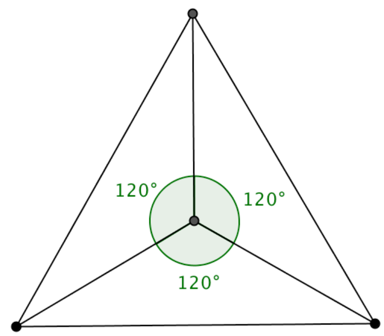
Review Questions for Final Examination

Unit 1

1.1 Sketch an example of a polygon with 120° rotational symmetry. Explain why it has rotational symmetry. An equilateral triangle. Students may observe that the altitudes (which are also medians) intersect at 120° angles.

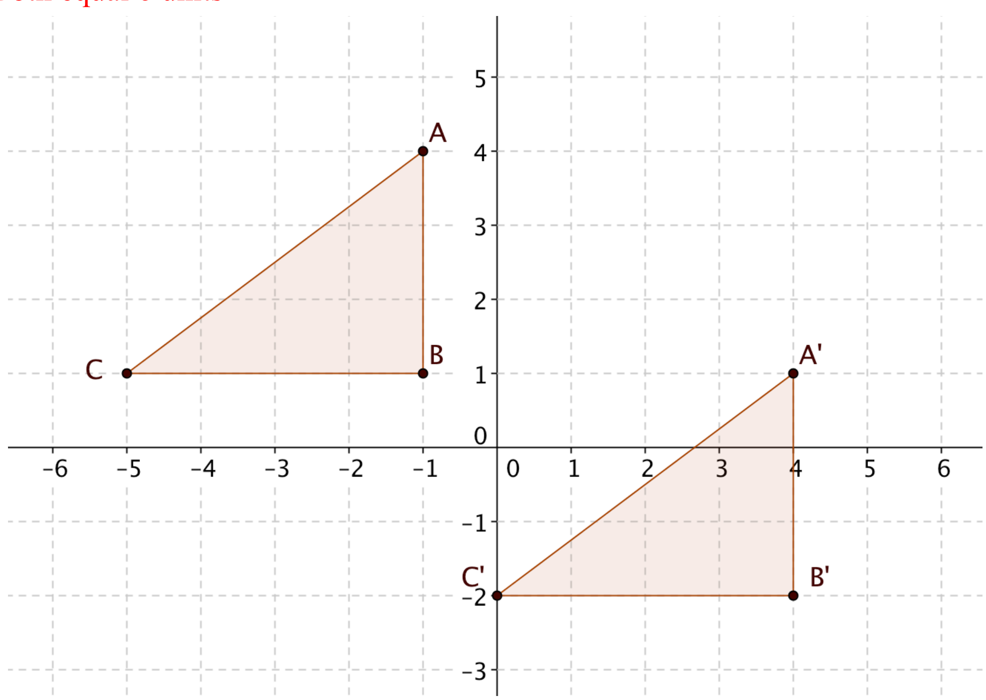


1.2 In the graph below, ΔABC is translated by the vector [5, –3].

a. Sketch $\Delta A'B'C'$ on the gird. See below

b. Find the perimeters of ΔABC and $\Delta A'B'C'$. What do you notice?
Both equal 12 units

c. Find the areas of ΔABC and $\Delta A'B'C'$. What do you notice?
Both equal 6 units^2



1.3 Suppose you reflect the dog over vertical line *g* and then again over vertical line *h*. If lines *g* and *h* are 6 units apart, what is one transformation you could have applied to the dog to obtain the same result as these two reflections? Translation to the right by 12 units

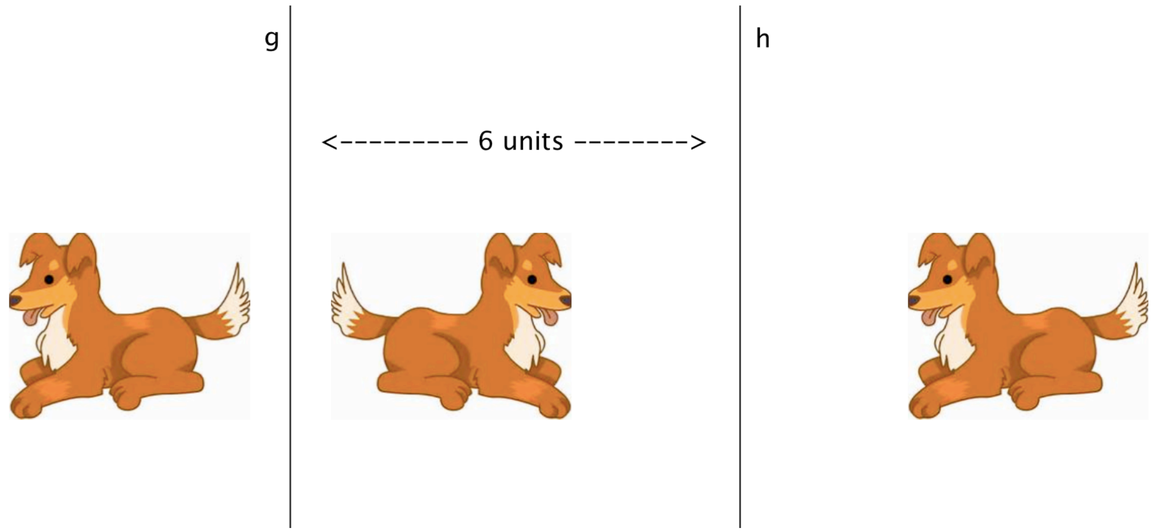
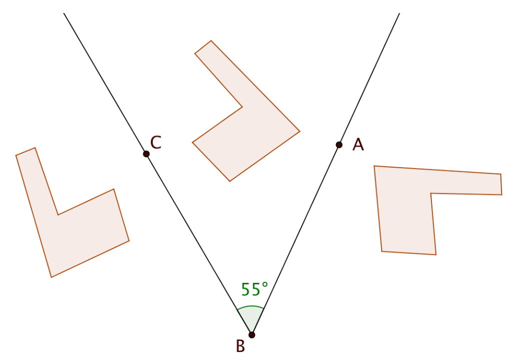


Image from clipartsheep.com

1.4 m $\angle ABC = 55°$. Suppose you reflect the "chair polygon" first over line $\overleftrightarrow{BC}$ then over line $\overleftrightarrow{BA}$. What is a single transformation that would result in the same final image? Rotation about point B 110° clockwise.



1.5 Write mapping rules for each of these transformations:

a. A 90° counter-clockwise rotation about the origin. $(x, y) \rightarrow (-y, x)$

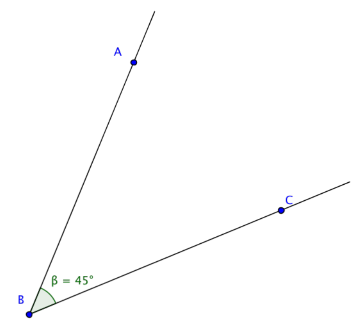
b. A reflection about the line $y = -x$. $(x, y) \rightarrow (-y, -x)$

c. A reflection about the x-axis, followed by a translation by the vector [2, –4].
$(x, y) \rightarrow (x + 2, -y - 4)$

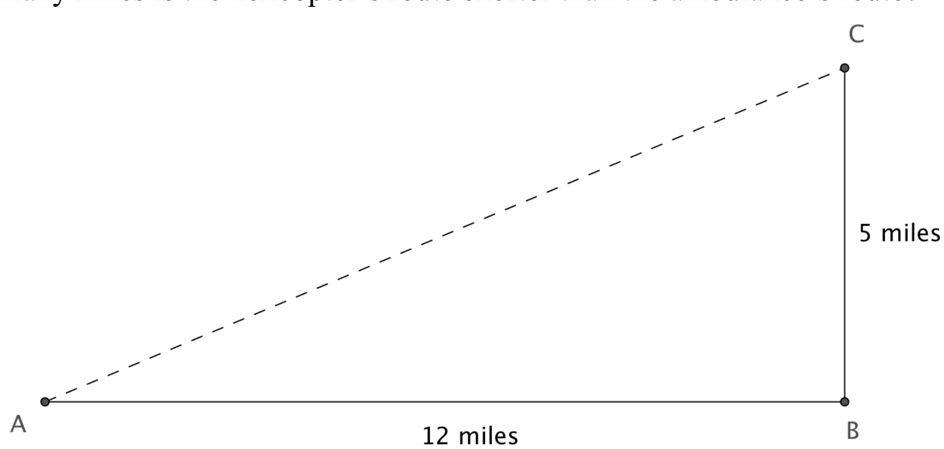
1.6 Suppose you are tutoring a student in geometry. Explain, in simple terms, what a glide reflection is. Answers will vary. A glide reflection combines translation and reflection, where the direction of the translation is parallel to the reflection line.

1.7 Give an example of a transformation that IS NOT an isometry. Explain why.
Answers will vary. A good example is a dilation. The measures of the angles remain unchanged but the lengths of segments are multiplied by the absolute value of the scale factor. Therefore unless the scale factor is 1 or –1, distances are not preserved and the transformation is not an isometry.

1.8 When you measure this angle with GeoGebra, you get 45° by clicking on *C*, *B*, and *A* in that order. What measure will you get if you select the points with the opposite order: *A*, *B*, *C*? Explain your reasoning. 315°. GeoGebra always measures angles in the counter-clockwise direction..



1.9 A person is critically injured in an automobile accident at point *A*. The nearest hospital is located at point *C*. The shortest route for an ambulance is to drive 12 miles due east from *A* to *B*, and then 5 miles due north from *B* to *C*. A helicopter can fly directly from *A* to *B*. By how many miles is the helicopter's route shorter than the ambulance's route?



By the Pythagorean Theorem, $AC = 13$ miles. $AB + BC = 17$ miles. Thus the shortcut saves 4 miles.

1.10 ΔABC lies in the coordinate plane with coordinates $A(-1, -6)$, $B(5, -4)$, and $C(-2,-3)$.

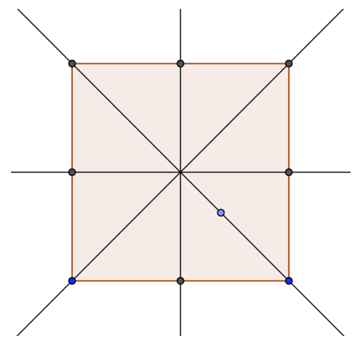
a. Find the slopes of the lines containing each of the three sides.

Slope of $\overleftrightarrow{AB} = \frac{-4-(-6)}{5-(-1)} = \frac{1}{3}$ Slope of $\overleftrightarrow{AC} = \frac{-3-(-6)}{-2-(-1)} = \frac{3}{-1} = -3$ Slope of $\overleftrightarrow{BC} = \frac{-3-(-4)}{-2-5} = -\frac{1}{7}$

b. Show that ΔABC is a right triangle. Since the slopes of $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are opposite reciprocals, the two lines are perpendicular and $\angle BAC$ is a right angle. Therefore ΔABC is a right triangle.

c. Find the length of the hypotenuse two ways: (1) by using the distance formula and (2) by the Pythagorean Theorem. (1) Using the distance formula $BC = \sqrt{7^2 + 1^2} = \sqrt{50}$.
(2) The distance formula also gives us $AB = \sqrt{40}$ and $AC = \sqrt{10}$.
$BC^2 = AB^2 + AC^2 = 40 + 10 = 50$. Therefore $BC = \sqrt{50}$.

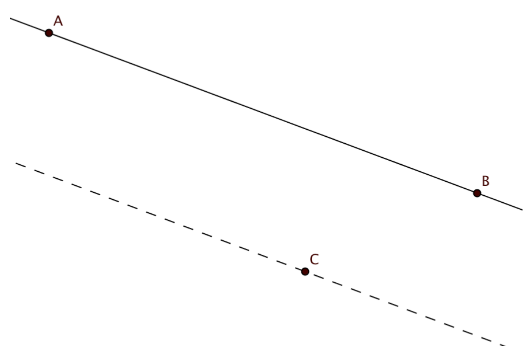
1.11. A quadrilateral has four lines of symmetry.



a. Sketch the quadrilateral showing the location of the lines of symmetry. Figure at the right.

b. What type of special quadrilateral must it be? A square.

1.12 In the diagram, the dashed line through C is parallel to $\overleftrightarrow{AB}$.



a. Suppose $\overleftrightarrow{AB}$ is reflected about the dashed line. Will the image $\overleftrightarrow{A'B'}$ be parallel to $\overleftrightarrow{AB}$? yes

b. Suppose $\overleftrightarrow{AB}$ is rotated counter-clockwise 180° about point C. Will $\overleftrightarrow{A'B'}$ be parallel to $\overleftrightarrow{AB}$? yes

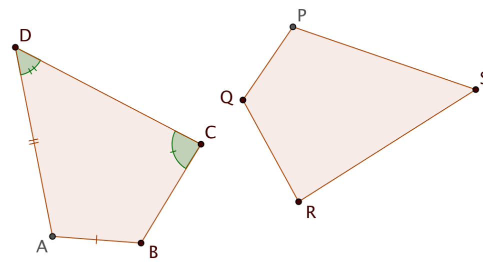
c. Suppose $\overleftrightarrow{AB}$ is rotated 90° counter-clockwise about point C. Will $\overleftrightarrow{A'B'}$ be parallel to $\overleftrightarrow{AB}$? no

d. Suppose $\overleftrightarrow{AB}$.is dilated with dilation center at C by a scale factor of 2. Will $\overleftrightarrow{A'B'}$ be parallel to $\overleftrightarrow{AB}$? yes

e. Suppose $\overleftrightarrow{AB}$ is translated by the vector from C to A. What happens to $\overleftrightarrow{A'B'}$? It's mapped onto a line parallel to $\overleftrightarrow{AB}$.

f. Suppose $\overleftrightarrow{AB}$ is translated by the vector from B to A. What happens to $\overleftrightarrow{A'B'}$? It is mapped onto itself.

Unit 2



2.1 Quadrilateral *ABCD* is congruent to quadrilateral *PQRS*.

a. Two sides and two angles of *ABCD* have been marked. Use the same marks for the corresponding sides of *PQRS*.

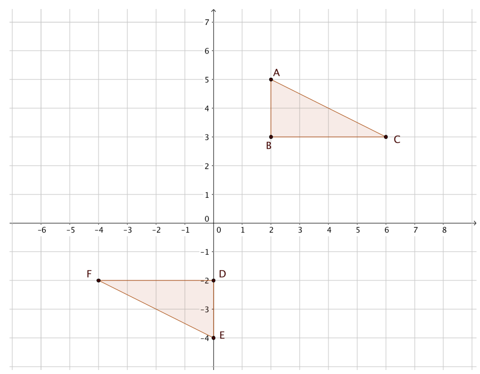
b. Fill in the blanks to show relations between the two figures.

$BC = QR$

$\angle PQR \cong \angle ABC$

$\text{m}\,\angle DAB = \text{m}\,\angle SPQ$

$\overline{RS} \cong \overline{CD}$



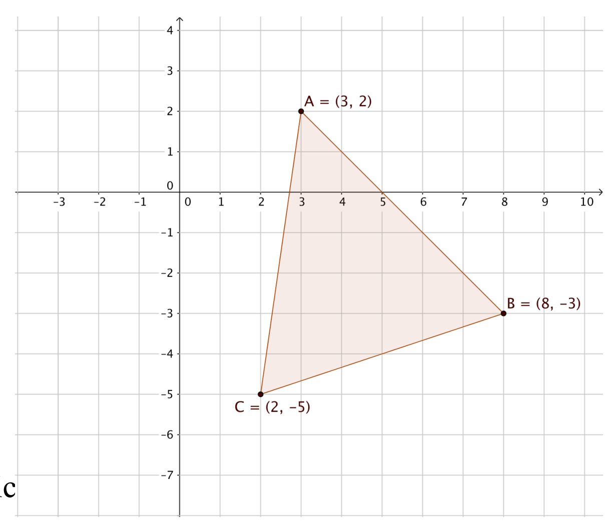
2.2 Describe a transformation or a set of transformations that will map Δ*ABC* onto Δ*DEF*.

Rotate 180° (clockwise or counter-clockwise) around the origin and then translate by the vector [2, 1] OR

Translate by the vector [–2,–1] then rotate 180° (clockwise or counter-clockwise) about the origin.

An alternative solution involves using two reflections followed by a translation.

2.3 Under what conditions are two circles congruent? Justify your answer. Two circles are congruent if their radii are equal. Show that one circle is the image of the other under a translation mapping the center of one circle on the other. Let the radii of the circles be *r*. Then every point on the first circle is mapped onto a point *r* units from the center of the second circle, so it lies on the second circle.

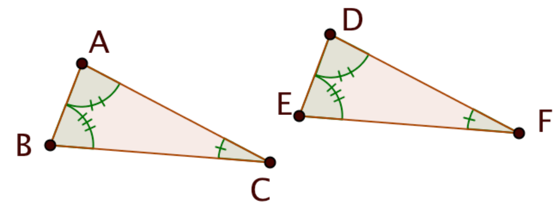


2.4 The coordinates of the vertices of ΔABC are given in the figure.

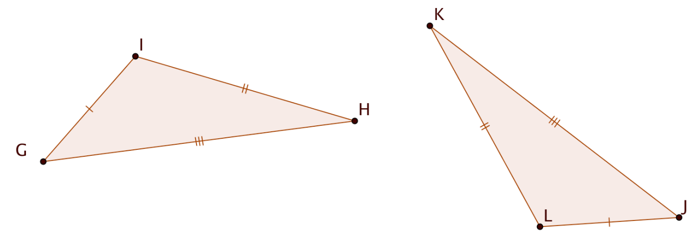
a. Is Δ*ABC* isosceles? Explain. By the distance formula $AB = \sqrt{50}$, $AC = \sqrt{50}$, and $BC = \sqrt{40}$. Since $AB = AC$, the triangle is isosceles.
b. Is Δ*ABC* equilateral? Explain. No. Not all sides are congruent.
c. Name two congruent angles. $\angle ACB \cong \angle ABC$ by the Isosceles Triangle Theorem.

2.5 Decide whether each pair of triangles may be proved to be congruent with the given information. If they can be proved congruent, identify the theorem you would use.

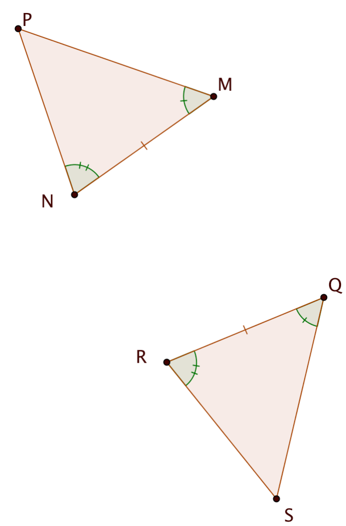
a. Given $\angle A \cong \angle D$
$\angle B \cong \angle E$
$\angle C \cong \angle F$
Can you prove $\Delta ABC \cong \Delta DEF$?
No.
If so, which theorem? AAA is not sufficient to prove congruence.



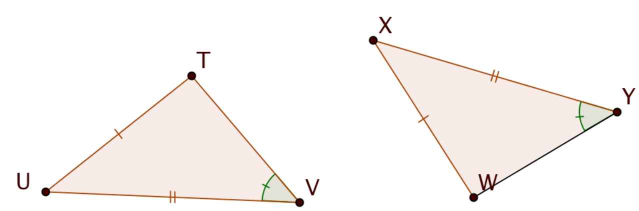
b. Given $\overline{IG} \cong \overline{LJ}$
$\overline{HI} \cong \overline{KL}$
$\overline{GH} \cong \overline{JK}$
Can you prove $\Delta GHI \cong \Delta JKL$?
Yes.
If so, which theorem?
SSS Congruence Theorem.



c. Given $\overline{MN} \cong \overline{QR}$
$\angle M \cong \angle Q$
Can you prove $\Delta MNP \cong \Delta QRS$?
Yes.
If so, which theorem?
ASA Congruence Theorem.



d. Given $\overline{TU} \cong \overline{WX}$
$\overline{UV} \cong \overline{XY}$
$\angle V \cong \angle Y$
Can you prove $\Delta TUV \cong \Delta WXY$?
No.
If so, which theorem?
SSA is not sufficient to prove congruence.

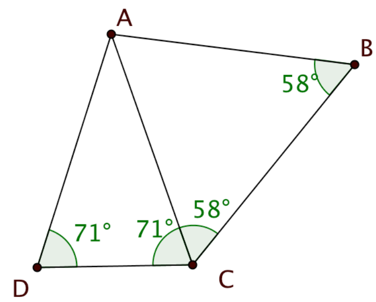


2.6

Given: m$\angle ADC$ = m$\angle ACD$ = 71°
m$\angle ACB$ = m$\angle ABC$ = 58°

Prove: $AD = AB$

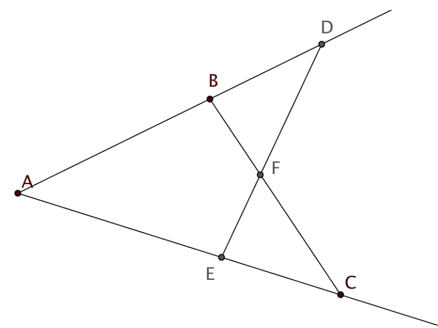
$AD = AC$ by the Isosceles Triangle Converse.
$AC = AB$ by the Isosceles Triangle Converse.
$AD = AB$ by the Transitive Property.



2.7

Given: $AB = AE$
$AC = AD$

Prove $\Delta ABC \cong \Delta AED$



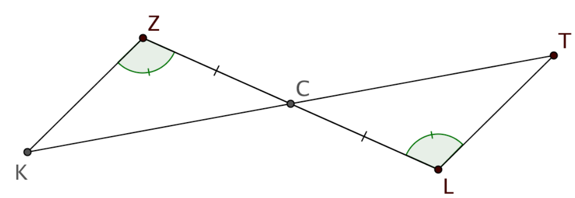
1. AB = AE is given
2. AC = AD is given
3. $\angle BAC$ and $\angle DAE$ are the same angle and are congruent by the reflexive property. This angle is included between pairs of congruent sides.

$\Delta ABC \cong \Delta AED$ by the SAS Congruence Theorem (steps 1, 2, and 3).

2.8

Given: m$\angle KZC$ = m$\angle TLC$
C is the midpoint of $\overline{LZ}$

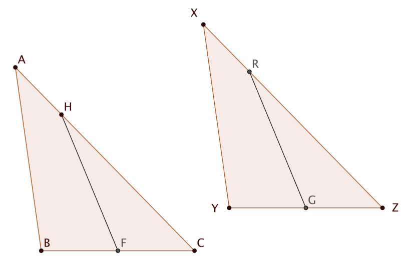
Prove: m$\angle ZKC$ = m$\angle LTC$



1. m$\angle ZCK$ = m$\angle LCT$ because vertical angles are congruent.
2. m$\angle KZC$ = m$\angle TLC$ was givsen
3. Because C is the midpoint of $\overline{LZ}$, $CZ = CL$. This pair of sides are included between the pairs of congruent angles.

$\Delta ZCK \cong \Delta LTC$ by the ASA Congruence Theorem (Steps 1, 2 and 3).
m$\angle ZKC$ = m$\angle LTC$ because corresponding parts of congruent triangles are congruent.

2.9

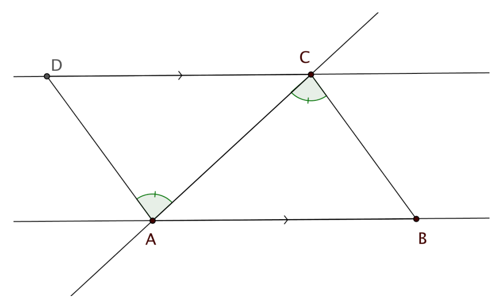


Given: $\Delta ABC \cong \Delta XYZ$
F is the midpoint of $\overline{BC}$
G is the midpoint of $\overline{YZ}$
$CH = ZR$
Prove: $\Delta HFC \cong \Delta RGZ$

1. Because corresponding parts of congruent triangles are congruent, $BC = YZ$. Therefore $\frac{1}{2}BC = \frac{1}{2}YZ$. Because F and G are midpoints $FC = \frac{1}{2}BC$ and $GZ = \frac{1}{2}YZ$. By substitution $FC = GZ$. This is one pair of sides.
2. We are given $CH = ZR$. This is a second pair of sides.
3. Because corresponding parts of congruent triangles are congruent, $\angle ACB \cong \angle XZY$. This pair of angles are included between the pairs of congruent sides.

Therefore $\Delta HFC \cong \Delta RGZ$ by the SAS congruence Theorem (Steps 1, 2 and 3).

2.10

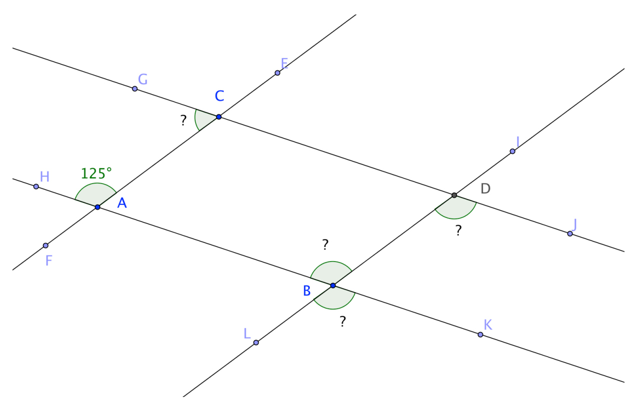


Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$
$\angle DAC \cong \angle ACB$
Prove: $\overline{AD} \cong \overline{CB}$

1. $\angle DCA \cong \angle BAC$ because they are alternate interior angles formed by parallel lines and a transversal.
2. $\angle DAC \cong \angle ACB$ is given.
3. $\overline{AC} \cong \overline{AC}$ by the reflexive property. This side is included between pairs of congruent angles.

$\Delta ABC \cong \Delta CDA$ by the ASA Congruence Theorem (Steps 1, 2 and 3).
$\overline{AD} \cong \overline{CB}$ because corresponding parts of congruent triangles are congruent.

2.11

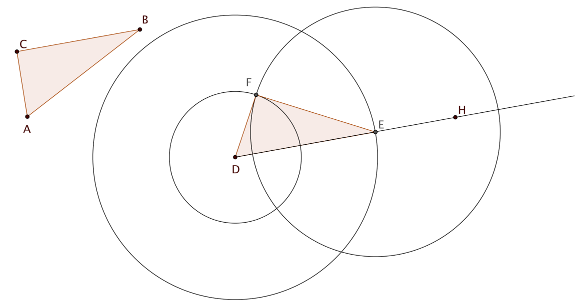


Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
$\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$
m $\angle HAC = 125°$

Find the measure of each angle and justify your answers.

a. m $\angle ABD = 125°$ (corresponding with $\angle HAC$; $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$)
b. m $\angle BDJ = 125°$ (alternate interior with $\angle ABD$; $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)
c. m $\angle LBK = 125°$ (vertical with $\angle ABD$ or corresponding with $\angle BDJ$; $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)
d. m $\angle GCA = 55°$ (same side interior with $\angle HAC$; $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)

2.12 A student was given ΔABC and asked to construct ΔDEF so that side $\overline{DE}$ lies along ray $\overrightarrow{DH}$.

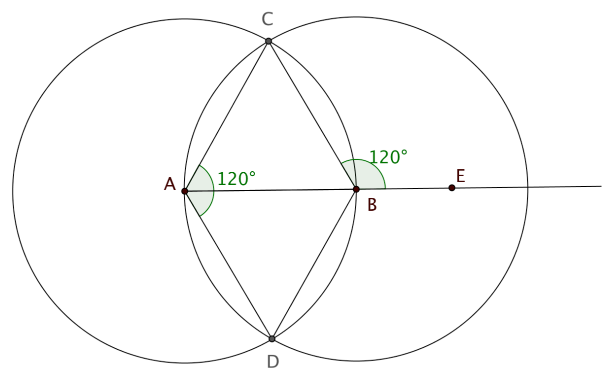


Here is what she did:

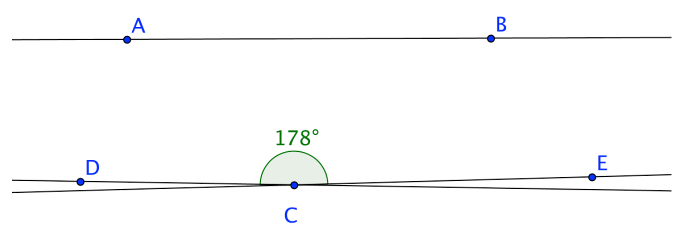
1. She drew a circle with center D and radius $= AB$. She labeled point E where this circle intersects $\overrightarrow{DH}$.
2. She drew a circle with center D and radius $= AC$.
3. She drew a circle with center E and radius $= BC$.
4. She labeled F as one of the points where the last two circles intersect and drew segments $\overline{DE}$ and $\overline{EF}$.

Prove that her construction works; that is, $\Delta DEF \cong \Delta ABC$.
By the construction $DE = AB$, $DF = AC$ and $EF = BC$. Thus, $\Delta DEF \cong \Delta ABC$ by the SSS Congruence Theorem.

2.13 With a compass and straightedge, construct an angle measuring 120° OR explain how you would construct an angle measuring 120° with a compass and straightedge. There are several different ways to do this. Here are two of them based on the classic construction of equilateral ΔABC. (1) Let D be the second point where the two circles intersect. Then ΔABD is also equilateral. Two adjacent 60° angles form an angle measuring 120° at A. (2) Extend side $\overline{AB}$ to form exterior $\angle EBC$. It is supplementary to the adjacent 60° interior angle and therefore measures 120°.

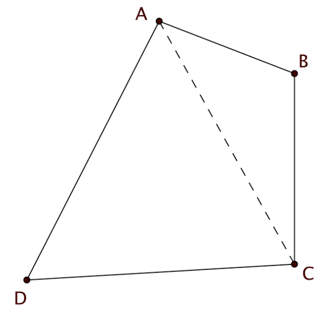


2.14 Two lines $\overleftrightarrow{DC}$ and $\overleftrightarrow{EC}$ intersect at point C and form an angle measuring 178°. Is it possible for both $\overleftrightarrow{DC}$ and $\overleftrightarrow{EC}$ to be parallel to $\overleftrightarrow{AB}$?
Explain. No. By the Parallel Postulate there is only one line through C parallel to $\overleftrightarrow{AB}$.



Unit 3

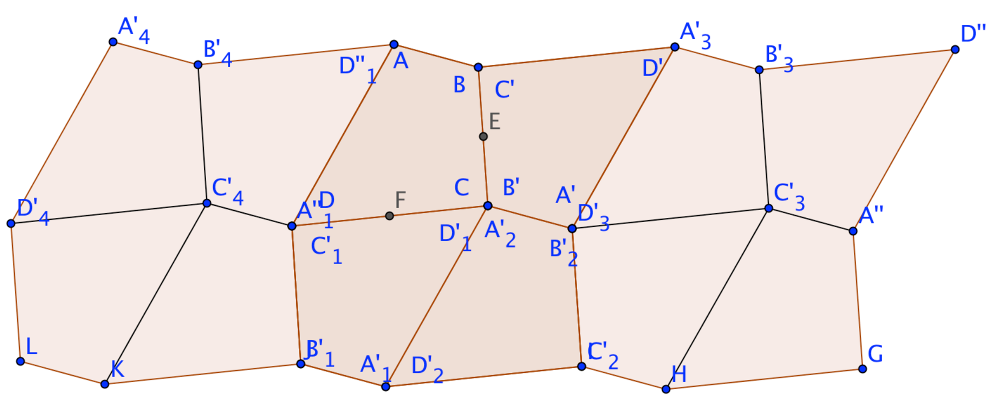
3.1 Use this diagram to explain how the Quadrilateral Sum Theorem can be proved using the Triangle Sum Theorem. The sum of the interior angle measures of each triangle is 180°. Diagonal $\overline{AC}$ splits $\angle DAB$ and $\angle BCD$ of the quadrilateral each into two angles, one from each triangle. Thus, the sum of the measures of the angles in two triangles is equal to the sum of the measures of angles in the quadrilateral: that is, 360°

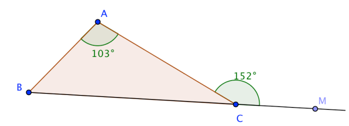


3.2 Here is another way to prove the Quadrilateral Sum Theorem:

(1) Start with Quadrilateral *ABCD*. (See figure at the bottom of the page.) Rotate it about *E*, the midpoint of $\overline{BC}$, to produce *A'B'C'D'*.
(2) Rotate *ABCD* about *F*, the midpoint of $\overline{CD}$, to produce $A'_1\ B'_1\ C'_1\ D'_1$.
(3) Translate *ABCD* by the vector from *A* to *C*.
a. Explain why $m\angle BCD + m\angle A'B'C' + m\angle B'_2A'_2D'_2 + m\angle C'_1D'_1A'_1 = 360°$
The sum of the measures of angles around a point is 360°.
b. Explain why $m\angle BCD + m\angle ABC + m\angle BAD + m\angle CDA = 360°$
Translations and rotations preserve angle measures. Therefore $m\angle A'B'C' = m\angle ABC$, $m\angle B'_2A'_2D'_2 = m\angle BAD$, and $m\angle C'_1D'_1A'_1 = m\angle CDA$. Substitute into the equation in (a) to get the desired result.

3.3 On the figure below show how additional copies of *ABCD* can be added to create a tessellation. (Show at least four more quadrilaterals)





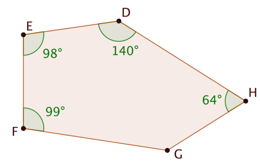
3.4 Find the measure of each indicated angle using information given on the figures. Justify each answer.

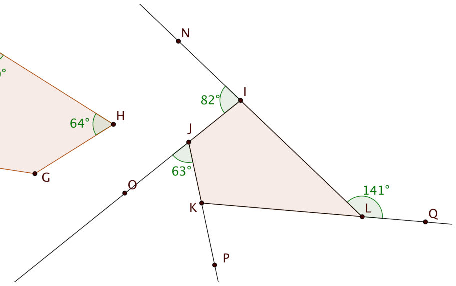
a. m $\angle ABC =$ 49° Measure of exterior angle of a triangle is sum of measures of remote interior angles.

b. m $\angle FGH =$ 139° Sum of interior angles of a pentagon is 540°

c. m $\angle LKP =$ 74° Sum of exterior angles of any convex polygon is 360°.

d. m $\angle JKL = 106°$. A linear pair of angles are supplementary.





3.5 Find the number of sides in a regular polygon when:

a. each interior angle measures 108°

$$\frac{(n-2)180°}{n} = 108°$$

$180°n - 360° = 108°n$

$72°n = 360°$

$n = 5$

b. each exterior angle measures 36°

$$\frac{360°}{n} = 36°$$

$360° = 36°n$

$n = 10$

c. the sum of the interior angles is twice the sum of the exterior angles.

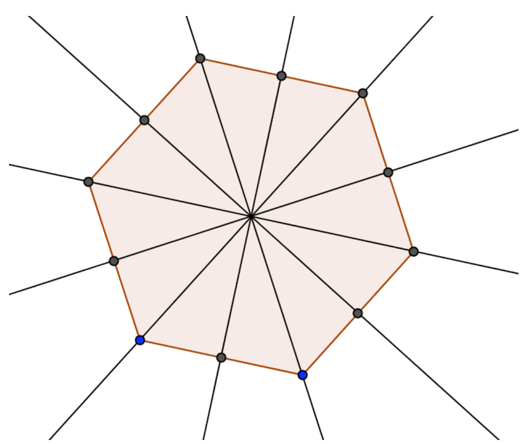
$(n-2)180° = 2 \times 360°$

$180°n - 360° = 720°$

$180°n = 1080°$

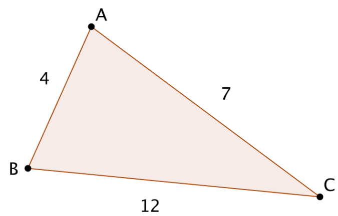
$n = 6$

3.6 Show all lines of symmetry for this regular hexagon.

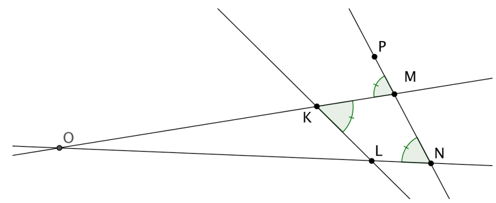


3.7 Something is wrong in the diagrams based on the information given. Explain the error in each case.

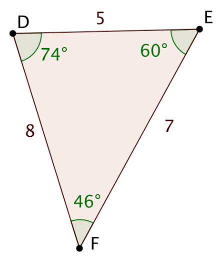
a.



b.



c.



d.

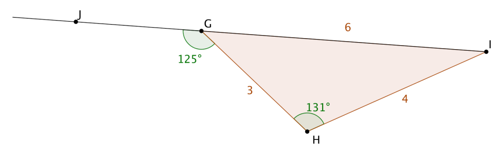
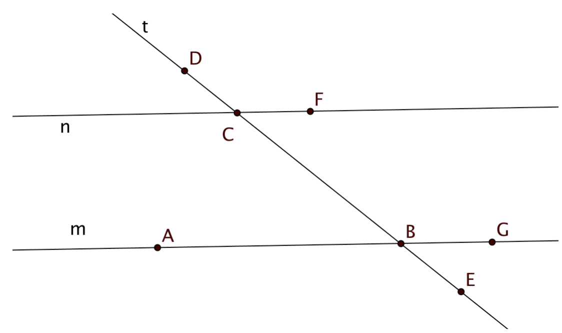


Figure (a) violates the Triangle Inequality. In (b) because a pair of corresponding angles are congruent, $\overleftrightarrow{KM}$ should be parallel to $\overleftrightarrow{LN}$, but it is not. In (c) the side opposite the 74° angle should be longer than the side opposite the 60° angle, but it is not. In (d) the exterior angle measure is less than the remote interior angle measure, violating the Triangle Exterior Angle Theorem.

3.8 Complete this proof of the statement: If two lines are intersected by a transversal and one pair of same side exterior angles are supplementary, then the lines are parallel.



Given: Lines *m* and *n* are intersected by transversal *t*.
m $\angle DCF$ + m $\angle EBG$ = 180°

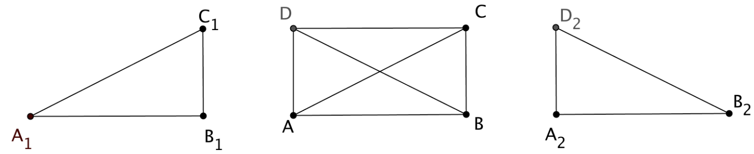
Prove: $m \parallel n$

Complete the proof here:

m $\angle CBG$ + m $\angle EBG$ = 180° because these angles form a linear pair.
m $\angle DCF$ + m $\angle EBG$ = 180° is given.
m $\angle CBG$ + m $\angle EBG$ = m $\angle DCF$ + m $\angle EBG$ by the transitive property (or substitution)
Subtract m $\angle EBG$ on both sides to get
m $\angle CBG$ = m $\angle DCF$
Since $\angle CBG$ and $\angle DCF$ are congruent corresponding angles formed by lines *n* and *m* with transversal *t*, $m \parallel n$ by the Parallel Lines Corresponding Angles Converse.

3.9 Given $ABCD$ is a parallelogram.
$\Delta ABD \cong \Delta BAC$

Prove: $ABCD$ is a rectangle.



(Hint: to help you visualize the congruent triangles, $\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$ have been drawn to the left and right of the parallelogram.)

We are given that $\Delta ABD \cong \Delta BAC$; therefore, m $\angle BAD$ = m $\angle ABC$ since corresponding parts of congruent triangles are congruent.

$ABCD$ is given to be a parallelogram, so by the definition of parallelogram, $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.

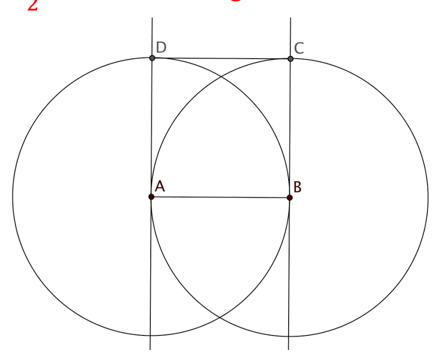
Consequently m $\angle BAD$ + m $\angle ABC$ = 180° since same-side interior angles formed by parallel lines and a transversal are supplementary.

Substitute m $\angle BAD$ for m $\angle ABC$ in the above equation and we have 2 m $\angle BAD$ = 180°. Divide both sides by 2, so m $\angle BAD$ = 90°. This means that one of the angles of parallelogram $ABCD$ is a right angle; therefore, parallelogram $ABCD$ is also a rectangle.

3.10 A middle school student says to you "We are learning about area. I know that the formula for the area of a triangle is $\frac{1}{2}$ base times height, but where does the $\frac{1}{2}$ come from? How would you answer her question?

Answers will vary. One approach is to show that when the triangle is rotated around the midpoint of one side the triangle and its image form a parallelogram. The altitude of the triangle to one of the other sides is also an altitude of the parallelogram. Since the area of the parallelogram is base times height, the area of one of the triangles is $\frac{1}{2}$ base times height.

3.11 A student constructs a square with side $\overline{AB}$ with the following steps. First he constructs a line perpendicular to $\overline{AB}$ at point A and another line perpendicular to $\overline{AB}$ at point B. Then he draws a circle with center A passing through B and another circle with center B passing through A. He labels points C and D where the circles intersect the perpendicular lines. He draws segment $\overline{CD}$ and claims that $ABCD$ is a square. Do you agree with his claim? Defend your answer.



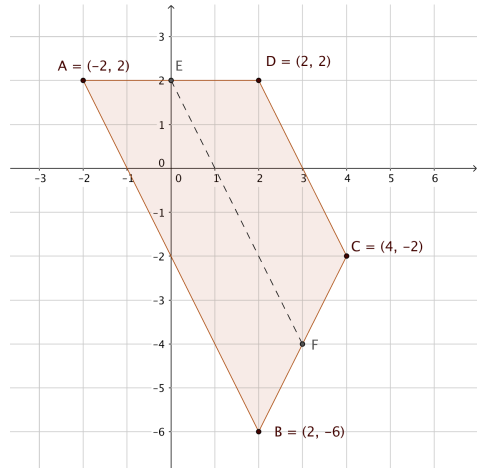
By the construction $DA = BC = AB$. Also since $\overline{DA} \perp \overline{AB}$ and $\overline{CB} \perp \overline{AB}$, $\overline{DA} \parallel \overline{CB}$ (Two lines perpendicular to the same line are parallel).

In quadrilateral $ABCD$ we have two sides, $\overline{DA}$ and $\overline{BC}$ that are both congruent and parallel. Therefore, by the One Pair Congruent and Parallel Theorem, $ABCD$ is a parallelogram.

Since $ABCD$ is a parallelogram, the other pair of opposite sides $\overline{AB}$ and $\overline{CD}$ are also congruent. Therefore, all four sides are congruent and $ABCD$ is a rhombus. But since $\angle DAB$ is a right angle and a rhombus with a right angle is a square, $ABCD$ is also a square.

3.12 Quadrilateral *ABCD* lies in a coordinate plane. The coordinates of the vertices are shown.

a. What kind of special quadrilateral is *ABCD*? Explain. *ABCD* is a trapezoid since one pair of sides are parallel. ($\overline{AB} \parallel \overline{DC}$ since the slopes of both lines are –2.)

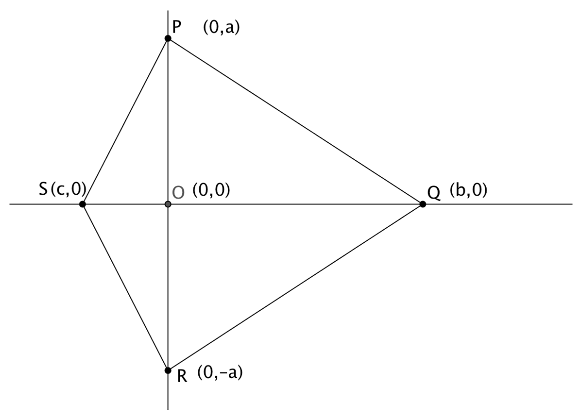


b. Let *E* be the midpoint of $\overline{AD}$ and *F* be the midpoint of $\overline{BC}$. Use the distance formula to show that $\frac{AB+CD}{2} = EF$.
Use the midpoint formula to find coordinates *E*(0,2) and *F*(3,–4). The distance formula then gives $AB = \sqrt{80}$, $CD = \sqrt{20}$ and $EF = \sqrt{45}$. Students who know how to simplify radicals can show that $\sqrt{45} = 3\sqrt{5}$ is the average of $4\sqrt{5}$ and $2\sqrt{5}$. Others may use decimal approximations to show that $EF \approx 6.708$ is the average of 8.944 and 4.472.

c. Show that $\overline{EF} \parallel \overline{AB}$.
The slope of $\overline{EF}$ is –2, which is the same as the slope of $\overline{AB}$ from (a) above.

d. What theorem is illustrated by the results in (b) and (c)?
Trapezoid Midsegment Theorem: The segment joining the two legs of a trapezoid is parallel to the bases and equal in length to the average of the lengths of the two bases.

3.13 Quadrilateral *PQRS* is placed in the coordinate plane with coordinates *P*(0, *a*), *Q*(*b*, 0), *R*(0, – *a*) and *S*(*c*, 0).



a. Prove that *PQRS* is a kite.
Use the distance formula to show that $PQ = RQ = \sqrt{b^2 + a^2}$ and that $PS = RS = \sqrt{c^2 + a^2}$.
PQRS is a quadrilateral with two distinct pairs of congruent consecutive sides. By definition it is, therefore, a kite.

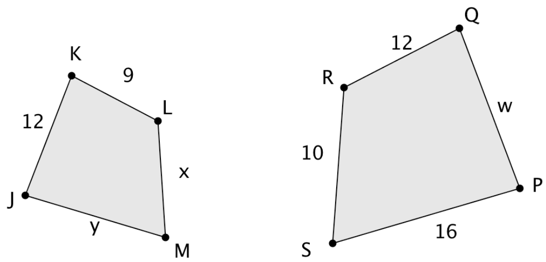
b. Suppose that $c = -b$. What kind of special quadrilateral is *PQRS* now? (Give the most specific category.) Explain.
When $c = -b$, $c^2 = b^2$. Substitute b^2 for c^2 in the equations above to get $PQ = RQ = PS = RS = \sqrt{b^2 + a^2}$. Since all four sides are congruent, kite *PQRS* is also a rhombus.

c. Suppose that $c = -b$ and $a = b$. What kind of special quadrilateral is *PQRS* now? Explain. The coordinates are now *P*(0, *a*), *Q*(*a*, 0), *R*(0, – *a*) and *S*(–*a*, 0). The slopes of the sides are now 1 and –1. Since the slopes of adjacent sides are opposite reciprocals all of the angles of the rhombus are 90°, so the rhombus is also a square.

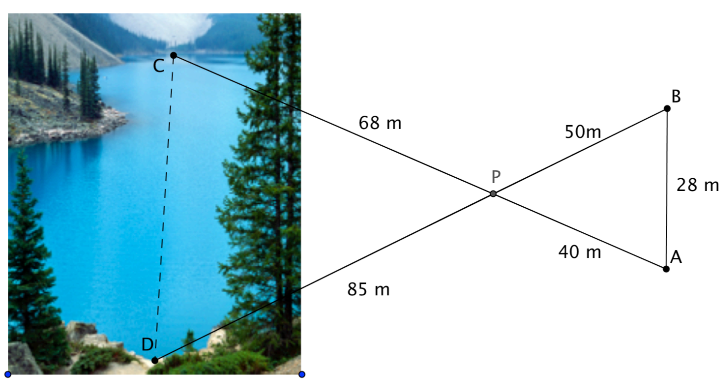
Unit 4

4.1 Quadrilateral $PQRS \sim$ quadrilateral $JKLM$.

a. Find the scale factor. $\frac{4}{3}$ (or $\frac{3}{4}$)
b. Find the values of w, x, and y. $w = 16$; $x = 7.5$; $y = 12$
c. Name an angle congruent to $\angle KLM$. $\angle QRS$



4.2 A group of scouts are camping near a pond. They would like to know how wide the pond is. From point P they use clinometers to measure the distances to points C and D. They extend rays $\overrightarrow{CP}$ and $\overrightarrow{DP}$ in straight lines to points A and B. Then they pace off the distances PA, PB, and AB. All their measurements are shown on the figure below.

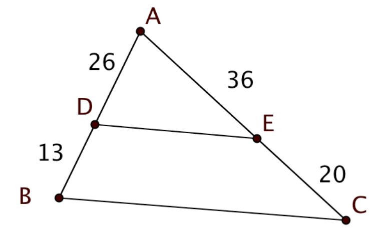


a. Prove that $\Delta ABP \sim \Delta CDP$.
$\frac{PA}{PC} = \frac{40}{68} = \frac{10}{17}$ and $\frac{PB}{PD} = \frac{50}{85} = \frac{10}{17}$
Therefore, $\frac{PA}{PC} = \frac{PB}{PD}$
$\angle DPC \cong \angle APB$ because vertical angles are congruent.
Therefore, $\Delta ABP \sim \Delta CDP$ by the SAS Similarity Theorem.

b. Find the width of the pond, CD, to the nearest meter.

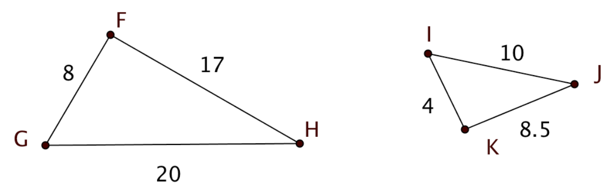
$\frac{AB}{CD} = \frac{10}{17}$. Therefore $\frac{28}{CD} = \frac{10}{17}$. Solving for CD we have $CD = 47.6 \approx 48$ meters.

4.3 In each case decide whether the given triangles are similar. Justify your answer.

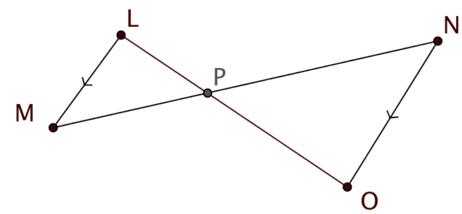


a. Is ΔABC similar to ΔADE? Explain.
No. Corresponding sides are not proportional.

b. Is ΔFGH similar to ΔKIJ? Explain.
Yes. All three pairs of corresponding sides are in a 2:1 ratio. The SSS Similarity Theorem applies.

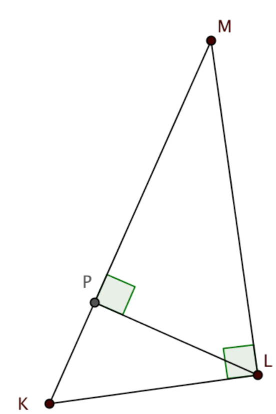


c. $\overleftrightarrow{LM} \parallel \overleftrightarrow{NO}$.
Is ΔLMP similar to ΔONP?
Explain.
Yes. Alternate interior angles formed by parallel lines and a transversal are congruent. Therefore $\angle MLP \cong \angle NOP$ and $\angle LMP \cong \angle ONP$. The AA Similarity Theorem applies. (We could also use the vertical angles at P for one of the angle pairs.)



4.4 In the figure, $\angle RSV \cong \angle RTU$.
$RS = 10$, $VS = 8$, and $UT = 12$.
Find RT and ratio $\frac{UV}{RV}$. Justify your answers.
Since one pair of angles is given to be congruent and the angle at R is common to both triangles, $\Delta VRS \cong \Delta URT$ by the AA Similarity Theorem. Since corresponding sides of similar triangles have the same ratio, $\frac{RS}{RT} = \frac{VS}{UT}$. Substituting the known side lengths and solving the proportion, we have $RT = 15$. $TS = RT - RS = 15 - 10 = 5$.
By the Parallel Lines Corresponding Angles Converse $\overline{SV} \parallel \overline{UT}$. Apply the Side-Splitter Theorem and we get $\frac{UV}{RV} = \frac{TS}{SR} = \frac{5}{10} = \frac{1}{2}$.

4.5 In the figure at the right, $\overline{LP}$ is the altitude to the hypotenuse of right triangle KLM.



Fill in the blanks to make the proportions correct:

a. $\frac{KL}{LM} = \frac{PK}{PL} = \frac{PL}{PM}$

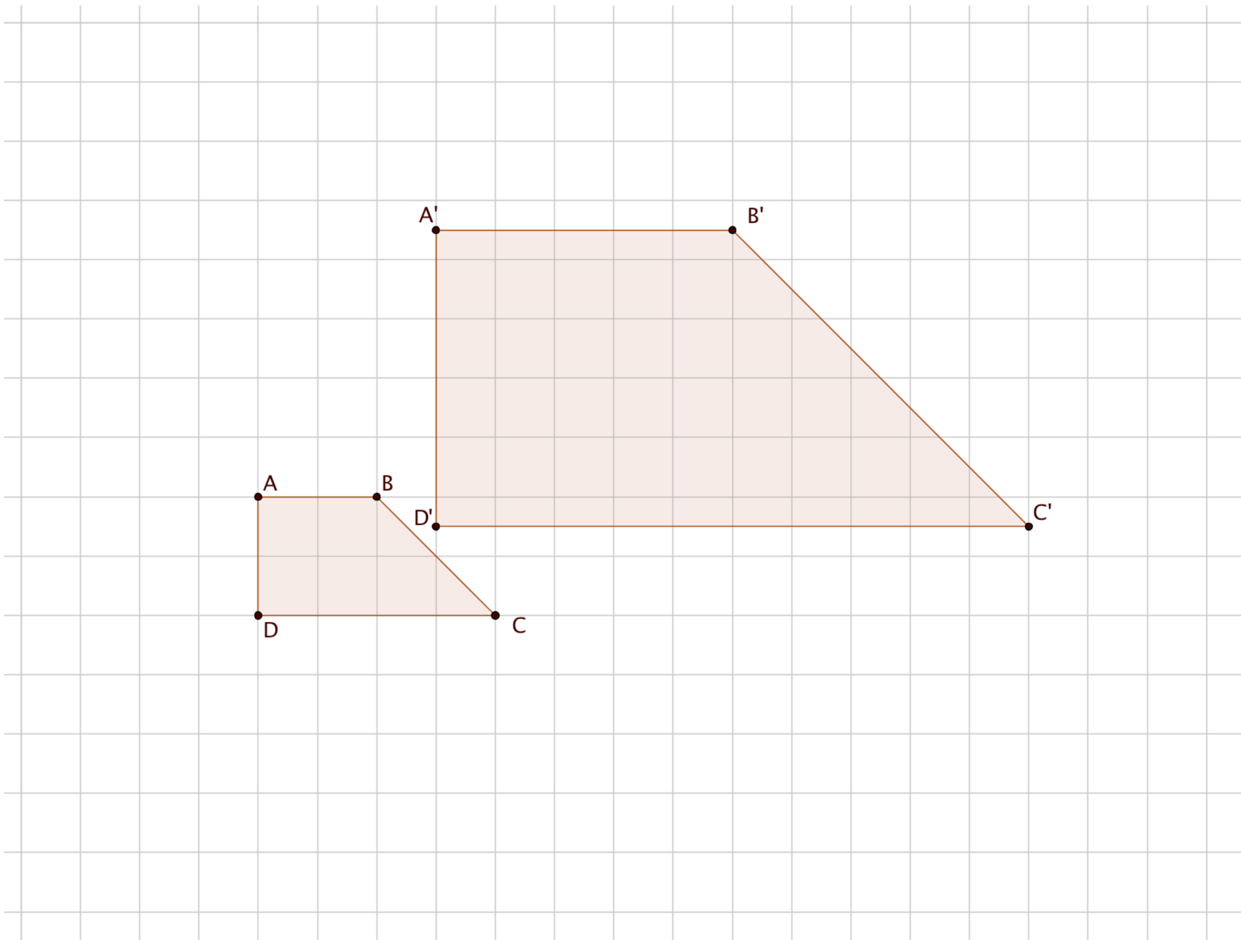
b. $\frac{KM}{KL} = \frac{KL}{PK} = \frac{ML}{PL}$

4.6 *A'B'C'D'* is the image of trapezoid *ABCD* under a dilation.

a. On the grid below, locate the center of dilation. Find the intersection of two of these lines $\overleftrightarrow{AA'}$, $\overleftrightarrow{BB'}$, $\overleftrightarrow{CC'}$, and $\overleftrightarrow{DD'}$.

b. Find the scale factor for this dilation. Explain your reasoning.

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{D'A'}{DA} = \frac{5}{2}$$



4.7 In right triangle *ABC*, *AB* = 13, *BC* = 5 and *AC* = 12. Find:

a. sin $A = \frac{5}{13}$

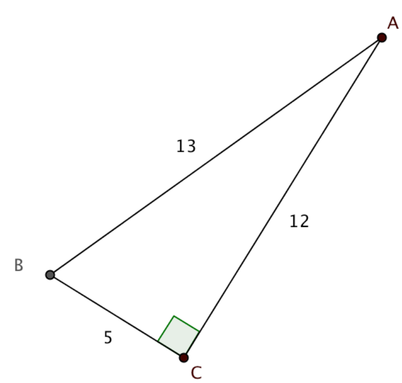
b. sin $B = \frac{12}{13}$

c. cos $A = \frac{12}{13}$

d. cos $B = \frac{5}{13}$

e. tan $A = \frac{5}{12}$

f. tan $B = \frac{12}{5}$



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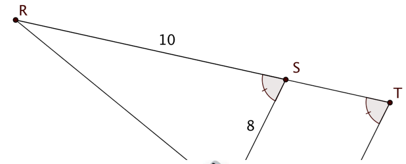


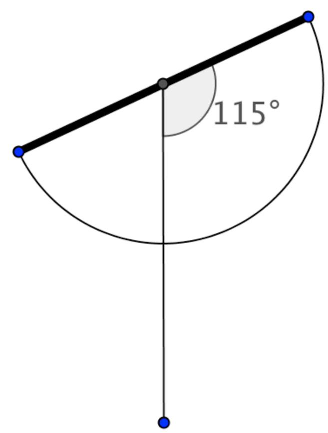
4.8 A 24 foot-long ladder is resting against the roof of a house that is 20 feet high. What is the angle at which the ladder meets the house? Answer to the nearest 0.1 degree.

Let θ represent the angle at which the ladder meets the house.

Then $\cos\theta = \frac{20}{24} \approx 0.833$. $\theta \approx 33.6°$.

www.clipartsheep.com

4.9 Jane is measuring the height of a flagpole. Her eyes are 5 ft 2 in above the ground. Jane is standing 121feet from the base of the pole looking at the top of the pole using her clinometer. The clinometer displays 115° as shown. Find the height of the pole to the nearest inch. Give your answer in feet and inches.



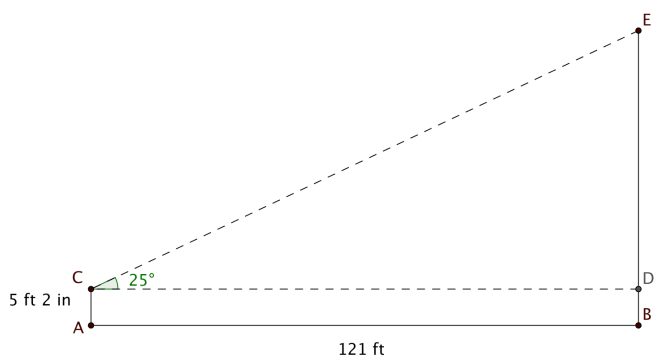
In the figure below, $m\angle CED = 180° - 115° = 65°$.

$m\angle ECD = 90° - 65° = 25°$.

$\tan 25° = \frac{ED}{CD} = \frac{ED}{121\text{ ft}}$.

$ED = 121\text{ ft} \times \tan 25° \approx 56.4232\text{ ft} \approx 56\text{ ft }5\text{ in}$.

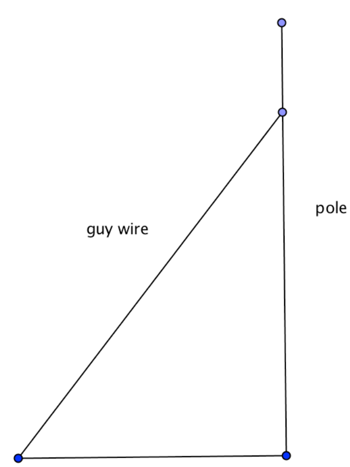
$EB = ED + DB = ED + AC = 56\text{ ft }5\text{ in} + 5\text{ ft }2\text{ in} = 61\text{ ft }7\text{ in}$.



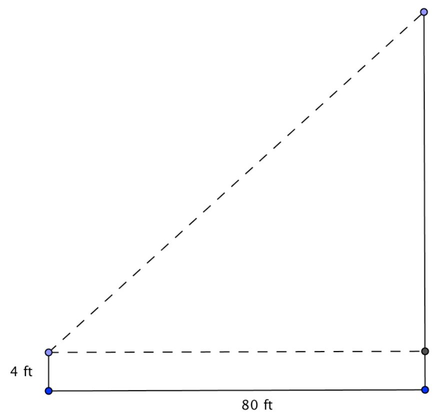
4.10 A 60-foot guy wire is attached to a pole 12 feet from the top of the pole. If the wire is at a 38° angle with the ground, what is the length of the pole?

Let x represent the distance from the top of the guywire to the ground. Then $\sin 38° = \frac{x}{60\text{ ft}}$. $x \approx 36.9$ ft.

Pole length $= x + 12 \approx 48.9$ ft.



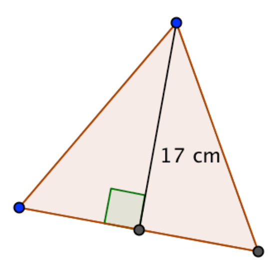
4.11 To find the height of a pole, a surveyor moves 80 feet away from the base of the pole and then, with a transit 4 feet tall, measures the angle of elevation from the top of the transit to the top of the pole to be 57°. What is the height of the pole? Round answer to the nearest tenth of a foot.



http://www.wikihow.com/Use-a-Surveyor's-Transit

Let the height of the pole be $x + 4$ ft. Then $\tan 57° = \frac{x}{80\text{ ft}}$.
$x = 80\text{ ft} \tan 57° \approx 123.2$ ft. Height of pole ≈ 127.2 ft.

4.12 An altitude of an equilateral triangle is 17 cm.



a. Find the length of a side to the nearest centimeter.
Let s represent the length of a side.
Using the Pythagorean Theorem, 30°-60°-90° triangle, or trigonometry, $17\text{ cm} = \frac{\sqrt{3}}{2}s$. $s \approx 19.6 \approx 20$ cm (to nearest cm).

b. Find the area of the triangle to the nearest square centimeter.
Area $= \frac{1}{2}$ base $\times$ altitude $\approx \frac{1}{2} \times 20\text{ ft} \times 17\text{ ft} = 170\text{ ft}^2$.

Unit 5

5.1 Find an equation of the circle in the coordinate plane with center at the origin and radius = 6.
$x^2 + y^2 = 36$

5.2 A circle in the coordinate plane has the equation $x^2 + 8x + y^2 - 12y = 48$. Find its center and radius.
$x^2 + 8x + 16 + y^2 - 12y + 36 = 48 + 16 + 36$
$(x + 4)^2 + (y - 6)^2 = 100$
Center is (–4, 6). Radius = 10.

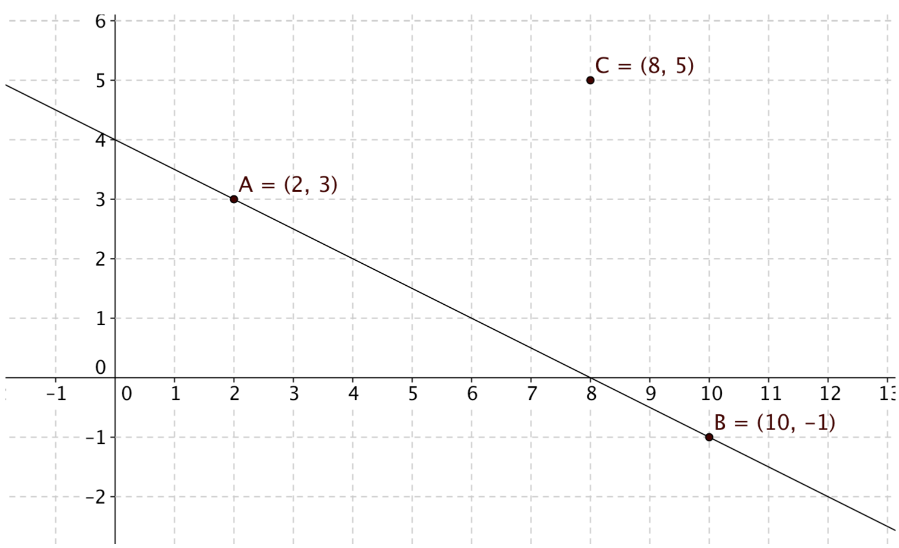
5.3 Coordinates of points *A*, *B*, and *C* are given in the figure below.

a. Find the midpoint of $\overline{AB}$. (6, 1)

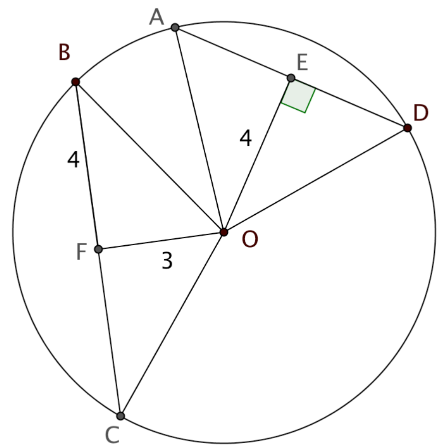
b. Find an equation for the perpendicular bisector of $\overline{AB}$.
Slope of $\overline{AB} = -\frac{1}{2}$. So slope of perpendicular bisector = 2. Using the point-slope form of the equation for a line, $y - 1 = 2(x - 6)$. This can be rewritten in slope-intercept form as $y = 2x - 11$.

c. Without performing any calculations, explain why the circle with center *C* passing through point *A* must also pass through point *B*.
Every point on the perpendicular bisector of a segment is equidistant from the endpoints. So *CA* = *CB*. The circle is the locus of points at distance *CA* from center *C*, so *B* must also be a point on the circle.

d. Find an equation for the circle described in part (c) and show that *B* lies on the circle.
By the distance formula, $CA = \sqrt{40}$; so an equation for the circle is $(x - 8)^2 + (y - 5)^2 = 40$. Substitute $x = 10$ and $y = -1$ to get $2^2 + (-6)^2 = 40$, which is a true equation.



5.4 Chords $\overline{AD}$ and $\overline{BC}$ are drawn in a circle with center O. $\overline{OE}$ is an altitude of ΔAOD. F is the midpoint of $\overline{BC}$. $OF = 3$, $BF = 4$ and $OE = 4$.



a. Find the radius of the circle.
By the Radius Chord Midpoint Theorem, $\overline{OF} \perp \overline{BC}$, so ΔOFB is a right triangle. Use the Pythagorean Theorem to show that radius $OB = 5$.

b. Find AD.
By the Radius Chord Midpoint Converse, since $\overline{OE} \perp \overline{AD}$, E is the midpoint of $\overline{AD}$. Use the Pythagorean Theorem to show that $AE = 3$. Therefore $AD = 6$.

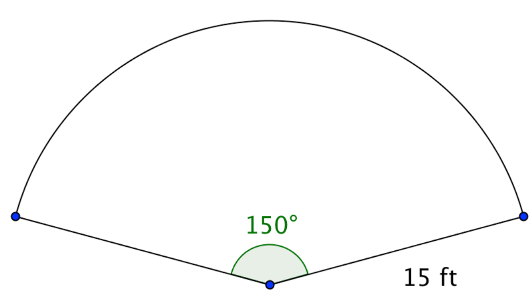
c. In this circle the longer chord is closer to the center than the shorter chord. Is this always true for two chords in the same circle? Explain. Yes. Explanations will vary. For example, the square of the distance from the center to the chord plus the square of half the length of the chord equals the square of the radius, which is constant. When one of the two quantities increases, the other one must decrease.

5.5 Vinyl records, which were popular in the mid-20th century, fell out of use when compact disks became available. They are now making a comeback. A "long playing" final record is placed on a turntable that makes $33\frac{1}{3}$ revolutions per minute. Through how many radians does the record pass in one second?

$33\frac{1}{3}$ revolutions per minute $= 2\pi \times 33\frac{1}{3}$ radians per minute $\approx$ 209.44 radians per minute. Divide by 60 to get 3.49 radians per second.

Image from www.clipartsheep.com

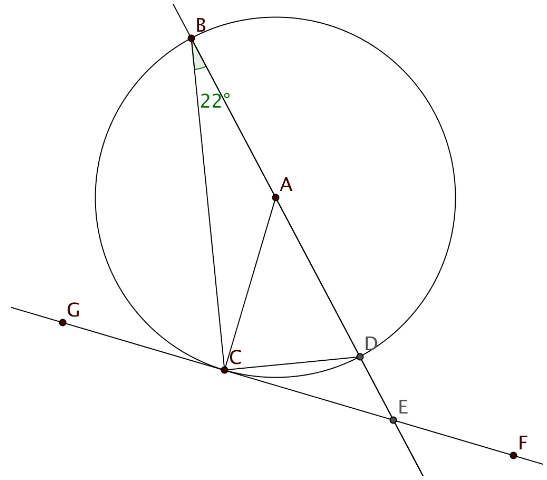
5.6 A flowerbed in front of a hotel has the shape of a sector of circle with a 150° central angle and a radius of 15 feet.



a. Find the area of the flowerbed.
Area of the circle = 225π ft^2.
Area of flowerbed $= \frac{150}{360} \times 225\pi$ ft$^2 \approx 295$ ft^2

b. Find the perimeter of the flowerbed
Circumference of circle = 30π ft.
Arc length $= \frac{150}{360} \times 30\pi \approx 39.3$ ft.
Add the two radii to get the entire perimeter of the sector = 69.3 ft.

5.7 In the figure m$\angle ABC$ = 22°. $\overleftrightarrow{GF}$ is tangent to circle A at point C. Secant $\overleftrightarrow{AE}$ intersects the circle at points B and D. Find the following:



a. The measure of central angle $\angle CAD$. 44° (Inscribed Angle Theorem)

b. m$\angle BCD$. 90° (angle inscribed in a semicircle)

c. m$\angle ACB$. 22° (isosceles triangle)

d. m$\angle DCE$. m$\angle DCA$ = 68°(radius is perpendicular to tangent). m$\angle DCE$ = 90° – 68° = 22°.

e. m$\angle BEC$. 90° + 22° = 112°.

f. Suppose the diameter of the circle is 14 cm. Find the lengths of minor arc CD and major arc DBC. Circumference = 14π ≈ 44.0 cm. Minor arc $CD = \frac{44}{360} \times 44.0 \approx 5.4$ cm. Major arc $DBC = \frac{316}{360} \times 44.0 \approx 38.6$ cm.

5.8 Describe the locus of points in a plane that satisfy each of these conditions:

a. Equidistant from the sides of an angle
The ray that bisects the angle

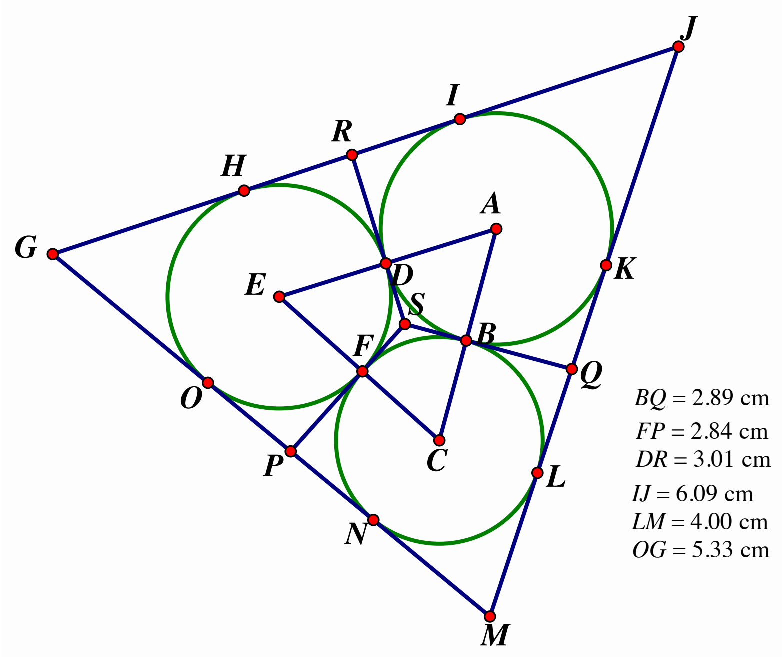
b. Equidistant from the endpoints of a segment
The perpendicular bisector of the segment

c. Equidistant from a given point and a given line
The parabola with the point as focus and line as directrix

d. The same distance from a given point.
The circle with the given point as center and given distance as radius

5.9 In the diagram below segments $\overline{JG}$, $\overline{JM}$ and $\overline{MG}$ are tangent to circles with centers A, E, and C as shown. Segments $\overline{RS}$, $\overline{SQ}$ and $\overline{SP}$ are also tangent to the circles. The lengths of some segments are given with the diagram. Find the perimeter of ΔGJM.

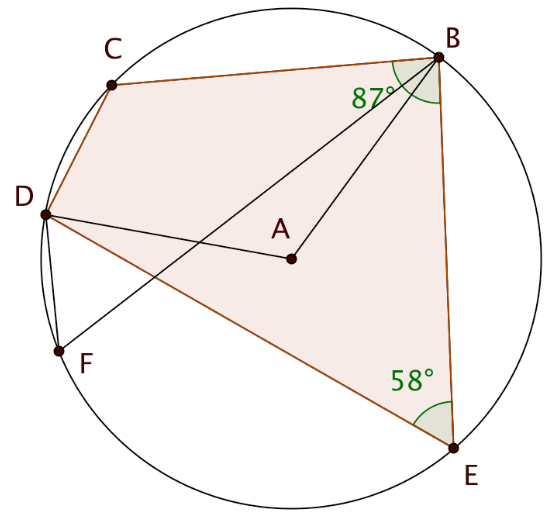
Use the Tangent Segment Theorem repeatedly to show that:
$LQ = KQ = BQ = 2.89$ cm
$OP = NP = FP = 2.84$ cm
$IR = HR = DR = 3.01$ cm
$KJ = IJ = 6.09$ cm
$LM = NM = 4.00$ cm
$OG = HG = 5.33$ cm
The perimeter is sum of the lengths of the sides of ΔGJM and is found by adding the lengths of the 12 segments along its sides =
$2(2.89 + 2.84 + 3.01 + 6.09 + 4.00 + 5.33)$ cm $= 48.32$ cm



5.10 Explain, step by step, how you would construct the circumscribed circle for a given triangle.

Answers will vary. Here is one possible response.
Suppose the vertices of the triangle are A, B, and C. We need to first find the center of the circumscribed triangle. It is the point where the three perpendicular bisectors meet. To perform the construction, however we only need two of them. To find the perpendicular bisector of $\overline{AB}$ draw a circle with center A passing through B and another circle with center B passing through A. Draw a line through the two points where these circles intersect. That line is the perpendicular bisector of $\overline{AB}$. Do the same to find the perpendicular bisector of $\overline{BC}$. Label D the point where the two perpendicular bisectors meet. Draw a circle with center D passing through A. This circle will also pass through points B and C and will be the circumscribed circle for ΔABC.

5.11 Points B, C, D, E, and F lie on circle A.
$m\angle CBE = 87°$ and $m\angle BED = 58°$.



Find

a. $m\angle BCD$ 180° – 58° = 122°
(Cylic Quadrilateral Theorem)

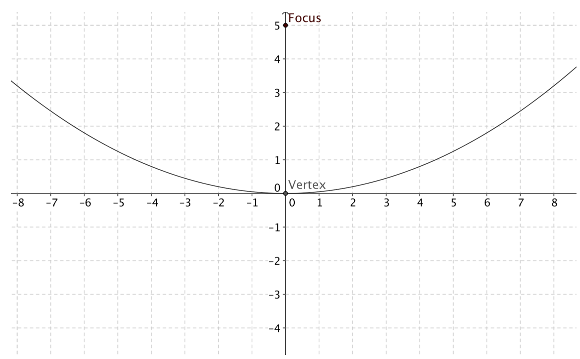
b. $m\angle CDE$ 180° – 87° = 93°

c. $m\angle DFB$ 58° (intercepts same arc as $m\angle BED$; Inscribed Angle Theorem)

d. Measure of reflex $\angle BAD$.
$360° - m\angle BAD = 360° - 116° = 244°$
(Inscribed Angle Theorem)

5.12 Prove or disprove: If the vertices of a rhombus lie on a circle, then the rhombus must be a square.
Every rhombus is a parallelogram and the opposite angles of a parallelogram are congruent. But if the rhombus is inscribed in a circle then, by the Cyclic Quadrilateral Theorem, the opposite angles are also supplementary. If the angles are congruent and supplementary they must be right angles. Since all the angles of this rhombus are right angles and all the sides are congruent, the rhombus must be a square.

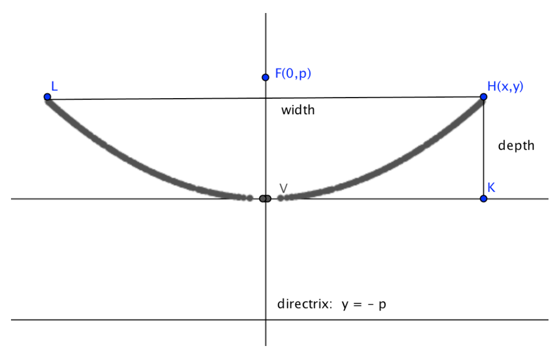
5.13 The vertex of a parabola is at the origin and its focus is at (0,5).



a. Find an equation for the parabola.
$p = 5$. An equation for the parabola is $4py = x^2$. Substituting for p, we have $20y = x^2$, or $y = 0.05x^2$.

b. Find an equation for its directrix.
$y = -5$

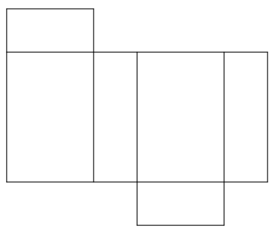
5.14 A reflecting telescope has a mirror with a cross-section shaped like a parabola. If the distance across the top of the mirror is 16 feet, and the distance from the vertex to the focus is 5 feet, how deep is the mirror in the center? Place a cross section of the mirror in the coordinate plane as shown. Then the equation in 5.13 applies so $y = 0.05x^2$. We are given that the width of the mirror is 16 ft, so the rightmost edge of the mirror has x-coordinate = 8. Substituting 8 into the equation we find the y-coordinate is 3.2. So the depth of the mirror is 3.2 ft.

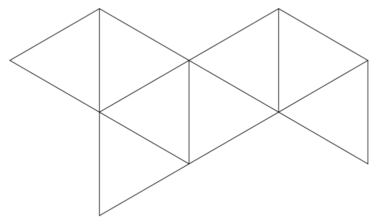


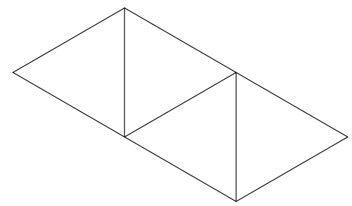
Unit 6

6.1 Identify the type of polyhedron formed by each of these nets:

a. rectangular prism b. regular octahedron c. regular tetrahedron

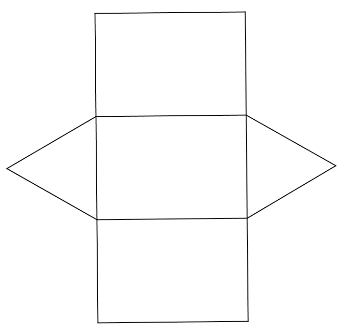




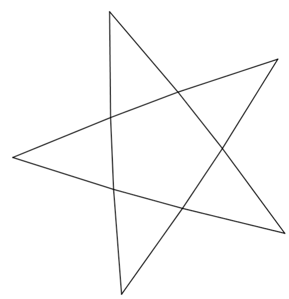


6.2 Sketch a net for each of these solid figures

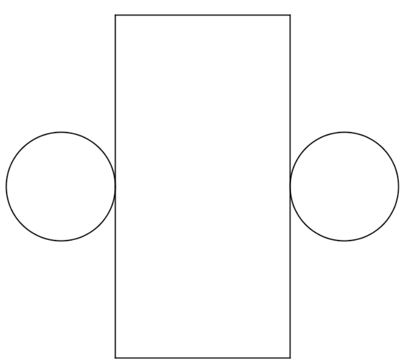
a. Triangular prism



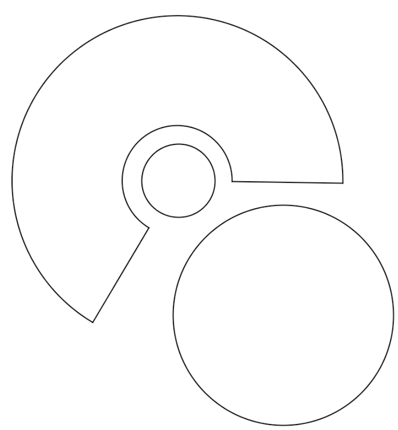
b. Pentagonal pyramid



c. Cylinder



d. Frustum of a Cone



6.3 Sketch each of these figures. Then find the surface area and volume. Leave π in your answer where appropriate.

a. A sphere with radius = 3 units
Surface area = $4\pi r^2 = 36\pi$ units^2
Volume = $\frac{4}{3}\pi r^3 = 36\pi$ units^3

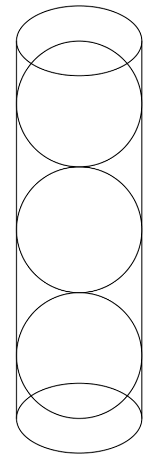
b. A cube with edge = 4 units
Surface area = $6e^2 = 96$ units^2
Volume = $e^3 = 64$ units^3.

c. A square pyramid with the side of the base = 6 units and height = 4 units
Area of base = 36 units^2
Triangular faces have slant height = 5 units (Pythagorean Theorem)
Area of each triangular face = 15 units^2. Total for 4 faces = 60 units^2
Total surface area = 96 units^2
Volume = $\frac{1}{3}Bh$ =48 units^3

d. A cone with the slant height = 13 units and the radius of the base = 5 units
Height of cone is 12 units (Pythagorean Theorem)
Lateral surface area = $\frac{5}{13} \times \pi \times 13^2 = 65\pi$ unit^2
Area of base = 25π unit^2
Total surface area = 90π unit^2
Volume $= \frac{1}{3}Bh = 100\pi$ unit^3

e. A prism with a height of 10 units and a regular hexagonal base that is 3 units on a side.
Lateral surface area = $6 \times 3 \times 10 = 180$ unit^2
Area of base = 6 × Area of equilateral triangle with side 3 = $6 \times \frac{9\sqrt{3}}{4} = \frac{27\sqrt{3}}{2} \approx 23.4$ unit^2
Total surface area = lateral surface area + 2 × area of base ≈ 226.8 unit^2.
Volume = $Bh \approx 234$ unit^3.

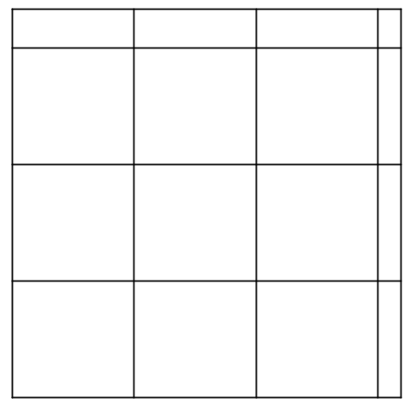
6.4 Is it possible to fit a 9-foot pole inside a closet that is 3 feet wide, 2 feet deep, and 8 feet high? Explain your reasoning. No. The length of the diagonal of the rectangular prism formed by the closet is given by $\sqrt{3^2+2^2+8^2} = \sqrt{77} \approx 8.77$ ft. The closet is too small to accommodate the 9-ft pole.



6.5 Three tennis balls are stacked on top of each other and fit tightly in a cylindrical can.

a. Find the surface area of the can if the diameter of each ball is 10 cm.
The radius of the cylinder is 5 cm and the height is 30 cm.
Area of each base is 25π cm^2.
Circumference of base is 10π cm, so the lateral surface is 300π cm^2.
Total surface area is therefore 350π cm$^2 \approx 1100.0$ cm^2.

b. The lateral surface is made from a clear plastic, which comes in square sheets that are 1 meter on a side. How many cans can be made with one sheet? Be sure to show not only that you have enough area, but that the lateral surfaces will all fit within the sheet.
9 cans may be made with one sheet. Each can has a rectangular lateral surface 30 cm by about 31.4 cm as shown.



6.6 For each plane figure, identify one or more solids for which it could be a cross-section. (Try to find as many as you can!)
Answers will vary.

a. square
cube, square prism, square pyramid, cylinder with height = diameter.

b. circle
cylinder, cone, sphere

c. triangle
any pyramid or prism, regular dodecahedron, cone

d. non-square rectangle
rectangular prism, cube, cylinder

e. trapezoid with only one pair of parallel sides
frustum of cone or pyramid

6.7 Explain how the entire surface of the earth may be divided into eight spherical triangles each with three 90° angles. One example is the set of 8 triangles formed by the equator, the prime meridian (0° longitude), international date line (180° longitude) and the 90°E and 90° W meridians.

6.8 Describe how each of these solid figures could be generated by revolving a plane figure about an axis of rotation.

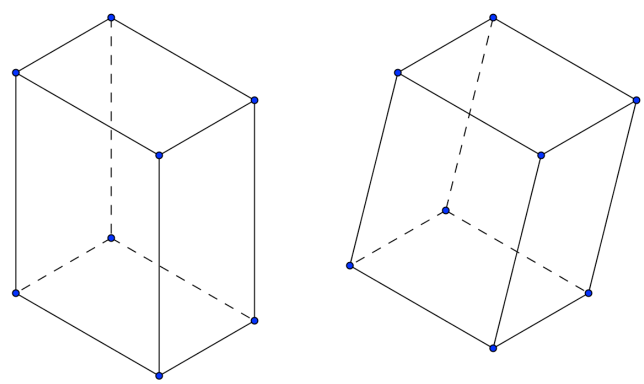
a. A cylinder Rotate a rectangle about one of its sides.

b. A cone Rotate a right triangle about one of its legs.

c. A sphere Rotate a semicircle about its diameter.

d. A frustum Rotate a trapezoid with two right angles about the side that is perpendicular to the two bases.

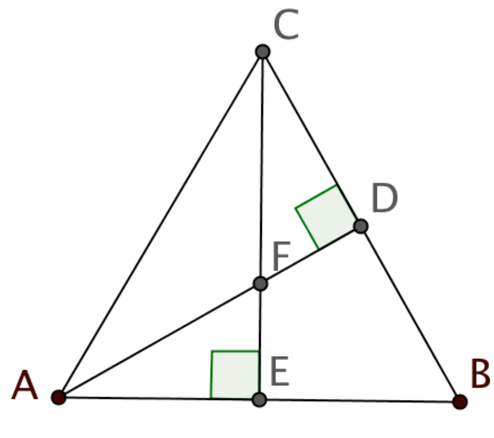
6.9 Two prisms have the same base and the same height, but one is a right prism and the other is an oblique prism. Explain how Cavalieri's principle is used to show that the two prisms have the same volume.



Suppose the bases of both prisms lie in a plane. Then any plane parallel to that plane will intersect the prisms in rectangles that are congruent to the bases. Since the bases are congruent to each other, these rectangles are also congruent and have the same area. As long as this is true for every plane intersecting the two figures (which it is), Cavalieri's principle states that the two figures will have the same volume.

6.10 The edge of a regular tetrahedron is 6 cm. Follow these steps to find the volume of the tetrahedron.

a. First consider the base of the tetrahedron, equilateral ΔABC. Altitudes $\overline{AD}$ and $\overline{CE}$ intersect at point F as shown. Use your knowledge of 30°-60°-90° triangles to find AF. From 30°-60°-90° ΔADB, $AD = 3\sqrt{3}$ cm. Since altitudes of equilateral triangles are also medians, F is the centroid. If students have learned that the centroid divides the median in a 2:1 ratio, then it follows that $AF = \frac{2}{3}AD = 2\sqrt{3}$ cm. Or students may use congruent triangles ΔFEA and ΔFDC to get the same result.

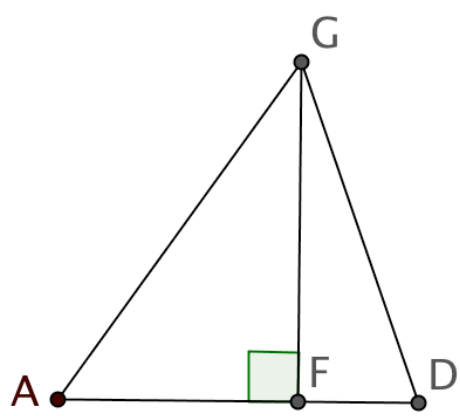


b. Now look at a cross section of the tetrahedron passing through vertex G and segment $\overline{AD}$. Use this figure to find GF, the height of the tetrahedron. Use the Pythagorean Theorem.

$AG^2 = AF^2 + GF^2$

$36 = 12 + GF^2$

$GF = \sqrt{24} = 2\sqrt{6}$ cm.



c. Find the volume of the tetrahedron.

Area of base $= \frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}$ cm^2.

Volume of pyramid $= \frac{1}{3}Bh = \frac{1}{3}9\sqrt{3} \times 2\sqrt{6} = 6\sqrt{18} = 18\sqrt{2} \approx 25.5$ cm^3.

d. A student says that the height of a regular tetrahedron is the same as the altitude of one of the faces. Is she correct? Explain. No. The height of the tetrahedron is slightly less than the altitude of one of its faces. This should be evident since the altitude of a face is also the slant height of the pyramid. In this case the altitude is $3\sqrt{3} \approx 5.2$ cm and the height is $2\sqrt{6} \approx 4.9$ cm.

Unit 7

7.1 Students formed a sample space from the room numbers/labels corresponding to rooms in a hallway of their school:

S = {Janitor, Girls, Boys, Media Center, 1, 2, 3, 4, 5, 6, 7, 8, 9}

a. Consider the following events:

- Event A consists of the even numbered rooms.
- Event B consists of the rooms that have labels rather than numbers.
- Event C consists of rooms numbered less than 6 or labeled with words that would appear in a dictionary before the word Excellent.

Specify the outcomes in each of these events using set notation.

A = {2, 4, 6, 8}; B = {Janitor, Girls, Boys, Media Center}; C = {1, 2, 3, 4, 5, Boys}

b. Events A, B, C and sample space S are represented by the Venn diagram in Figure 1. Enter the outcomes from the sample space S in the appropriate regions on the Venn diagram.
See diagram below.

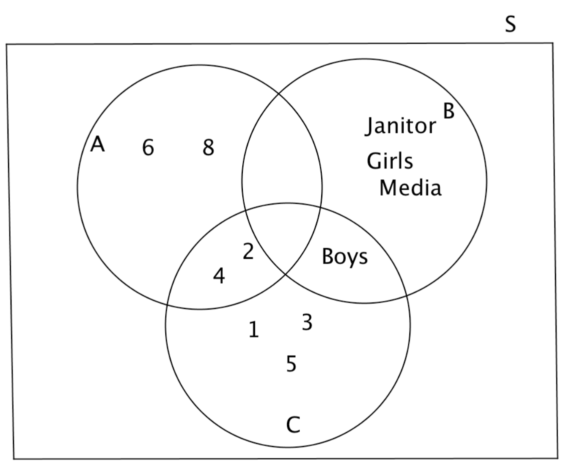


Figure 1. Venn diagram representing events A, B, and C.

Use your answer in (b) to find the following events. Write the events using set notation.

c. C^C = {6, 7, 8, 9, Janitor, Girls, Media}

d. $A \cup B$ = {2, 4, 6, 8, Janitor, Boys, Girls, Media}

e. $(A \cup B) \cap C^C$ = {6, 8, Janitor, Girls, Media}

7.2 The entire student body of a high school consists of 75 9th-graders, 82 10th-graders, 70 11th-graders, and 65 12th-graders. A student is randomly selected from the school. Find the following probabilities. Show how you determined your answers.

a. *P*(Student is in the 12th grade) = $\frac{65}{292} \approx 0.223$

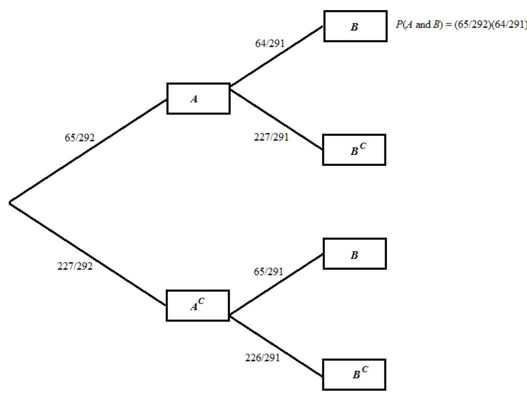
b. *P*(Student is not in the 12th grade) = 1 – *P*(Student is in the 12th grade) = $1 - \frac{65}{292} = \frac{227}{292} \approx 0.777$. We used the complement rule.

c. *P*(Student is either in the 9th or 10th grade) = *P*(Student is in the 9th grade) + *P*(Student is in the 10th grade) = $\frac{75}{292} + \frac{82}{292} = \frac{157}{292} \approx 0.538$. We used the addition rule for mutually exclusive events.

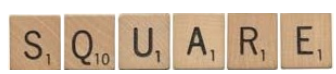
7.3 Return to the information on the high school in question 7.2. Suppose that two students were randomly chosen from this high school. Find the probabilities below. Show how you arrived at your answers.

a. What is the probability that both students were in the 12th grade? We can calculate this answer using combinations: *P*(both students in 12th grade) = $\frac{_{65}C_2}{_{292}C_2} \approx 0.049$.

We can also calculate this probability as follows using a tree diagram. Represent the random sample as Student 1, Student 2. Let *A* be the event that Student 1 is in 12th grade and *B* be the event that Student 2 is in 12th grade. We can represent this situation with the tree diagram below in which probabilities are written along the branches. Hence, *P*(Student 1 is in 12th grade and Student 2 is in 12th grade) = $P(A \cap B) = P(A) \times P(B|A) = \frac{65}{292} \times \frac{64}{291} = \frac{4160}{84{,}972} \approx 0.049$.



b. What is the probability that at least one of the two students was in the 12th grade? Use either method in (a) to show that *P*(neither student is in 12th grade) ≈ 0.604. Then *P*(At least one of the 2 students in in 12th grade) = 1 – *P*(neither student is in 12th grade) $\approx 1 - 0.604 = 0.396$.

7.4 Suppose that you have the following six Scrabble tiles: 

a. What is the total number of arrangements of all six letters? 6! = 720

b. How many 4-letter sequences can be made from these letters? $_6P_4 = \frac{6!}{(6-4)!} = 360$

c. According to an online Scrabble Word Finder, there are 18 real words that can be made from 4 of the scrabble tiles that spell SQUARE. Suppose that you randomly select four of the tiles from SQUARE and lay them down in the order you selected them. What is the probability that your sequence of tiles makes an actual word? The probability is $\frac{18}{360} = 0.05$.

7.5 A high school swim team consists of 15 female swimmers and 21 male swimmers. Six team members are randomly selected as a delegation to represent the team at a state swim meet.

a. How many different delegations are possible? ${}_{36}C_6 = \frac{36!}{(36-6)!\times 6!} = 1{,}947{,}792$

b. What is the probability that the delegation that is chosen is all male? The number of six-person all male delegations is ${}_{21}C_6 = \frac{21!}{(21-6)!\times 6!} = 54{,}264$. The probability that an "all male" delegation will be chosen is $\frac{54{,}264}{1{,}947{,}792} \approx 0.028$.

c. How many delegations are half male and half female? What is the probability that the randomly selected six-person delegation will be half male and half female? The number of delegations that are half male and half female are ${}_{21}C_3 \times {}_{15}C_3 = 605{,}150$. The probability that the delegation is half male and half female is $\frac{605{,}150}{1{,}947{,}792} \approx 0.311$.

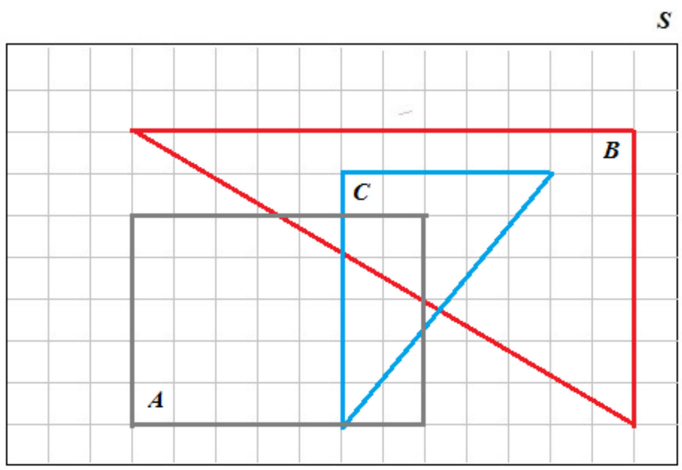


Figure 2. Area model for three events *A*, *B*, and *C*.

7.6. Figure 2 shows an area probability model for three events:

- *A*, the rectangular region
- *B*, the large triangular region
- *C*, the smaller triangular region

Determine the probabilities for (a) – (d):

a. $P(A)$, $P(B)$, and $P(C)$. $P(A) = \frac{35}{160} \approx 0.219$; $P(B) = \frac{42}{160} \approx 0.263$; $P(C) = \frac{15}{160} \approx 0.094$.

b. $P(A \cap B) = \frac{3.5}{160} \approx 0.022$

c. $P((A \cap B)^C) = 1 - \frac{3.5}{160} \approx 0.978$

d. $P(B \cup C) = P(B) + P(C) - P(B \cap C)$. B's area is 42 squares and C's area is 15 squares. The area of $B \cap C$ may be split into a triangle and a trapezoid. A good estimate for this region's area is 10.5 squares. $P(B) + P(C) - P(B \cap C) \approx \frac{42}{160} + \frac{15}{160} - \frac{10.5}{160} = \frac{46.5}{160} \approx 0.291.$

e. What fraction of B's outcomes overlap with A? What probability does this fraction give?
$\frac{P(A \cap B)}{P(B)} = \frac{0.022}{0.263} \approx 0.84$. Alternatively the ratio of areas is $\frac{3.5}{42} \approx 0.83$. The discrepancy is due to round off area. This is also the conditional probability of A given B, or $P(A|B)$

f. What fraction of A's outcomes overlap with B? What probability does this fraction give?
$\frac{P(A \cap B)}{P(A)} = \frac{0.022}{0.219} \approx 0.10$. Alternatively the ratio of areas is $\frac{3.5}{35} = 0.1$, exactly. This is also the conditional probability of B given A, or $P(B|A)$

7.7 A survey of pet owners was conducted to see if gender affects what type of pet they own. The results of the survey are recorded in the table below.

	Animal Owned			
Pet Owner	Cat	Dog	Cat and Dog	Other
Female	75	250	400	20
Male	50	280	320	30

a. How many pet owners completed the survey?
1425

b. What percentage of the pet owners who completed this survey were female? What percentage were male? Show your calculations.

Female: $\frac{745}{1425} \times 100\% \approx 52.3\%$; Male: $\frac{680}{1425} \times 100\% \approx 47.7\%$

c. Does gender affect what type of animal is owned? To answer this question, calculate the conditional percentages for animals owned for each gender and enter your percentages into the table below. Show your calculations.

Pet Owner	Cat only	Dog only	Cat and Dog	Other
Female	$\frac{75}{745} \times 100\%$ 10.1%	$\frac{250}{745} \times 100\%$ 33.6%	$\frac{400}{745} \times 100\%$ 53.7%	$\frac{20}{745} \times 100\%$ 2.7%
Male	$\frac{50}{680} \times 100\%$ 7.4%	$\frac{280}{680} \times 100\%$ 41.2%	$\frac{320}{680} \times 100\%$ 47.1%	$\frac{30}{680} \times 100\%$ 4.4%

d. Discuss the gender differences in pet ownership that you find most striking.
Answers will vary. Males (41.2%) were more likely to own only a dog compared to females (33.6%). However, females (53.7%) were more likely to own both a cat and a dog compared to males (47.1%).

7.8 Joanne wanted to find out how likely it was to win her town's lottery. She purchased 120 scratch cards at $5.00 a card and recorded her results in the table below.

	Frequency	Relative Frequency
Lost	65	$\frac{65}{120} \approx 0.542$
Purchase price returned	43	$\frac{43}{120} \approx 0.358$
Win $15	12	$\frac{12}{120} = 0.10$

a. Compute the relative frequencies (round to 3 decimal places) and enter them into the table above. See table.

b. Use the relative frequencies as estimates for the probabilities. Determine the expected value of her town's lottery game from the player's point of view. On average, how much will Joanne win or lose per ticket if she continues to play the game?
Expected value = (–$5)(0.542) + (0)(0.358) + ($10)(0.100) = –$1.71.

c. On average, how much will the town win or lose per ticket)? How much would the town expect to receive in revenue from its lottery in a week when 1000 scratch cards were sold?
The town wins, on average, $1.71 per ticket sold. The town would expect to receive $1,710 in a week when 1000 scratch cards were sold.

7.9 Let A and B be two independent events with $P(A) = 0.7$ and $P(B) = 04$. Find the following probabilities. Justify your answers.

a. $P(A \cap B) = P(A) \times P(B) = 0.7 \times 0.4 = 0.28$. (Multiplication rule for independent events)

b. $P(A|B) = P(A) = 0.7$. Since A and B are independent, the fact that B has occurred will not influence the probability that A occurs.

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.4 - 0.28 = 0.82$.

d. $P(A^C \cap B^C) = P((A \cup C)^C) = 1 - P(A \cup C) = 1 - 0.82 = 0.18$. (De Morgan's law)
Students can also use the fact that if A and B are independent events, then so are their complements. Hence, $P(A^C \cap B^C) = P(A^C) \times P(B^C) = (1 - 0.7)(1 - 0.4) = 0.18$.

7.10 The table below gives probabilities of the birth weights of babies. (These probabilities are based on Massachusetts hospital data.)

Weight	Probability
0 lb – 4 lb 15 oz	0.046
5 lb – 5 lb 15 oz	0.077
6 lb – 6 lb 15 oz	0.227
7 lb – 7 lb 15 oz	0.350
8 lb – 8 lb 15 oz	?
9 lb – 9 lb 15 oz	0.066
10 lb or more	0.010

a. Determine the probability that a randomly selected baby weighs between 8 pounds and 8 pounds 15 ounces. Explain how you determined your answer.
The probabilities must add to 1. The given probabilities sum to 0.776. Therefore, the missing probability is 1 – 0.776 = 0.224.

b. What is the probability that a randomly selected baby weighs less than 7 pounds?
0.227 + 0.077 + 0.046 = 0.350

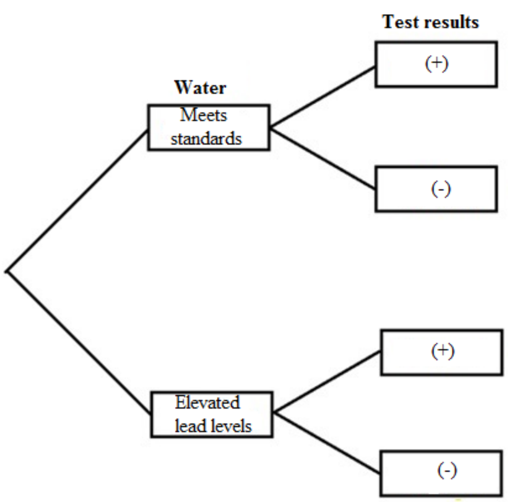
c. What is the probability that a randomly selected baby weighs 5 pounds or more?
1 – 0.046 = 0.954

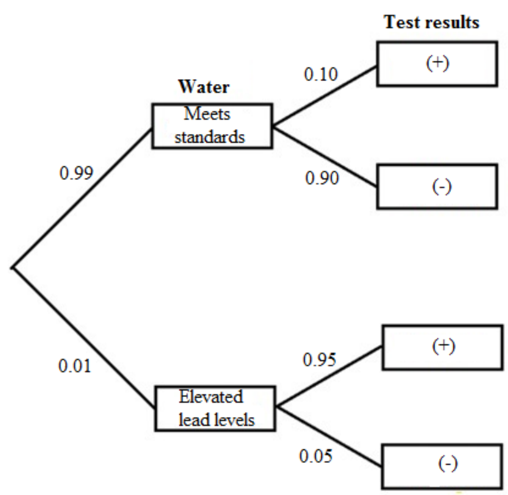
d. Given a randomly selected baby weighs 5 pounds or more, what is the probability that the baby weighs under 7 pounds? Explain how you got your answer.
Let A be the event that a baby weighs 5 pounds or more and B the event that a baby weighs under 7 pounds. We need $P(B|A) = \frac{P(B \cap A)}{P(A)}$. $P(A) = 0.954$ from (c). $P(A \cap B) = 0.077 + 0.227 = 0.304$ (from the table). Therefore $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.304}{0.954} \approx 0.319$ (using $P(A)$from (c)).

e. Explain the difference between the probability you calculated in (d) and the probability that a baby weighs 5 pounds or more given the baby weighs under 7 pounds.
The information given about the baby differs in this question and (d). In this case, you know that the randomly selected baby weighs under 7 pounds and you want to calculate $P(A|B) = \frac{P(B \cap A)}{P(B)}$. The numerator in both the probability calculations remains the same but in this question, we must divide by $P(B)$ instead of $P(A)$. From (b), we know that $P(B) = 0.350$. $(A|B) = \frac{P(B \cap A)}{P(B)} = \frac{0.304}{0.350} \approx 0.869$.

7.11 Flint Michigan made the news in 2016 due to elevated levels of lead found in the drinking water. In response to Flint's water crisis, Connecticut's Department of Public Health sent out a press release saying that 99% of the public water systems in Connecticut are in compliance with federal standards for lead levels (less than 15 parts per billion (ppb) of lead in public water systems). Suppose the public water system in your area is being tested. Your water system could meet federal standards or have elevated lead levels. The test for lead in your area's water system could turn out to be positive (+) indicating lead levels are elevated and hence, the drinking water is unsafe or negative (-). However, these tests are not perfect. False positives as well as false negative test results are possible.

a. Draw a tree diagram showing the possibilities for the lead levels in the drinking water and the test results. See diagram on left below.





b. Explain what a false positive means.
A false positive means that the test results come back positive for elevated levels of lead when, in fact, the water has a lead concentration below 15 ppb.

c. Suppose that the probability of getting positive test result given the levels of lead are elevated is 0.95 and the probability of getting a negative test result given the levels of lead meet federal standards is 0.90. Assume that the probability of having lead in Connecticut water supplies is 0.01. Add probabilities to your tree diagram in (a). See diagram on right above.

d. Suppose a water system is tested and the results are positive. What is the probability that this water system actually has elevated levels of lead? In other words, find $P(\text{elevated lead} \mid (+))$. Show how you can use your diagram to help find this probability. Interpret your results.

We want $P\left(\text{elevated lead}\middle|(+)\right) = \frac{P(\text{elevated lead} \cap (+))}{P((+))} = \frac{(0.01)(0.95)}{(0.01)(0.95)+(0.99)(0.10)} \approx 0.088.$

Even though the water test was positive, there is a relatively low probability that the lead level is actually elevated.

e. Suppose you are in an area where the likelihood of lead in the drinking water is much higher than in Connecticut. Assume that the probability of lead in the water supply is 0.30. Rework your answer to (d).

$$P\left(\text{elevated lead}\middle|(+)\right) = \frac{P(\text{elevated lead } \cap (+))}{P((+))} = \frac{(0.30)(0.95)}{(0.30)(0.95)+(0.70)(0.10)} \approx 0.803.$$