**Unit 4: Investigation 1 (2 Days)**

**WHAT MAKES A FUNCTION LINEAR?**

**CCSS:** FLE-1, FIF-7A

**Overview**

Students distinguish non-linear functions from linear functions by exploring distance as a function of time in verbal, graphical and tabular form. Students learn that linear functions are characterized by a constant rate of change.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Interpret distance-time graphs and tables in terms of the motion of an object.
* Write a verbal description of a distance-time function, sketch its graph, and construct a table of values.
* Distinguish between linear and non-linear functions by recognizing that linear functions have a constant rate of change whether the function is given verbally, graphically, or in table form. (Note: Calculation of slopes will be developed more fully in Investigation 3.)
* Identify distance-time functions with slopes of different magnitudes from the verbal description, the graph, and the table.
* Distinguish between distance-time functions with positive slopes (increasing functions) and functions with negative slopes (decreasing functions) given a verbal, graphical or tabular representation of the function.

**Assessment Strategies: How Will They Show What They Know?**

**Exit Slip 4.1.1** asks students to draw qualitative graphs of someone walking with the motion detector.

**Exit Slip 4.1.2** asks students to draw quantitative graphs of someone walking with the motion detector given speed as well as direction.

**Journal entry** asks students to draw a distance-time graph based on a story.

**Launch Notes**

The theme of this investigation is motion. To stimulate student interest in the activity, you may play the video of Carole King singing her classic song “Do the Locomotion” with Slash in concert as students enter class. The video is located at <http://www.youtube.com/watch?v=ehVaMey2e7A&feature=related> . Locomotion refers to how animals, including people, move.

Ask the students to guess what today’s lesson is about as you show any of the several videos available on the internet about how fast and/or how slowly animals move. A quick internet search for “fastest animals” videos yielded:

* A TED video by scientist Sheila Patek showing how she measures the speed of the feeding strike of a shrimp using slow motion film <http://www.ted.com/speakers/sheila_patek.html>,
* Another TED video by Paralympics runner and model Aimee Mullens at <http://www.ted.com/talks/lang/en/aimee_mullins_prosthetic_aesthetics.html>,
* A cheetah at the Cincinnati Zoo, <http://www.thetravelalmanac.com/lists/videos/animals-speed.htm> or in the wild <http://www.extremescience.com/cheetah.htm> .
* If you use to the Futures Channel, you will find the Maglev Train <http://dsc.discovery.com/videos/extreme-engineering-season-1-shorts-maglev-train.html>

Choose your motion video according to your students’ interests or the interests that you want to encourage among your students. Next, challenge students to find how fast a person walks on average, leading them to understand the units of speed to be units of distance divided by units of time. To test their hypotheses, show them how to create a distance-time graph first by gathering data by hand, then by using a motion detector. Be sure to display tables and graphs of a variety of walks: slow, fast, forward, backward, constant rate, standing still, slowing down, or speeding up. The students will analyze the relationship between the graph, the tables, and the verbal description of the person’s motion. During the lesson you can test the students’ hypotheses about how fast a person walks, on average, by looking at the time-distance table and by observing the steepness of the distance-time graph.

**Closure Notes**

This investigation culminates with students being able to produce graphs and descriptions for distance-time functions. Students will identify that a constant rate of change creates a linear graph. Students will understand that steepness of a linear graph is determined by the speed of the object. They will identify increasing, decreasing and constant lines as corresponding to objects that move away from the motion detector, come toward the motion detector, or stand still, respectively.

**Teaching Strategies**

1. Engage the class in a discussion about the velocity of people, animals, or the objects depicted in the launch videos. You may inform the class that they are going to be studying the velocity of their own walk during these activities. The magnitude and direction of velocity correspond to the two independent questions you can ask the students: how fast is the object moving, and in what direction is the object moving? Slope need not be defined yet, because Investigation 3 will formalize the concept of slope.

Depending on the needs of your class, you can start with the motion detector lesson right away, or you can first have students gather and plot distance-time data by hand before you show them the motion detector. To demonstrate how to measure and display distance as a function of time without a motion detector, have one student walk very slowly and at a constant rate in a line along a tape measure spread on the floor or along chalk marks that indicate feet or meters on the floor. Have another student call out “time” every two or three seconds, and have a third person observe or mark with a chalk the position of the walker along the measured line. Record the time and distance data in a table, using function vocabulary students learned in unit 3. Then describe the person’s walk, and create a distance-time graph by hand. Ask students to determine, on average, how far the person walks in 6 seconds, and in 1 second.

To use the motion detector you will need either a computer or graphing calculator with classroom display so that the class can see the motion graph of a person walking. Explain to the students that the motion detector works using the principles of sonar, so they will hear the detector emit a series of clicking sounds that will then bounce off the object it points toward. The equipment will measure how much time elapses between emitting the sound and receiving the sound echo.

Draw set of axes for a first quadrant graph on the board. Let time be represented on the *x-*axis and distance from the motion detector on the *y*-axis. Ask for a volunteer and have the student stand at the motion detector and walk away from it when you tell them to start. Trace the path created by the volunteer and the *x*- and *y*-axes on the board. Ask the class if someone can walk a steeper line and have a volunteer try. Trace this path on the same set of axes. Ask the class if someone can walk a less steep line and have a volunteer try. Trace this path. Ask the class what determined how steep the line was? If the path is a line, then ask what about the person’s walk created the line. If it isn’t a line, then ask why not and what might we change so that the path is a line.

Now collect data from a variety of walks, encouraging the class to ask creative questions, test their hypotheses, and generally explore distance-time graphs with the motion detector. You may lead the discussion by asking the following questions and allowing students opportunities to imitate certain motions.

* How could a walker create a line that slants from the bottom left to the top right?
* How could a walker create a line that slants from the top left to the bottom right?
* Why do we talk about the line going from left to right?
* How they would create a steeper line or a less steep line?
* What determines the steepness of the line?
* What does it take to walk a straight line?
* What is the domain and range from the graph?
* Can a person walk a wave?

Have the students calculate how fast one of the students walked by using the table of values or by using the trace feature on the graph of the line to display coordinates of two points.

1. **Activity 4.1.1 Motion Graphs – What Makes a Function Linear?**, students predict what distance-time graphs will look like before collecting distance-time data using a motion detector. Students can use a motion detector to verify that they have the correct graphs in question 3. This activity can be completed without using a motion detector. By the end of the activity, students should have concluded that a person walking at a steady pace will produce a graph of a line; faster walks produce steeper lines; walking forward creates an increasing function and backwards a decreasing one, standing still produces a horizontal line; and from the table you can determine the average velocity of the person walking.

**Exit Slip 4.1.1** and **Activity 4.1.2 Motion Graph Scenarios** can be assigned in class or for homework to reinforce and assess student understanding.

Alternative Activity If Motion Detectors Are Not Available: If you do not have a motion detector, you may want to discuss materials and data collection ideas with the physics teacher at your school. Then you can adapt the problems from the motion detector part of this investigation by having students roll a ball or toy car at different speeds along a paper tape on a table, marking the position of the object every few seconds. For example, one could lay a strip of paper (such as a cash register roll of paper) alongside the path of something that moves such as a rolling ball or a toy car. As the object moves along the paper tape, one person marks the position of the object with a dot as the timer calls out the seconds. The disadvantage of this method is that linear data is not possible since the ball or toy car will slow down unless you have a motorized toy.

Another way to collect distance-time data is to fix the distances (the dependent variable) and record the time (the independent variable) as a person or toy reaches a certain distance marker. One could mark distances on a floor or table with a tape measure, and then record the time when the object reaches every 6 inches or 10 cm. Observe what happens to the distance between dots as the object speeds up or slows down. Note that dot pattern of objects moving at constant velocity will give equally spaced dots; and the distance between dots divided by the interval of time gives the velocity of the object. An object that accelerates will have dots further and further apart, and an object that slows down will have dots closer together. A strobe photograph of a drip coming out of a faucet gives analogous information about the speed of the falling drip.

1. **Activity 4.1.3** **More Motion Graphs** provides students additional practice with distance-time functions. As students complete this activity, you may use Data Match, a built in application on the Logger Lite and the TI-calculator motion detector programs. Data Match allows students to attempt to walk in such a way that produces a graph identical to one displayed on the screen. Students should be looking at the screen as they produce their walks so that they can make adjustments during their walk. The graphs in Data Match are piecewise linear functions. When using Data Match, prompt each student group to first describe the walk to be matched, present their description to the class for discussion, and then send a representative to the front to test out their idea with the motion detector in front of the class. As you circulate among the groups, elicit answers to the following sample questions:

* Where is walker going to start?
* What do we know about the distance and time from the graph?
* What is the walker going to do? What’s her plan?
* How fast should she walk?
* How could we figure out exactly how fast she should walk?
* What is a situation in real life that could be modeled by a graph that looks like this?

Whole class discussions can be lively as students argue for and refine their ideas about how to describe each walk for the data matches. Assess whether students are able to describe the underlying mathematical concepts of constant rate of change or describe the magnitude and direction of the rate of change from data tables and graphs.

Ask the class if they could determine the speed at which the person walked and if so, how. Using the table feature, display the data points that correspond with the last graph created and have the students calculate the walker’s rate. Some students may make the connection that they are actually calculating the slope. Explain that slope is the rate at which distance changes as time changes. Pay attention to the units of measure of rate: meters per second, or feet per second. Connect the idea to the units on the vertical and the horizontal axis of the distance time graph. Ask students if they remember the formula RT=D, and have them solve for R.

Ask the class to create a horizontal line. Why did their approach create or not create a horizontal line? What must be true to have a horizontal line and why? Challenge the students to make a horizontal line higher on the display or lower. What is true about the horizontal lines (they are parallel)? Ask the students to calculate the rate of change of the person who created a horizontal line. Make the connection that the rate of change or slope of a horizontal line is zero.

Ask the class to create a vertical line. Many students will have ideas of how to do this, but ultimately they will not be successful. Let them experiment with this. Challenge the class to explain why this cannot be done. Have a discussion about the fact that it is physically impossible to be in two places simultaneously. Connect this with the idea that a vertical line is not a function. Ask the class what the slope of a vertical line would be. Make the connection that the slope of a vertical line is undefined. (See question 4 in **Activity 4.1.3**.)

Continue with questions 5–7 in **Activity 4.1.3,** which direct students to think more precisely about rate of change in terms of meters per second by graphing distance and time on a coordinate plane with precise values labeled on the distance and time axes.

Assess student understanding at the end of the class by displaying a new graph from Data Match and having students write an exit slip describing the walk of the person that would match that graph. Or, you may have students do complete **Exit Slip 4.1.2** requiring them to sketch a graph given a verbal description of a person’s walk. **Activity 4.1.4 Stories and Graphs** may be assigned for homework.

Feel free to modify the activities so that they contain topics of greater interest to your students. Optional ideas include asking students to give a rough description and graph a common destination, such as the walk or car ride from school to a favorite soda shop, or an amusing occurrence, such as a person leaving home heading to school, discovering they left their homework on the kitchen table, returning home, and then proceeding to school at a faster rate.

**Differentiated Instruction (For Learners Needing More Help)**

Before asking students to describe the motion depicted in a graph, have students brainstorm a list of words to choose from such as: move steadily, speed up, slow down, walk at a constant rate, move away from the motion detector, move toward the motion detector, increasing/decreasing the distance from the motion detector, velocity, steep line, flat line, etc.

To facilitate students trying to translate from one representation of a function to another, you may provide four verbal descriptions, four graphs and four tables, and ask students to match the descriptions, graphs and tables.

Have students annotate a graph with descriptions of the walk or movement rather than write a paragraph describing the walk.

**Differentiated Instruction (Enrichment)**

Challenge students to show how to walk a lower case letter “m”.

Can you make an upper case letter “M” with sharp angles? (No, because you can’t change direction instantly – you always need to slow down turn change direction.) What letters can you walk? Students should notice which letters are functions, which ones aren’t, which have sharp points, and which are rounded.

When students compute velocity by looking at the table for time distance data, a) have them compute the rate of change between each time interval, then have them compute the averages of these rates, and b) have them convert feet per second to miles per hour (or meters per second to kilometers per hour.)

Alternatively, you can distribute the **Activity 4.1.5** **Motion Graph Challenge Problems.**

**Group Activity**

Data match can be done as a group activity whereby each group discusses their ideas about how to walk to match a given graph, writes down their idea and then tests the idea with the motion detector.

Collecting distance-time data by hand can be done in groups. Students can go into a hallway to time and measure distances they walk. Have them record the data for distance as a function of time in table form, then draw a graph by hand.

If you have access to several motion detectors, you might choose to have students work in small groups to “do the walks”.

**Journal Entry**

Sketch a motion-time graph of your teacher’s drive to school this morning. Your teacher describes their drive below:

“I started out at 6:30 AM, driving on local streets. Just before I got onto the highway, I stopped at the convenience store to get coffee for $1.05 because I use my eco friendly reusable travel mug (cream no sugar, thank you). After a minute or two on the highway, I realized I forgot my backpack, returned home, retracing my path. With my backpack next to me, I finally get back onto the highway, and was enjoying NPR Morning Edition, when I saw police helping a distressed motorist whose car had broken down in the right lane. All of the traffic was forced to slow down considerably for a short while. Soon enough, I was able to go the normal speed limit of 65mph. The school is only 5 minutes from the highway, so I arrived at school at 7:30, too late for homeroom, but just in time for algebra class.”

**Resources and Materials**

* **Activity 4.1.1** Motion Graphs-What Makes a Function Linear?
* **Activity 4.1.2** Motion Graph Scenarios
* **Activity 4.1.3** More Motion Graphs
* **Activity 4.1.4** Stories and Graphs
* **Activity 4.1.5** Motion Graph Challenge Problems
* **Exit Slips 4.1.1** and **4.1.2**
* Motion detector such as those by Vernier, together with either (a) a calculator with view screen to project to class OR (b) a computer that projects to entire class.

The calculator software is under the APPS key. You may see CBL/CBR or Ranger, for example. You can also download programs like RANGER, HIKER or DATA MATCH from the website [www.Education.TI.com](http://www.Education.TI.com) .

The software for the computer called LOGGER LITE is free on the Vernier website <http://www.vernier.com/products/software/logger-lite/> . To connect the motion detector to the computer, you will need a printer cable commonly used for many printers. The same printer cable is also sold as the Go Motion cable at Vernier.

* Timer, tape measure and chalk to mark floor location as person walks
* Bulletin Board for key concepts
* Student Journals
* <http://illuminations.nctm.org/LessonDetail.aspx?ID=U188> for an NCTM Illuminations lesson entitled “Movement with Functions”
* Time-Distance lessons under “Classroom Activities” on the Texas Instruments website: <http://education.ti.com/calculators/downloads/US/Activities/Search/Keywords?k=time+distance+graphs>
* Workbooks from Texas Instruments such as activities 1 and 13 in *Real World Math Made Easy* by Chris Brueningsen et. al. or *CBR Explorations: Math and Science in Motion* by Brueningsen et. al. or *CBR Explorations: Modeling Motion: High School Activities with the CBR* by Linda Antinone et. al.