**Unit 7: Investigation 4 (2 Days)**

**MODELING EXPONENTIAL DATA**

CCSS: F-LE 2, F-LE 5

**Overview**

Students perform two experiments, collect data, and apply their knowledge of exponential functions to analyze the data. Data from the M&M experiment may be modeled with exponential growth and data from the bouncing ball experiment may be modeled with exponential decay. An additional activity concerning the growth of Facebook does not require the collection of data and may be assigned for homework.

**Assessment Activities**

**Evidence of Success: What will students be able to do?**

* Collect data from an experiment, make a table and a graph, and then fit an exponential function to the data.
* Reflect on the accuracy of the exponential model given the nature of the experiments.

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 7.4** asks students to fit an exponential function to a set of data from a bouncing ball experiment.
* **Journal Prompt** asks students to reflect on the outcome of the experiments and assess how will their models fit the data.

**Launch Notes**

Begin by asking students to recall examples of real-world situations that can be modeled with exponential functions, and identify which ones represent growth or decay. Ask them to brainstorm about other situations that might b**e** exponential. Then explain that for the next two days we will conduct experiments and fit exponential functions to the data collected.

**Closure Notes**

Ask students to reflect on the reasons why not all students got the same function in each of the experiments. For the M&M experiment, students should recognize the role that chance plays. For the bouncing ball experiment students should recognize that the “bounciness” of the ball depends upon the material it is made of and the surface on which it lands.

**Teaching Strategies**

**Group Activity**

Both experiments in Activities 7.4.1 and 7.4.2 lend themselves to having students work in groups of two or three, at least in the data collection phase.

1. Introduce the M&M experiment in **Activity 7.4.1 Tossing M and Ms.** Make sure students understand the rules by walking through the example on the first page. Students may work in groups to collect the data. Then have students work together to answer questions 2–6. Question 4 asks students to find differences and ratios of successive values from the table in question 1. Make sure they understand that we compare the new trial to the previous trial by finding *new minus previous* and *new divided by previous*. As you circulate around the room you should tell at a glance if they are doing this correctly – the differences should be non-negative and the ratios should be greater than or equal to one.

**Differentiated Instruction (For Learners Who Need More Help)**

For question 4 in Activity 7.4.1 some students may need more scaffolding. You can give them a chart like this to help them.

|  |  |  |  |
| --- | --- | --- | --- |
| Trial | # of M&Ms | Difference | Ratio |
| 0 | f(0) = 2 | -------- | -------- |
| 1 | f(1) = | f(1) – f(0) = | f(1) ÷ f(0) = |
| 2 | f(2) = | f(2) – f(1) = | f(2) ÷ f(1) = |
| 3 | f(3) = | f(3) – f(2) = | f(3) ÷ f(2) = |
| 4 | f(4) = | f(4) – f(3) = | f(4) ÷ f(3) = |
| 5 | f(5) = | f(5) – f(4) = | f(5) ÷ f(4) = |

When students get to questions 7–13 they will apply what they learned in previous investigations about the parameters of an exponential function. Because the data are subject to random fluctuations, however, the table on page 2 will not give them an exact growth factor. However the ratios should suggest that the growth factor is a number between 1 and 2. Most students will probably come up with an estimate close to 1.5.

For question 14 make sure students use the equation they discovered in question 13, not the data from the table on page 1. They should then observe that the differences in successive values for an exponential function grow as the number of trials increases. Question 17 asks students again to summarize the differences between linear and exponential functions. Their responses should reflect a maturing understanding of this important concept.

Questions 18–20 provide an opportunity for students to make predictions based on their model and to further deepen their number sense. Students will be surprised at the number of M&Ms needed for 50 trials. In most cases they will need at least a million. Depending upon their estimate for the growth factor it may be in the hundreds of millions. Discussing the different results obtained here will lead to an appreciation of how a slight change in the growth factor can result in a big difference over time.

**Differentiated Instruction (Enrichment)**

Assign questions 21–25 to students who would like the challenge of fitting an exponential function to a set of data. The Exponential Regression feature of the calculator is analogous to the Linear Regression feature introduced in Unit 5. A full explanation of how it works will wait until a later course when students study logarithms.

**Activity 7.4.3 Facebook** **Users** may be assigned at anytime during this investigation. Since this activity does not require the collection of data, it is suitable for a homework assignment. In this activity students try different values of the parameters to find a curve that appears to fit the data. Question 10 merits discussion since the predicted value for 2020 may exceed the world’s population. This example illustrates the limitations of exponential models, since in most situations a quantity can grow exponentially for only a limited period of time.

1. **Activity 7.4.2 Bouncing Balls** has students collect data that can be modeled by an exponential decay function. Before getting started discuss with students ideas about how they can improve the accuracy of their measurements. They should tape a yardstick, meter stick, or tape measure to a wall to measure height. Height is best measure when the observer’s eyes are level with the ball when it reaches the top of its bounce. It will take some experimentation to achieve this. You may also want to suggest that they take several measurements from the same drop height and average the results. They may also want to concentrate on only one bounce for each trial, e.g. first measure just the height on the first bounce, then the height on the second bounce, etc.

You should have a variety of different balls and possible different surfaces, which will produce different results. Try to avoid carpeted surfaces, which absorb more energy and hence lead to a smaller bounce. If your school permits you may want to take students into the hallway or cafeteria to find a good hard surface.

Questions 6–11 ask students to think about the effects of the parameters. If time is limited part of the class may perform the experiment in questions 6 – 8 (focusing on *a*) and the other students perform the experiment in question 9 – 11 (focusing on *b*).

**Differentiated Instruction (Enrichment)**

Challenge students to design an experiment with M&M’s that is modeled by an exponential decay function. (One approach is to start with a large number of M&Ms and remove those with the logo facing up on each toss of the cup.)

**Exit Slip 7.4** presents students with a “clean” set of data from a bouncing ball experiment and asks them to fit an exponential function and use it to make a prediction.

**Journal Prompt**

Describe the two experiments that you conducted in Activities 7.4.1 and 7.4.2. How were they alike and how were they different? How accurate were the mathematical models you found? What aspects of the experiments may have influenced their accuracy?

**Resources and Materials**

* **Activity 7.4.1 Tossing M and Ms**
* **Activity 7.4.2 Bouncing Balls**
* **Activity 7.4.3 Facebook Users**
* **Exit Slip 7.4 Bouncing Ball**
* Bags of M&Ms, enough for 50 per student
* Paper plates and paper cups
* Balls of various kinds and materials
* Yard or meter sticks, or tape measures
* Masking tape for mounting the above
* Graphing calculators
* Bulletin board for key concepts