**Activity 1.3.1 Linear and Nonlinear Growth**

One of the most important ways to study functions is to look at how they grow. Suppose you have a table of values from a function where the inputs are always increasing by 1. There are three types of function growth we will study in this Activity:

* If the **difference** between successive outputs is constant, the function has **linear growth**.
* If the **ratio** between successive outputs is constant, the function has **exponential growth**.
* If the **difference between the successive differences** in outputs (the “second differences”) is constant, the function has **quadratic growth**.

Follow these steps to complete the tables in this Activity.

* Fill-in the output column.
* In the “difference between outputs” column, find the difference between successive outputs when the inputs increase by 1. If the differences in this column are constant, the function has **linear** growth.
* If not, then in the “ratio of outputs” column, find the ratios between successive outputs when the inputs increase by 1. If the ratios in this column are constant, the function has **exponential** growth.
* If not, then in the “differences in differences” column, find the difference in successive differences between outputs. If the differences in this column are constant, the function has **quadratic** growth.
* If none of these characteristics are found, write **other** for the type of growth in the function.
1. $f\left(x\right)=x^{2}+2x+1$

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| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference in differences** |
| 1 | 1 |  |  |  |
| 2 | 4 | 4 – 1 = 3 | 4/1 = 4 |  |
| 3 | 9 | 9 – 4 = 5 | 9/4 = 2.25 | 5 – 3 = 2 |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

1. What type of function growth does $f\left(x\right)=x^{2}+2x+1$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=-2x+5$

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| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference in differences** |
| 1 | 3 |  |  |  |
| 2 | 1 | 1 – 3 = -2 | 1/3 = 0.33 |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
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1. What type of function growth does $f\left(x\right)=-2x+5$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=2^{x}-4$

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| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference in differences** |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=2^{x}-4$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=x^{3}-1$

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| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference in differences** |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=x^{3}-1$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=x^{2}+5x$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| --- | --- | --- | --- | --- |
| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference in differences** |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=x^{2}+5x$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=\frac{x-4}{3}$

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| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference in differences** |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=\frac{x-4}{3}$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.



1. $f\left(x\right)=3^{x}+2$

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| **Input*****x*** | **Output*****f (x)*** | **Difference between outputs** | **Ratio of outputs** | **Difference in differences** |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

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1. What type of function growth does $f\left(x\right)=3^{x}+2$ have? Explain why.
2. Sketch a graph of $f\left(x\right)$ on the axes below.

