**Activity 2.3.3a Complex Numbers (+)**

Now that you know that *i* is the imaginary number equivalent to $\sqrt{-1}$ ,and that *i* is not a real number, you are ready to put real numbers together with imaginary numbers to obtain *Complex Numbers.* An example of a complex number is 2+3*i* . The real part is the real number 2, the imaginary part is the 3 that multiplies the imaginary number *i.* Complex numbers are of the form a+bi : the real part is \_\_\_ the imaginary part is \_\_

Because the reals are a subset of the complex numbers, we can extend some of our operations on the real numbers to the complex numbers, but the there is a twist : *i2* is always equal to -1, and you must always write the square root of a negative number with as an imaginary number before operating on the square root of a negative number. For example, write$ \sqrt{-5 }$ as$ \sqrt{5 }i$ before operating with $\sqrt{-5 }$ .

Start with what you know about real numbers to reason how to operate with Complex Numbers:

1. Add and Subtract Complex numbers:

Show you know how to do this addition:

a. (3+2x) +(5-3x) = \_\_\_\_\_\_

Now try this. Be sure to write your answer in a+bi form even if the real or imaginary part is 0.

b. (3+2i) + (5-3i) c. (3+2i) + (3-2i)

d. (8 – 2i) – (-2+13i) -10 e. 3i – (3+2i)

2. Multiply Complex Numbers

a. Show you know how to do this multiplication: (3+2x)(5-3x)

Now try multiplying these complex numbers. It is similar to the multiplication in part a, but the only extra piece you have to remember is to rename *i2* as -1, then simplify. As before, write your answer in a+bi form.

b. (3+2i)(5-3i) c. (3+2i)(3-2i)

d. 10(8 – 2i) (-2+13i) e. (3i)(3+2i)

3. Conjugate of a Complex Number

The conjugate of a+bi is a-bi and is denoted with a bar over the top:

“z” is often used to denote a+bi , so z-bar (written z ) is the conjugate of z.

The conjugate of a complex number is the complex number that has the same real part and the opposite of the imaginary part.

3+5i: The real part is\_3\_\_ the imaginary part is \_5\_\_ The conjugate of 3+5i is \_\_3-5i\_\_

a. 4-6i: The real part is\_\_\_\_\_ the imaginary part is \_\_\_\_\_ The conjugate of 4-6i is­­­­­­­­­­­­­­­­­\_\_\_\_\_\_\_

b. -3-7i:The real part is\_\_\_\_\_ the imaginary part is \_\_\_\_\_ The conjugate of -3-7i is \_\_\_\_\_\_

c. -8i : The real part is\_\_\_\_\_ the imaginary part is \_\_\_\_\_ The conjugate of -8i is \_\_\_\_\_\_\_

d. 9 : The real part is\_\_\_\_\_ the imaginary part is \_\_\_\_\_ The conjugate of 9 is \_\_\_\_\_\_\_

e. When you did part d., did you notice something? If a number equals its complex conjugate, then it is not only a complex number, but also a \_\_\_\_\_\_\_\_\_\_ number.

f. Add a complex number and its conjugate:

(3+5i)+(3-5i) (-5-7i)+(-5+7i)

What kind of number do you obtain when you add a complex number to its conjugate?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_

g. Multiply a complex number and its conjugate:

(3+5i)(3-5i) (-5-7i)(-5+7i)

What kind of number do you obtain when you multiply a complex number by its conjugate?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_

h. True or false: Complex numbers are closed under addition, subtraction and multiplication.

4. Divide Complex Numbers : (rationalize the denominator),

a. Rewrite the following expressions by eliminating radicals in the denominator.

3÷$\sqrt{5}$ = $\frac{ 3 }{\sqrt{5}}$ = and this: 3÷$(2+\sqrt{5)}$ =$\frac{3}{2+\sqrt{5}}$

Now do this: write final answer in a+bi form

b. 3÷5i = $\frac{3}{5i}$ =

c. 3÷$(2-\sqrt{5}$) =$\frac{3}{2-\sqrt{5}}$ =

(hint: multiply the denominator by the conjugate)

d. 3÷(2-5i) =

e. 1÷ i =

f. Are the Complex Numbers closed under division? Explain.

g. Note: Rationalizing denominators is a useful technique, but there is nothing wrong with having a radical in the denominator, except in high school math competitions where you are told to write radicals in “simplified form” . In fact, in calculus, it will be useful to rationalize the numerator. Challenge: why do people consider rationalized denominators to be simplified form? Hint: consider the different scales on a slide ruler. Can you divide a square root by another square root on the slide ruler?