**UNIT 2 INVESTIGATION 3 (4 days)**

**Combining Like Terms to Solve Equations**

*CCSS: 8EE 7, A-SSE 3, A-CED 1, A-REI 1, A-REI 3*

**Overview**

Students solve multi-step equations in a variety of real-life contexts. To solve the equations students must combine like terms on one side of an equation, and collecting variable terms on one side and collect constants on the other side. Students also solve equations that have no solution or an infinite number of solutions.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

Students will be able write linear equations that model real world scenarios, solve equations with variables on both sides, and justify their steps using the properties of equality. Students will also recognize equations for which there is no solution or an infinite number of solutions.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 2.3.1** asks students to model a contextual problem with a linear equation and then solve the equation, and to solve a multi-step equation. To solve both equations students must combine like terms.
* **Exit Slip 2.3.2** asks students to identify the steps which occurred in the process of solving an equation.
* **Journal Entry 1** asks students to identify how the commutative and associative properties allow us to solve a multi-step linear equation.
* **Journal Entry 2** asks students to explain how they can tell if an equation has no solution or an infinite number of solutions.

**Launch Notes**

Inform students that they will encounter equations which require that they combine like terms. To describe the process of combining like terms, you may introduce this investigation by one of the following discussions.

1. Discuss a scenario involving students attending a concert. Tickets cost $30 but there is also a $5 for handling. Suppose *n* students attend the concert. Write an expression for how much money they will spend altogether. There are two possible approaches. One is to find the cost for one person, which is $35. The total spent is therefore 35*n* dollars. The other approach is to find how much is spent for the tickets (30*n* dollars) and add it to the amount spent on handling fees (5*n* dollars). Hopefully both methods will be suggested; if not, you can lead a discussion to discover the other one. Conclude that the two algebraic expressions, 35*n* and 30*n* + 5*n* represent the same situation. Inform students that in this investigation we will combine like terms to simplify expressions when we solve equations.
2. Ask students why 2*x* + 3*x =* 5*x*? Is this really true for all values of *x*. Use a table to show that 2*x* + 3*x =* 5*x*, for every value of *x*.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***x*** | **2*x*** | **3*x*** | **2*x + 3*x** | **5*x*** |
| 3 | 6 | 9 | 15 | 15 |
| 7 |  |  |  |  |
| -5 |  |  |  |  |
| -2 |  |  |  |  |
| 0 |  |  |  |  |

Students can discover that for whatever values are chosen for *x* in the first column, the values in the last two columns are the same. Discuss other examples of combining like terms such as $8x-5x=3x$ or $x-6x=-5x$. Use these examples to introduce the procedure “combining like terms.” Stress the fact that when we combine like terms we have two *equivalent* expressions containing the same variable(s). This means that no matter what values are substituted for the variable, the values of the expressions are equal.

**Closure**

At the end of this investigation we may step back and look at the bigger picture. Much of what we do in solving an equation is to find a simpler equation. For example, combining like terms makes the expression on one side of the equation simpler. When a variable appears on both sides of an equation, we can eliminate the variable from one side. This also makes an equation simpler. The underlying problem solving strategy which we have used is sometimes called “solve a simpler problem.” Explain to students that even though some equations may look complicated, if we keep this principle in mind we can often transform a complex equation into a simpler equation.

**Teaching Strategies**

1. In **Activity 2.3.1 Combining Like Terms with Algebra Tiles**, students use algebra tiles to simplify algebraic expressions by combining like terms. Students could use algebra tiles or draw pictures of algebra tiles to model and simplify expressions.

**Differentiated Instruction (For Learners Needing More Help)**

Algebra tiles (**Activity 2.3.1**) provide students a hands-on activity for modeling and simplifying algebraic expressions.

**Group Activity**

Students can work together in small groups using algebra tiles (**Activity 2.3.1)** to model and simplify algebraic expressions.

1. In **Activity 2.3.2 Solving Equations that Contain Like Terms**, students model contextual problems with linear equations of the form $ax+bx=c$ or the form $ax+bx+c=d. $ The first problem in this activity asks students to solve an equation using a table, by graphing the expressions on both sides and finding the intersection point, and by solving the equation.

 **Note**: If you still have students who use the flow-chart method for solving equations, that is fine. Make sure to advise them they can continue to use that method, but they must first simplify the equation (distribute, combine like terms, and get the variable on only one side of the equation) *before* they put the equation into the flowchart.

1. In **Activity 2.3.3 Solving Equations with Variables on Both Sides**, students model contextual problems with linear equations of the form $ax+b=cx+d.$ The first problem in this activity asks students to solve an equation using a table, by graphing the expressions on both sides and finding the intersection point, and by solving the equation. The final problem allows students an opportunity to create their own an equation and solve it.

Prior to distributing this activity, you may present students the following **strategy** for solving multi-step equations that contain variable terms on both sides.

1. *Simplify each side of the equation by combining like terms.*
2. *Collect all the variable terms on one side, and collect all constants on the other side.*
3. *Multiply or divide by the same number on both sides to solve for x.*

Discuss how to solve an equation with variables on both sides. Show an equation like $3x+10=7x-6$. Ask the class, “What can we do to collect the *x* terms on one side? You may want to show that the above equation may be solved in at least two ways.

|  |  |
| --- | --- |
| $$3x+10=7x-6 $$ – 3*x* – 3*x*$$10=4x-6$$ + 6 + 6$$16=4x$$$$\frac{16}{4}=\frac{4x}{4}$$$$4=x$$ | $$3x+10=7x-6 $$ – 7*x* – 7*x*$$-4x+10=-6$$ – 10 – 10 $$-4x=-16$$$$\frac{-4x}{-4}=\frac{-16}{-4}$$$$x=4$$ |

 Students should be able to justify each step in terms of the addition, subtraction, multiplication, and division properties of equality. Most students will prefer the first method and will chose the first step so that the coefficient of the *x* term is positive. Also, point out that the associative and commutative properties of addition allow us to combine like terms on each side of an equation. For example, the equation

 1 – 2*w* + 3 = –8 can be rewritten as (1 + 3) – 2*w* = –8. This simplifies to 4 – 2*w* = –8.

**Group Activity**

Put students in small groups to write their own step-by-step procedures for solving multi-step equations. Then have them share their procedures with the class.

1. In **Activity 2.3.4 Practice Solving Equations**, students solve non-contextual multi-step equations.
2. In **Activity 2.3.5 Solving Equations with Balance Scales**, students are introduced to an online virtual manipulative, *Algebra Balance Scales*, at the National Library of Virtual Manipulative (NLVM) website. Students can use the applet to model and solve linear equations of the form $ax+b=cx+d$. The applet emphasizes keeping an equation balanced through performing the same operation to both sides of an equation.

**Differentiated Instruction (For Learners Needing More Help)**

Algebra Balance Scales (**Activity 2.3.5**) introduces students to a visual manipulative for solving multi-step equations. The applets are located on the National Library of Virtual manipulative (NLVM) website. One applet only models equations with positive coefficient, and the other models equations with negative coefficients.

1. In **Activity 2.3.6 How Many Solutions**, students practice solving equations which contain no solutions or infinite number of solutions. Before distributing the activity, you may introduce this topic by asking students to solve the following equations:

 *Solve*:

 (1) 5*x* – 6 = 5*x* – 2 – 4

 (2) 94 + 22*x* = 670 – 20*x* – 30*x*

 (3) 2*x* + 8 – 14*x* = –12*x* + 2

 Have a class discussion about what happened with each equation.

 For equation (1), have students input $Y\_{1}=5x-6$ and $Y\_{2}=5x-2-4$ into their graphing calculators. Then, using the ASK feature in the table, have students evaluate both functions (*Y*1, *Y*2) try different for several whole numbers, integers, decimals and fractions . They should see that for all values of *x* the two expressions are equal. The solution of equation (1) is the set of all real numbers. We call this type of equation an **identity**.

 Repeat the process for equation (2). In most cases the two sides of the equation are not equal. However if you try *x* = 8 you will find that *Y*1 and *Y*2 are both equal to 270. This is the typical situation in which we find **one unique solution** to the equation.

 Repeat the process for equation (3). When we make a table for this equation, we find that the two expressions are never equal. In fact they always differ by 6. This type of equation is called a **contradiction**.

 Students will notice that in solving equations (1) and (3) the *x*-terms end up “cancelling out” to give 0*x*. For equation (1) in the last step we have 0*x* = 0. Since 0 times any number is equal to zero, every possible value of *x* will satisfy this equation. For equation (3) we end up with 0*x* = –6. There is no value of *x* that satisfies this equation.

1. In **Activity 2.3.7 Comparing Cab Fares**, students model the cost of using several taxicab companies and identify when two companies charge the same amount. The equations are of the form $ax+b=cx+d$.

**Differentiated Instruction (For Enrichment)**

Some students may want to consider problem situations which lead to equations whose solutions are valid mathematically, but do not make sense in the context of the problem. For example, in **Activity 2.3.7,** suppose the fuel surcharge for Fast Cabs is $6.00 (and all other charges are the same). For how many miles will the cost of a ride with Fast Cabs be equal to the cost of a ride with Speedy Cabs? The equation would be 0.7*x* + 5.5 = 0.95*x* + 6. The solution is *x* = –2, which does not make sense. For all non-negative values of *x,* Speedy Cabs is the less expensive option. Challenge students to modify some of the other problems encountered in this investigation to produce similarly inappropriate “solutions.”

**Journal Entry**

1. When you solve 5 + 2*x* + 7 = 5*x* + 4, how can the commutative property and the associative property for addition help you simplify the left side of the equation?
2. How can you tell if an equation has no solution or an infinite number of solutions?

**Resources and Materials**

* **Activity 2.3.1** Combining Like Terms with Algebra Tiles
* **Activity 2.3.2** Solving Equations that Contain Like Terms
* **Activity 2.3.3** Solving Equations with Variables on Both Sides
* **Activity 2.3.4** Practice Solving Equations
* **Activity 2.3.5** Solving Equations with Balance Scales
* **Activity 2.3.6** How Many Solutions
* **Activity 2.3.7** Comparing Cab Fares
* **Exit Slip 2.3.1** Tickets for a Play
* **Exit Slip 2.3.2** What Did They Do?
* Student Journals
* Graphing calculators
* Algebra tiles
* *Algebra Balance Scales* (NLVM) applet (positive coefficients):

<http://nlvm.usu.edu/en/NAV/frames_asid_201_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html>

* *Algebra Balance Scales* (NLVM) applet (negative coefficients):

<http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html>.