**Linear and Nonlinear Growth**

One of the most important ways to study functions is to look at how they grow. Suppose you have a table of values from a function where the inputs are always increasing by 1. There are three types of function growth we will study in this Activity, called linear, exponential, and quadratic growth.

* If the **difference** between successive outputs is constant, the function has **linear growth**.
* If the **ratio** between successive outputs is constant, the function has **exponential growth** and no constant term (in other words, it is of the form f(x) = ax or f(x) = c(ax)).
* If the **difference between the successive differences** in outputs (the “second differences”) is constant, the function has **quadratic growth**.
* If the ratio between the successive differences in outputs is constant, the function has **exponential growth** with a constant term (in other words, it is of the form f(x) = ax + k or f(x) = c(ax) + k).

Follow these steps to complete the tables in this Activity.

* Fill-in the output column.
* In the “difference between outputs” column, find the difference between successive outputs when the inputs increase by 1. If the differences in this column are constant, the function has **linear** growth.
* If not, then in the “ratio of outputs” column, find the ratios between successive outputs when the inputs increase by 1. If the ratios in this column are constant, the function has **exponential** growth.
* If not, then in the “differences in differences” column, find the difference in successive differences between outputs. If the differences in this column are constant, the function has **quadratic** growth.
* If not, then in the “ratio of differences” column, find the ratio between successive differences between outputs. If the ratios in this column are constant, the function has **exponential** growth.
* If none of these characteristics are found, write **other** for the type of growth in the function.

The tables in #1 and #2 have been partially filled in for you to help you get started on this Activity. Remember—a function can only have one type of growth. You should fill in the tables below one column at a time. If you find for example that the third column in a table is constant, which means the function has linear growth, you do not have to complete the other columns in the table!

1. $f\left(x\right)=x^{2}+2x+1$

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| --- | --- | --- | --- | --- | --- |
| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** | **Ratio of differences** |
| 1 | 1 |  |  |  |  |
| 2 | 4 | 4 – 1 = 3 | $\frac{4}{1}$ = 4 |  |  |
| 3 | 9 | 9 – 4 = 5 | $\frac{9}{4}$ = 2.25 | 5 – 3 = 2 | $\frac{5}{3}$ = 1.666… |
| 4 | 16 | 16 – 9 = 7 | $\frac{16}{9}$ = 1.777… | 7 – 5 = 2 | $\frac{7}{5}$ = 1.4 |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |

1. What type of function growth does $f\left(x\right)=x^{2}+2x+1$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=-2x+5$

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| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** | **Ratio of differences** |
| 1 | 3 |  |  |  |  |
| 2 | 1 |  |  |  |  |
| 3 | -1 |  |  |  |  |
| 4 | -3 |  |  |  |  |
| 5 | -5 |  |  |  |  |
| 6 | -7 |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=-2x+5$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=2^{x}-4$

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| --- | --- | --- | --- | --- | --- |
| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** | **Ratio of differences** |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=2^{x}-4$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=x^{3}-1$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| --- | --- | --- | --- | --- | --- |
| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** | **Ratio of differences** |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=x^{3}-1$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=x^{2}+5x$

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| --- | --- | --- | --- | --- | --- |
| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** | **Ratio of differences** |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=x^{2}+5x$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=\frac{x-4}{3}$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| --- | --- | --- | --- | --- | --- |
| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** | **Ratio of differences** |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=\frac{x-4}{3}$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=3^{x}+2$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| --- | --- | --- | --- | --- | --- |
| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference between differences** | **Ratio of differences** |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=3^{x}+2$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.
3. 

8. Look back on questions #1-7. In the table below, copy the equations of the functions that exhibited linear growth, quadratic growth, and exponential growth, as well as those that did not exhibit any of these. In the space below the table, describe what the graphs and equations of each type of function share in common.

|  |  |
| --- | --- |
| **Type of growth** | **Functions** |
| Linear |  |
| Exponential |  |
| Quadratic |  |
| None |  |

What the graphs and equations of the linear functions share in common:

What the graphs and equations of the exponential functions share in common:

What the graphs and equations of the quadratic functions share in common: