**Activity 2.3.1 Closure and Sets of Numbers**

A. The Desire for Closure: In mathematics, we like to be able to get an answer. We want closure. If we try to operate on a set of numbers and can’t get an answer in that set, we might want to create a new set of numbers so that there is an answer. We say a set is closed under an operation if and only if an operation performed on any numbers in the set has an answer in the same set.

1. Is the set of Natural numbers closed under addition? i.e. if you add any two natural numbers, do you always get another natural number? \_\_\_\_\_\_\_\_\_\_\_

Give an example if yes. Give a counterexample if not. \_\_\_\_\_\_\_\_\_

2. a. If you subtract the two Natural numbers, 16 and 5, (16 – 5) do you get another natural number? \_\_\_\_

b. If you subtract 5 and 16 (5 – 16), is the result a natural number? \_\_\_\_

c. Is the set of Natural numbers closed under the subtraction?\_\_\_\_

d. What kinds of numbers are needed so that there is an answer to 5 – 16?

3. a. Are the Natural numbers closed under multiplication? \_\_\_\_\_

Give an example if yes, or a counter example if not.\_\_\_\_\_\_\_\_\_\_\_\_

b. Are the Integers closed under multiplication? \_\_\_\_\_

Give an example if yes, or a counter example if not.\_\_\_\_\_\_\_\_\_\_\_\_

c. Are the Natural numbers closed under division ? \_\_\_\_\_

Give an example if yes, or a counter example if not.\_\_\_\_\_\_\_\_\_\_\_\_

d. Are the Integers closed under division? \_\_\_\_\_

Give an example if yes, or a counter example if not.\_\_\_\_\_\_\_\_\_\_\_\_

4. What kind of numbers are needed so that the Natural numbers and the Integers will be closed under division?

5. Are the Rational Numbers closed under:

Operation: Example if yes, Counter example if not

Addition? \_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Subtraction? \_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Multiplication? \_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Division? \_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

6. a. Find the length of the diagonal of a square that has sides equal to 1 cm. (Show your work using the Pythagorean Theorem.)

b. Is the length of the diagonal of a square with sides = 1 a rational number or an irrational number? \_\_\_\_\_\_\_\_\_\_\_\_

c. Is the length of the diagonal a real number?\_\_\_\_\_\_\_

d. Where on the number line will you place $\sqrt{2}$ ? Find out by squeezing $\sqrt{2}$ between two numbers that have more and more accuracy. We start with squeezing $\sqrt{2}$ between two numbers to the nearest tenth 1.4<$\sqrt{2} $<1.5, note that 1.42 = \_1.96\_ and 1.52= \_2.25\_\_\_

the nearest hundredth 1.41 <$\sqrt{2}$ < 1.42 1.412 = 1.9881\_ and 1.422= 2.0164

nearest thousandth: 1.414 <$\sqrt{2}$ <\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

ten thousandth: \_\_\_\_\_\_<$ \sqrt{2} $ < \_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Label the number line with two approximations of $\sqrt{2}$ to the nearest thousandth: one approximation that is too low and one that is too high

 1.4 $\sqrt{2}$ 1.5

Note: Irrational numbers “fill in” the gaps between the rational numbers.

The 1.41 is just an approximation for $\sqrt{2}$ ; it is not exact.

Together, the rational and the irrational numbers make up the whole number line.

The real numbers consist of the rational numbers and the irrational numbers.

There are more irrational numbers than there are rational numbers.

7. a. Find the square roots of these positive rational numbers:$\sqrt{\frac{36}{25}}=\\_\\_\\_\\_\\_\\_$. $\sqrt{1}=\\_\\_\\_\\_\\_$ and $\sqrt{0.64}$ =\_\_\_\_\_\_

 b. Give a counter example to prove that the set of Positive Rational numbers is not closed under the Square Root operation:

 c. What set of numbers do we need to allow the square rooting of positive rational numbers?

d. Give a counterexample to prove that the set of negative rational numbers is not closed under square rooting:

e. What set of numbers do we need to allow the square rooting of all negative real numbers?

8. Write a definition for each of the following sets of numbers in words or by example or both for each of the following sets of numbers.

1. Natural Numbers (also called the Counting Numbers)

2. Whole Numbers

3. Integers

4. Rational Numbers

5. Irrational Numbers

6. Real Numbers

7. For the 6 types of numbers you just defined, make a Venn Diagram showing what sets of numbers are subsets of each other and what sets do not overlap.

8. Express the following rational numbers as a decimal. Show your long division work. Note that every rational number can be written as either a terminating or a repeating decimal.

a. $\frac{35}{4}$ b. $\frac{15}{11}$

c. Write the terminating decimal 1.23 as a repeating decimal by affixing infinitely many zeros after the digit “3”:

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

9. Irrational numbers are nonterminating and non-repeating. Put the following numbers in the correct column: (the columns will not necessarily be the same length)

.5, 3.0330333033330 … (Increase the number of 3’s in each block) , 1.234444…, $\overbar{.66}$ , 7.1254364 …(assume these digits continue randomly, without end) , $7.1\overbar{232}$

|  |  |
| --- | --- |
| Rational Numbers (can be expressed as a repeating decimal)  | Irrational numbers (can be expressed as non-repeating, non-terminating decimals) |
|  |  |
|  |  |
|  |  |
|  |  |

Warning: a decimal number that follows a pattern in not necessarily a rational number.

11. Fill in the blank with a >, < or = symbol to make a true sentence.

a. .333\_\_\_\_ $0.\overbar{3}$

b. .6667 \_\_\_ $\frac{2}{3}$

c. $0.\overbar{9}$ \_\_\_\_ 0.99999

d. $0.\overbar{9}$ \_\_\_\_\_ 1

e. 0.23232222…..\_\_\_\_\_ $0.\overbar{23}$

f. 3.1415\_\_\_\_\_ π

f. 3.142 \_\_\_\_\_ π

12. True or False? If true, give an example, if false, give a counter example.

a. The sum of an integer and a whole number is always a whole number.\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_

b. The sum of two rational numbers is always rational . \_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_

c. The product of two natural numbers can never be 0. \_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_

d. The square root of a natural number is always a real number \_\_\_\_\_, \_\_\_\_\_\_\_\_

e. If you add a rational number and an irrational number the answer could be rational. \_\_\_\_\_, \_\_\_\_\_\_\_\_\_

f. The sum of an integer number and its opposite is always 0. \_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_

g. The difference of two irrational numbers is always an irrational number.\_\_\_\_\_\_, \_\_\_\_\_\_\_\_

h. The sum of an irrational number and a rational number is always irrational \_\_\_\_\_\_, \_\_\_\_\_\_\_\_

i. The product of an irrational number and a non-zero rational number is always irrational\_\_\_\_\_\_. \_\_\_\_\_\_\_

13. Example: Prove that the product of a non-zero rational and an irrational is irrational.

We will do a proof by contradiction. We shall assume the product of a rational and an irrational is RATIONAL. We will see that this assumption leads to an obviously false statement. Then we conclude that the assumption must be false since it led us astray.

Assume that a rational number ‘a’ times an irrational number ‘b’ has a rational product ‘c’: ab=c.

Divide both sides of the equation by the rational number ‘a’ to get ab/a = c/a

That simplifies to b=c/a . Because c/a is a rational divided by a rational, and rationals are closed under division, then c/a is a rational number. However, b is an irrational number. That leads to the false statement that an irrational number ‘b’ equals a rational number ‘c/a’ .

Since that is impossible, our original assumption that “ab” is rational must be false.

Therefore the product of a rational and an irrational is irrational.

Now you try it: Prove that the sum of a rational and an irrational number is irrational.