**Exploring the Meaning of Rational Exponents**

1. What is a rational number? Why are they called rational numbers?

Now let’s figure out what a rational exponent means. We start with a specific example.

Look at the exponential pattern in the table below.

1. What do you think $9^{\frac{1}{2}}$ means? What do you think is its value?

|  |  |  |  |
| --- | --- | --- | --- |
| ***x*** | **9*x*** | **Meaning** | **Value** |
| –3 | 9–3 | $$\frac{1}{9}∙\frac{1}{9}∙\frac{1}{9}$$ | $$\frac{1}{729}$$ |
| –2  | 9–2 | $$\frac{1}{9}∙\frac{1}{9}$$ | $$\frac{1}{81}$$ |
| –1  | 9–1 | $$\frac{1}{9}$$ | $$\frac{1}{9}$$ |
| 0 | 90 | 1 | 1 |
| 1/2 | 9 1/2 |  |  |
| 1 | 91 | 9 | 9 |
| 2 | 92 | 9∙9 | 81 |
| 3 | 93 | 9∙9∙9 | 729 |

1. Does your estimate for$ 9^{\frac{1}{2}}$ in question 2 fit the pattern in the table?
2. Does your estimate for $9^{\frac{1}{2}}$ in question 2 fit with the rules for working with exponents?

Review some of the exponent rules on the next page and then return to this question.

**Recall:**

**EX:** $5^{2}∙5^{4}=\left(5∙5\right)∙\left(5∙5∙5∙5\right)=5∙5∙5∙5∙5∙5=5^{6}$

 **EX:** $\left(4^{3}\right)^{2} = \left(4∙4∙4\right)^{2} = \left(4∙4∙4\right)∙\left(4∙4∙4\right)= 4∙4∙4∙4∙4∙4= 4^{6}$

1. Simplify each expression by writing out what it means first. Leave your answer in exponential form.

a. $6^{4}∙6^{3}$

b. $\left(9^{3}\right)^{2}$

c. $2^{3}∙2^{5}$

d. $3^{4}∙3^{7}$

e. $\left(8^{5}\right)^{3}$

f. $\left(12^{2}\right)^{4} $

1. Describe the two ‘rules’ for working with exponents that you see in the patterns above.
2. Based on the rules, what should $\left(9^{\frac{1}{2}}\right)^{2}$mean? What should be its value?
3. Return to question 4. Does your estimate for $9^{\frac{1}{2}}$ in question 2 fit with the rules for working with exponents? Does that fit with the pattern in the table?
4. Discuss these ideas with your class and come up with your final estimate for the meaning of $9^{\frac{1}{2}}$.
5. What is the meaning of $25^{\frac{1}{2}}$? What is the meaning $49^{\frac{1}{2}}$? Check your answers on a calculator. (Hint: Because the calculator uses the order of operations you will need to enter 25^(1/2) for $25^{\frac{1}{2}}$.)
6. You have seen that exponential growth may be modeled with the function $f\left(x\right)=ab^{x}$, where *a* is the initial value and *b* is the growth factor. Suppose the number of bacteria in a laboratory beaker after *x* days is modeled by $f\left(x\right)=100∙4^{x}$. Find how many bacteria are in the beaker:

 a. after 3 days. b. after $\frac{3}{2}$ days. c. after $\frac{1}{2}$ day.