Connecticut Common Core Algebra 1 Curriculum

Professional Development Materials

Unit 6 Systems of Equations

Contents

* These items appear only on the password-protected web site.

Will the Women Catch the Men?

Income Bracket Earnings by Gender (2002 – 2005)

Number of years since 2000	% of men earning $50,000 - $74,999	% of women earning $50,000 - $74,999
2	20.1	13.0
3	20.2	13.3
4	20.5	14.2
5	20.7	15.1

Source: U.S. Census Bureau

Based on the data given, will the percent of women earning $50,000 to $74,999 ever catch up to the percent of men earning $50,000 to $74,999? And if it will, when?

In the space below, write notes based on the class discussion.

Will the Women Catch the Men?

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Analyzing the Data

1. What is the independent variable?

2. What is the dependent variable?

3. Draw scatter plots for both the men and the women on the same set of axes. (You may use a calculator, the graph at the right, or another sheet of graph paper.)

4. Fit a trend line for the men.

5. Fit a trend line for the women.
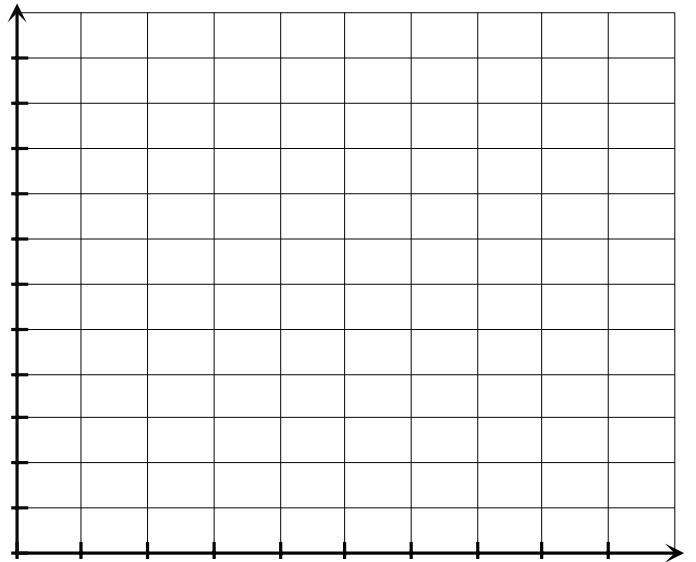
6. Find the point of intersection for the two trend lines.

7. Interpret the meaning of the point of intersection.

8. Make a prediction: will the percent of women ever equal the percent of men earning $50,000-$74,999. If so, when?

Choosing a Gym

1. There are two gyms in Groton, CT advertising promotional plans. The membership plans are as follows:

   Gym A: $120 one time membership fee plus $10 per month

   Gym B: No membership fee and $20 per month

   a. Mark's father is stationed at the Naval Submarine Base for the next nine months and his family lives in Groton. Which gym should Mark join during their nine months in Groton? Why?

   b. José is planning to attend the University of Connecticut at Avery Point to earn his Bachelor's Degree in Coastal Studies. José will live in off-campus housing in Groton and he is trying to decide which of the two gyms to join during his four years of college. Which gym should José join? Why?

   c. For how many months would it be best to join Gym A? When would it be best to join Gym B? Explain.

   d. When would it not matter which gym a person joined? Explain how you made your decision.

2. Susan is trying to choose between two gyms. Susan loves Zumba (the latest cardio/aerobic craze) and must decide between two gyms that both offer Zumba. The gyms in her neighborhood offer the following membership plans:

    Phoenix Gym: $40 one time membership fee, $10 one time towel fee, and $25 per month
    Rocker Spa: $50 one time membership fee, $15 per month for general membership, and an additional $10 per month for Zumba classes

  a. If you had to choose a gym without doing any math, which would you choose? Why?

  b. Write the equations for both gyms. Think about the meaning of each slope in context of the problem.

<div align="center">

Phoenix Gym   Rocker Spa

</div>

  c. Solve the system by graphing.
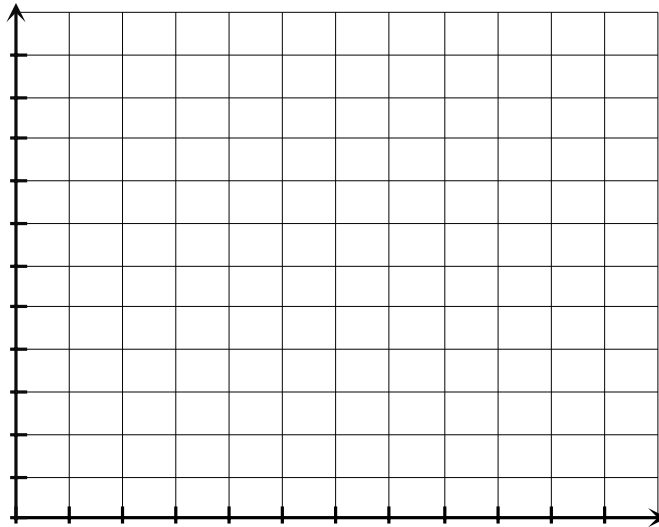
  d. What is the solution?
  e. Which Gym should Susan join? Why?

3. Natasha is also trying to decide between two gyms, but her passion is kickboxing. Should she join Ellie's Gym, a typical gym with kickboxing classes, or The Kick Box, a kickboxing gym, if she only wants to attend kickboxing classes and wants the best deal?

> **Ellie's Gym**: $60 one time membership fee and $10 per month for general membership and an additional $10 per month for kickboxing classes.
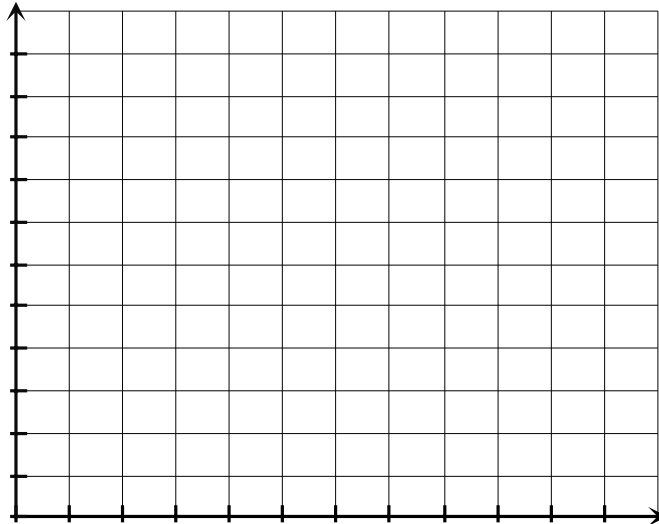> **The Kick Box**: $75 one time membership fee and $20 per month

a. If you had to choose a gym without doing any math, which would you choose? Why? (no wrong answers)

b. Write the equations for both gyms. Think about the meaning of each slope in context of the problem.

<u>Ellie's Gym</u> <u>The Kick Box</u>

c. Solve the system by graphing.

d. What is the solution?
e. Which gym should Natasha join? Why?

Passing on the Gift

One of the major tenets of the Heifer Project is "Passing on the Gift". In this lesson you will learn what gifts cost, and how many gifts will be paid forward next year.

1. Village Gardens in Portland, Oregon is a group of public housing residents, many of whom are refugees or immigrants. Heifer International will provide the group goats for milk and chickens for eggs. Each goat costs $120 and each flock of chickens cost $20. Portland's middle school and high school musicians raised $1,000 by putting on a charity band and orchestra concert for Village Gardens. The village has requested 10 gifts, so the total number of goats and flocks of chicken will be 10. How many goats and flocks of chickens will be sent to the village?

    a. Define the variables in this problem.

    b. Develop an equation that represents the number of goats and chickens.

    c. Develop an equation that represents the cost of the goats and chickens.

    d. Solve the system of equations using the substitution method.

2. Eight animals will be sent to Cameroon. The animals will be cows and pigs. A cow costs $500 and a pig costs $120. The total cost of all animals is $2,100. Find how many of each animal will be sent to Cameroon.

    a. Define the variables in this problem.

    b. Develop an equation that represents the number of cows and pigs.

    c. Develop an equation that represents the cost of the cows and pigs.

(Problem 2 continued)

    d. Solve the system of equations using the substitution method.

3. This year, a family in Kosovo that received goats and chickens from Heifer plans to breed their animals to produce 351 new animals to share with their neighbors. One goat produces 3 offspring per year, and one chicken produces 42 chicks a year. In addition to breeding the animals, the family will gain 88 servings of milk and eggs from the animals each day. One goat will yield 16 cups of milk per day, and one chicken will produce 1 egg a day. A serving consists of one cup of milk or one egg. The milk that they don't drink may be made into yogurt and cheese. Any eggs or milk that they don't use may be given to a food bank or sold to supplement family income. How many goats does the family receive from Heifer to start the breeding program? How many chickens did the family have at the start?

    a. Define the variables in this problem.

    b. Develop an equation that represents the number of offspring from the goats and chickens each year.

    c. Develop an equation that represents the number of servings of food from the goats and chickens.

    d. Solve the system of equations using the substitution method.

Solving Systems by the Substitution Method

1. Solve the following systems of equations by the substitution method. Show your work.

   a. $8x + 5y = -14$
       $y = -3x$

   b. $6x - 4y = 38$
       $x + y = 5$

   c. $x - 3y = 9$
       $6x - 5y = 2$

2. To eliminate the problem of World Hunger we need to understand what constitutes nutritious meals. This is important in order to ensure that people eat the food they need to stay healthy. The average 128-pound person needs to consume about 36 grams of protein and 2,200 calories each day.

    An egg has 6 grams of protein and 77 calories. A 6-ounce glass of goat's milk has 8 grams of protein and 168 calories. If a person has enough eggs and goat's milk each day, they will meet the required 36 grams of protein per day. How many eggs and how many 6-ounce glasses of goat's milk will a person need to eat each day if they want their total protein intake to be 36 grams and their total calorie intake from these two sources to be 658 calories? (Assume that there are other foods available that do not contain protein but will supply the remaining calories needed.)

    a. Develop an equation that represents the number of calories consumed from eggs and goat's milk each day.

    b. Develop an equation that represents the amount of protein consumer from eggs and goat's milk each day.

    c. Solve the system of equations using the substitution method. Explain what your solution represents in the context of the problem.

3. A serving of rice contains 251 calories, and a serving of beans has 227 calories. How many servings of rice and how many servings of beans will provide a total of 2,187 calories if the person eats twice as many servings of rice per day as beans?

   a. Define your variables and then write an equation that represents the number of calories consumed from rice and beans each day.

   b. Write an equation that represents the number of servings of rice and beans consumed each day.

   c. Solve the system of equations using the substitution method. Explain what your solution represents in the context of the problem.

More Practice with the Substitution Method

Solve each system of equations using the substitution method.

1.
$$3x + y = 15$$
$$y = 3$$

2.
$$4a - b = 20$$
$$a = 4$$

3.
$$c + 3d = 23$$
$$d = -10$$

4.
$$j = 2k$$
$$j + 3k = 5$$

5.
$$6m - n = -5$$
$$n = m + 5$$

6.
$$3g - 2h = 12$$
$$-g + 9 = h$$

7. The perimeter of Mrs. McCord's rectangular garden is 100 feet. This can be represented with the equation $2w + 2l = 100$, where w is the width of the garden (in feet) and l is the length of the garden (in feet). If the length of the garden is 1.5 times longer than the width ($l = 1.5w$), what are the dimensions of the garden?

8. Is $(5,6)$ a solution to the following system? How do you know?

$$3x + 2y = 27$$
$$x = y + 1$$

Break-Even Analysis: Popcorn

The business club is going to sell popcorn at hockey games. Since they are astute business men and business women, they know that they will not make a profit right away because they have to pay the cost of buying a popcorn machine. They need to know how many bags of popcorn they must sell in order to cover the set-up costs. In other words, what is the **break-even point** for their popcorn business?

The red and glass popcorn carts often seen at carnivals and fairs costs $450. This is the **fixed cost**. Regardless of the number of bags of popcorn they make and sell, the machine cost will not change.

1. The popcorn, butter, salt, and serving bags cost $15 for every 100 bags of popcorn. What is the cost per bag for these consumables?

The **variable cost** changes depending on how many bags of popcorn they make. The more popcorn they make, the more they spend on popcorn, butter, salt and bags. The variable cost is $0.15 times the number of bags of popcorn.

The **total cost** is the variable cost plus the fixed cost.

$Total\ Cost = Variable\ Cost + Fixed\ Cost$

2. Write an equation for the Total Cost as a function of the number of bags of popcorn made. Use the notation $C(x)$ for total cost, and let x be the number of bags of popcorn they make.

Each bag of popcorn sells for $1.00. The **revenue** is the amount of money they receive from selling bags of popcorn. If they sell 20 bags of popcorn, they will receive $20, since each bag sells for $1. The revenue they take in is the price per bag of popcorn multiplied by the number of bags of popcorn sold.
$Revenue = (Price\ per\ Item)(Number\ of\ Items)$

3. Write an equation for the Revenue as a function of the number of bags of popcorn sold. Label the revenue function $R(x)$.

The **break-even point** occurs when the amount of money they receive from selling popcorn is equal to the amount of money they spent to make the popcorn. It is when Revenue = Total Cost. The break-even point tells how many items they must create and sell in order to recover their expenses.

4. Take the Total Cost and Revenue functions that you developed above, and sketch the graph of the two functions on one coordinate plane. Label the axes appropriately.
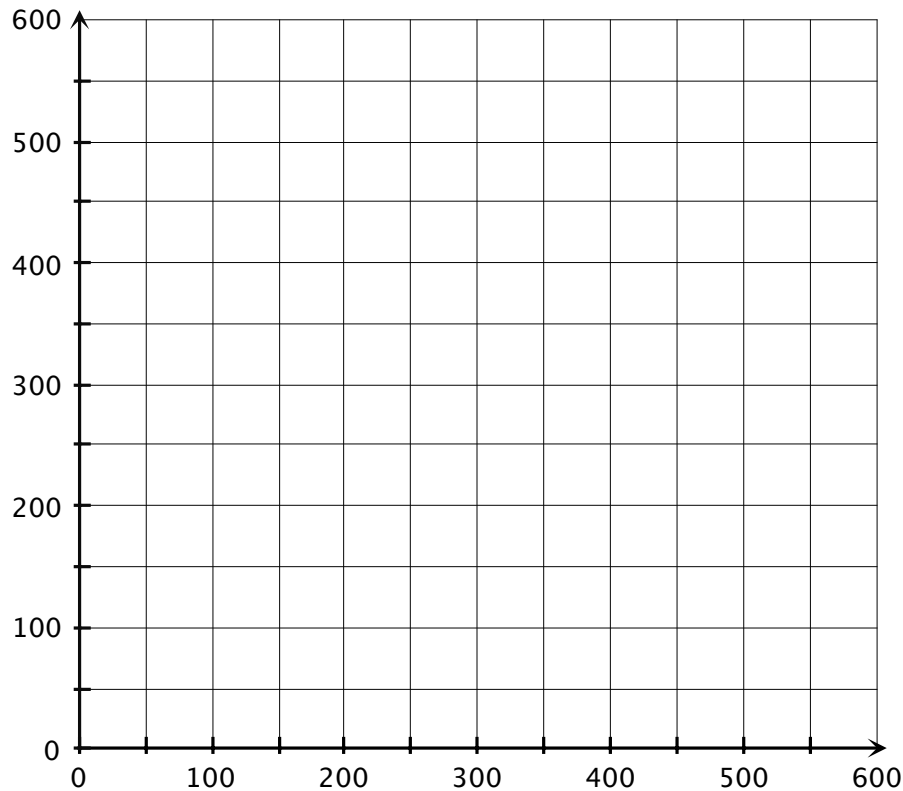
5. Estimate the break-even point graphically.

6. To find the break-even point algebraically, write R(x) = C(x).

7. Solve the equation R(x) = C(x) for x.

8. Check your graphical estimate with your algebraic solution. Explain any difference.

9. Now that you found *x*, what does it mean in terms of the popcorn business?

10. The business will earn a profit when revenue is greater than total cost.

    a. Use an inequality to represent the number of bags of popcorn that must be made and sold to make a profit.

    b. Show on a number line the number of bags of popcorn that must be made and sold to make a profit?
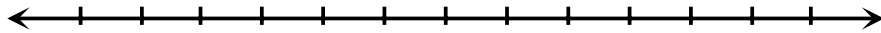
Introduction to the Elimination Method

1. Solve the following problem: The sum of two numbers is 20, and their difference is 2. Find the two numbers.

   a. The system of equations below models this problem if _____ represents the larger number and ____ represents the smaller number.

   $x + y = 20$
   $\underline{x - y = 2}$
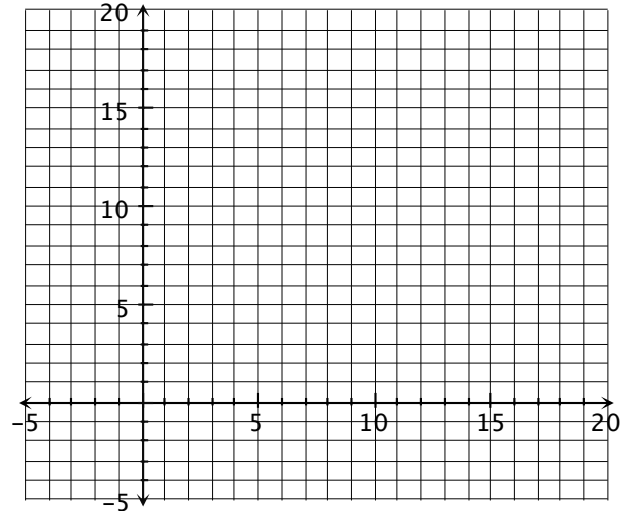
   b. In the space above, add the two equations together (left side added to left side, right side added to right side). You now have an equation in one variable. Which variable is it, x or y?

   c. Solve for the variable in part (b). Then substitute it into one of the original equations to solve for the other variable.

   d. Graph both equations by using the x-intercept and y-intercept on the coordinate plane above. Identify the intersection point.

2. Solve each of these systems by adding the two equations together to eliminate one of the variables.

   a. $-x + 2y = 5$
      $x + y = 7$

   b. $x - 2y = 7$
      $3x + 2y = 13$

3. Consider the following system of equations:

$$5x - 2y = 19$$
$$3x + y = 7$$

   a. What happens when you add these equations together? Is one of the variables eliminated? Show your work below.

   b. Go back to the initial system. Multiply both sides of the second equation by 2. Now add this new equation to the first equation. What happens?

   c. Complete the solution to this system of equations.

4. Write a system of equations for this problem and solve it by elimination:

   The sum of two numbers is 12. When 3 times the smaller number is subtracted from the larger number, the result is 4. Find the two numbers.

5. Find the sum of these two fractions: $\frac{2}{3} + \frac{1}{4}$.

6. Solve this system by first eliminating the variable y. (Hint: You will have to find one number to multiply the first equation by and a second number to multiply the second equation by.)

$2x + 3y = 9$
$5x - 4y = 11$

7. How are questions 5 and 6 related?

8. Explain how you used the addition and multiplication properties of equality to solve systems of equations using the elimination method.

Exploring the Number of Solutions

Solve each system of equations using the elimination method. (Be prepared for some surprises!)
Then graph the system.

1. $3x + 2y = 24$
        $x + 2y = 20$

2. $x + y = 4$
        $3x + 3y = 18$
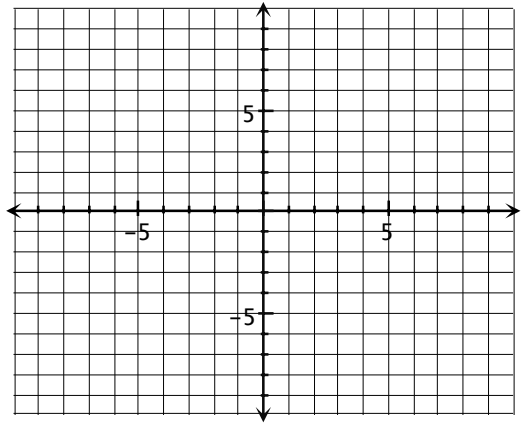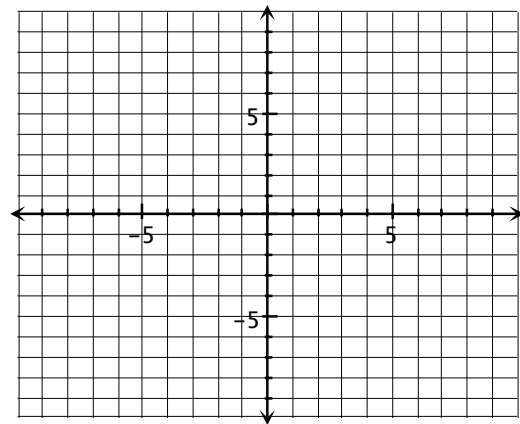
3. $-5x - 3y = 30$
        $10x + 6y = -60$
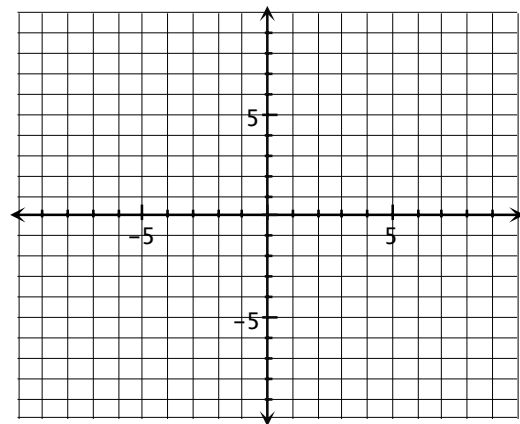
4. Describe what the graphs look like if your system has

    a. No solution

    b. One solution

    c. An infinite number of solutions

5. Describe what happens when you eliminate one variable if the system has:

    a. No solution

    b. One solution

    c. An infinite number of solutions

6. Which system in Questions 1–3 is inconsistent? Which system is dependent? Explain.

Applications of the Elimination Method

For each problem, write a system of linear equations and solve them using the elimination method.

1. You are planning a picnic for Memorial Day. You need to buy enough hot dogs and hamburgers so that each of your 10 guests can have two servings. You determined that hot dogs cost $0.40 each and hamburgers cost $0.80 each. You have $12 to spend on the hamburgers and hot dogs. How many hot dogs and how many hamburgers should you buy?

2. Your family is planning to take the Amtrak train from Hartford to New York City for a day trip. As a result of some research you learn that your friend Jackie took the train with a group of 3 adults and 5 children and it cost them $269.50. A cousin also took the train to the city with a group of 2 adults and 3 children and it cost them $171.50. Find the price of an adult's ticket and the price of a child's ticket.

3. During the 2008-2009 basketball season the UConn women's team had an incredible undefeated season (39-0) and won the NCAA championship. Maya Moore and Renee Montgomery were the top scorers during the year and together they scored 1,398 points. If Maya scored 110 more points than Renee, how many points did each player score during the season?

4. During the 2008-2009 men's basketball season, UConn's Hasheem Thabeet and Jeff Adrien had a total of 746 rebounds. Jeff had 30 fewer rebounds than Hasheem. How many rebounds did Hasheem and Jeff each have during the season?

5. At the upcoming school fair, your class is planning to raise money for a class trip to
    Washington, DC. You plan to sell your own version of Connecticut Trail Mix. After doing
    research on the cost of various ingredients, you find you can purchase a mixture of dried fruit
    for $3.25 per pound and a nut mixture for $5.50 per pound. The class plans to combine the
    dried fruit and nuts to make their unique Connecticut Trail Mix that sells for $4.00 per
    pound. After researching the number of people who attended last year's fair, you anticipate
    you will need 110 pounds of trail mix. Suppose the cost of making 110 pounds is exactly
    equal to the revenue from selling the trail mix. How many pound of dried fruit and how
    many pounds of mix nuts were used?

6. In question 5, how many pounds will you need to sell in order to make a reasonable profit?
    Explain your reasoning.

7. Find a system of equations that you solved in Investigation 2 and now solve it using the
    elimination method.