**Activity 1.7.2a Understanding Isometries**



1. On a sheet of graph paper draw a set of axes and parallelogram *ABCD* with these vertices: *A =* (0,2), *B* = (4,2), *C =* (6,4) and *D* = (2,4).
2. Use the distance formula or a centimeter ruler to find the lengths of the sides.

*AB* = \_\_\_\_\_\_\_ *BC* = \_\_\_\_\_\_\_\_

*CD* = \_\_\_\_\_\_\_ *DA* = \_\_\_\_\_\_\_\_

1. Use a protractor to measure the angles.

m$∠DAB$ = \_\_\_\_\_\_\_\_ m$∠ABC$ = \_\_\_\_\_\_\_\_ m$∠BCD$ = \_\_\_\_\_\_\_\_ m$∠CDA$ = \_\_\_\_\_\_\_\_
2. On the same sheet of graph paper plot the image of parallelogram ABCD under reflection over the *y*-axis. Recall that the mapping rule is (*x, y*) 🡪(–*x, y*).

a. Use the distance formula or a centimeter ruler to find the lengths of the sides.

*A’B’* = \_\_\_\_\_\_\_ *B’C’* = \_\_\_\_\_\_\_\_ *C’D’* = \_\_\_\_\_\_\_ *D’A’* = \_\_\_\_\_\_\_\_

b. Use a protractor to measure the angles.

m$∠D'A'B'$ = \_\_\_\_\_ m$∠A'B'C'$ = \_\_\_\_\_ m$∠B'C'D'$ = \_\_\_\_\_ m$∠C'D'A'$ = \_\_\_\_\_

1. On the same sheet of graph paper plot the image of parallelogram ABCD under the translation with mapping rule (*x, y*) 🡪(*x –* 6*, y* –5).

a. Use the distance formula or a centimeter ruler to find the lengths of the sides.

*A’B’* = \_\_\_\_\_\_\_ *B’C’* = \_\_\_\_\_\_\_\_ *C’D’* = \_\_\_\_\_\_\_ *D’A’* = \_\_\_\_\_\_\_\_

b. Use a protractor to measure the angles.

m$∠D'A'B'$ = \_\_\_\_\_ m$∠A'B'C'$ = \_\_\_\_\_ m$∠B'C'D'$ = \_\_\_\_\_ m$∠C'D'A'$ = \_\_\_\_\_

1. On the same sheet of graph paper plot the image of parallelogram *ABCD* under a 270° counter-clockwise rotation about the origin. Recall that the mapping rule (*x, y*) 🡪(*y*, *–x*).

a. Use the distance formula or a centimeter ruler to find the lengths of the sides.

*A’B’* = \_\_\_\_\_\_\_ *B’C’* = \_\_\_\_\_\_\_\_ *C’D’* = \_\_\_\_\_\_\_ *D’A’* = \_\_\_\_\_\_\_\_

b. Use a protractor to measure the angles.

m$∠D'A'B'$ = \_\_\_\_\_ m$∠A'B'C'$ = \_\_\_\_\_ m$∠B'C'D'$ = \_\_\_\_\_ m$∠C'D'A'$ = \_\_\_\_\_

1. On the same sheet of graph paper plot the image of parallelogram *ABCD* under the glide reflection with mapping rule (*x, y*) 🡪(*x* + 3*, –y*).

a. Use the distance formula or a centimeter ruler to find the lengths of the sides.

*A’B’* = \_\_\_\_\_\_\_ *B’C’* = \_\_\_\_\_\_\_\_ *C’D’* = \_\_\_\_\_\_\_ *D’A’* = \_\_\_\_\_\_\_\_

b. Use a protractor to measure the angles.

m$∠D'A'B'$ = \_\_\_\_\_ m$∠A'B'C'$ = \_\_\_\_\_ m$∠B'C'D'$ = \_\_\_\_\_ m$∠C'D'A'$ = \_\_\_\_\_

1. Summarize you results: translations, rotations, reflections and glide reflections are all examples of **isometries**.

a. How does the length of a segment and the length of its image compare under an isometry?

b. How does the measure of an angle and the measure of its image compare under an isometry?

1. Now let’s look at some other transformations.

a. On a fresh sheet of graph paper, draw a set of axes and the parallelogram *ABCD* with these vertices: *A =* (0,2), *B* = (4,2), *C =* (6,4) and *D* = (2,4), the same as before.

b. Assign each member of your group one of these transformations:

 Dilation: (*x*, *y*) 🡪 (1.5*x*, 1.5*y*)
 Vertical stretch: (*x*, *y*) 🡪 (*x*, 3*y*)
 Horizontal stretch: (*x*, *y*) 🡪 (2*x*, *y*)
 Shear: (*x*, y) 🡪 (*x* + *y*, *y*)

c. Plot the image of parallelogram *ABCD* under the transformation you were assigned.
2. Record the results for your transformation:

a. Use the distance formula or a centimeter ruler to find the lengths of the sides.

*A’B’* = \_\_\_\_\_\_\_ *B’C’* = \_\_\_\_\_\_\_\_ *C’D’* = \_\_\_\_\_\_\_ *D’A’* = \_\_\_\_\_\_\_\_

b. Use a protractor to measure the angles.

m$∠D'A'B'$ = \_\_\_\_\_ m$∠A'B'C'$ = \_\_\_\_\_ m$∠B'C'D'$ = \_\_\_\_\_ m$∠C'D'A'$ = \_\_\_\_\_

1. Compare your results with those of other members of your group. Summarize your findings here:
2. How do the last four transformations (dilation, vertical shift, horizontal shift, and shear) differ from the four transformations we call “isometries” (translation, rotation, reflection, and glide reflection)?
3. An additional question: Find the area of the original parallelogram *ABCD* and the areas of each of the image parallelograms, *A’B’C’D’.* For which transformations is the area unchanged?