transformations of the form kf(x)). In the next Activity, we will put the two kind of stretches together and consider only stretches of the form kf(x).

**Exit Slip 1.4** can be used after students complete Activity 1.4.4.

**Activity 1.4.5** **Transformations Made Easy (Seriously)** puts the results from Activities 1.4.3 and 1.4.4 together. According to this Activity, we can think of any transformation as a horizontal translation, followed by a vertical stretch, followed by a reflection across the x-axis, followed by a vertical translation. Not every transformation will consist of all four of these translations or stretches, but if we consider them in this order, we can determine how to start with the graph of a function and determine the graph of the transformed function. This Activity could be given as a homework assignment after completing the previous two Activities in class.

**Activity 1.4.6 Greenhouse Gas Emissions** involves the application of the ideas in this Investigation to the context of greenhouse gases and its impact on global warming. This Activity is optional, but it is strongly recommended that students complete it as a way of seeing the application of the ideas in this Investigation. Students may complete **Journal Entry 1.4** after this Activity.

**Journal Entry 1.4**

Consider all of the functions you have used in Algebra 2 so far this year. Would it be possible to describe all of them in terms of a “base function” and some set of transformations? Give examples to show why or why not.

**Closure Notes**

Tell your students you have just granted them an incredible amount of power! They can now take **any** “parent” (simple) function and transform it into **any** other function in the same family of functions. In addition, they can take functions from more than one family and combine them by adding, subtracting, multiplying, and/or dividing them. Using just the tools from Investigation 4, a student can take a (very!) small number of parent functions and essentially build just about any function they will ever need for any purpose in mathematics. Your students can now do it all!!

**Vocabulary**

Absolute value function

Dependent variable

Family of functions

Horizontal shift/translation

Independent variable

Inside change

Isometry

Outside change

Parent function

Piecewise function

Reflection

Transformation

Vertex

Vertical stretch

Vertical shift/translation

**Resources and Materials**

Graphing calculator/computer software with a graphing utility for all activities

Graph paper for all activities

Online access for “roller coaster activity”

**Activities 1.4.1 through 1.4.5 are essential for Algebra 2 and should be completed by all students, in the order given. Activity 1.4.6 is optional but it is strongly recommended to be completed, to demonstrate some of the usefulness of considering transformations of functions in context.**

Activity 1.4.1 Putting Functions Together

Activity 1.4.2 Inside Change, Outside Change

Activity 1.4.3 Move It! Part One

Activity 1.4.4 Stretch It! Part One

Activity 1.4.5 Transformations Made Easy (Seriously)

Activity 1.4.6 Greenhouse Gas Emissions