

# **Connecticut Common Core Algebra 1 Curriculum**

## **Professional Development Materials**

### **Unit 4 Linear Functions**

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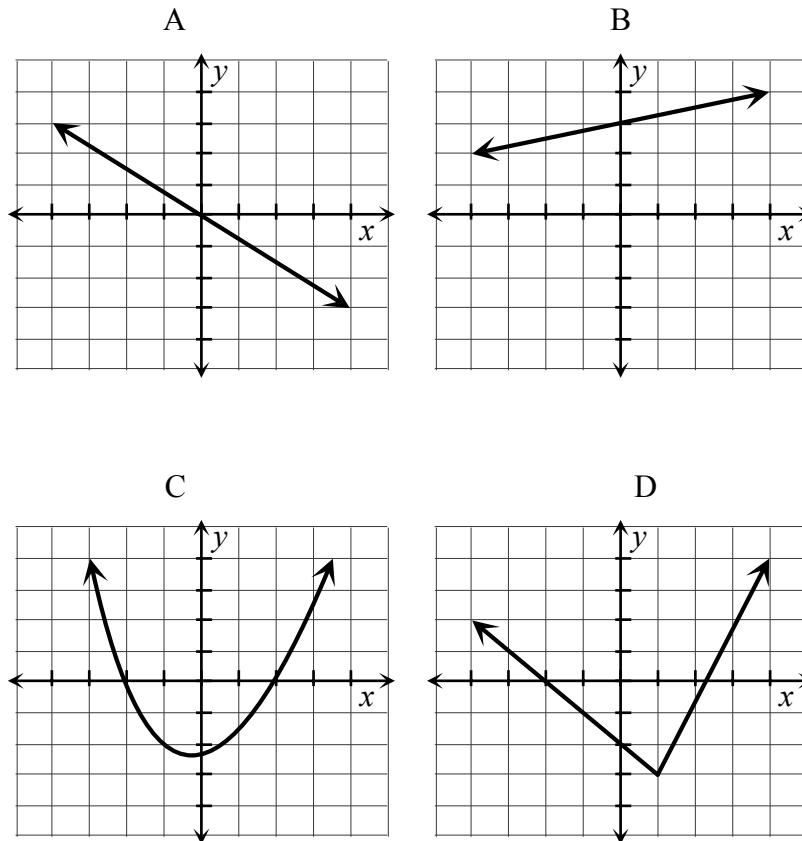
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**\* These items appear only on the password-protected web site.**

## What Makes a Function Linear?

**Linear functions** have graphs that are straight lines while **nonlinear functions** have graphs that are NOT straight lines. If a graph is made up of two or more pieces of lines, then that graph is a special type of linear function called a **piecewise linear function**.

1. Determine which graphs are linear and which graphs are nonlinear.



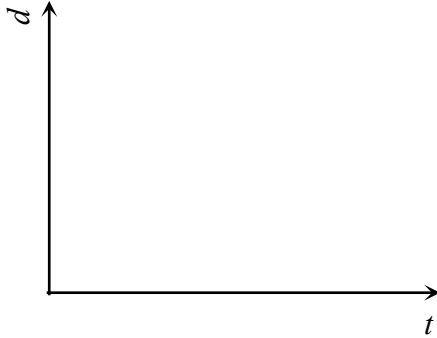
**Distance-time functions** describe the distance between a person and an object over time. A distance-time function may be linear or non-linear, increasing, decreasing, or constant, depending on the type of movement. To describe a distance-time function, tell (a) where the object starts, (b) what direction it moves, (c) how fast it moves and (d) whether it is speeding up, slowing down or moving at a steady rate.

2. Suppose a person's distance from a motion detector is changing over time.
- Identify the independent variable in this situation.
  - Identify the dependent variable in this situation.

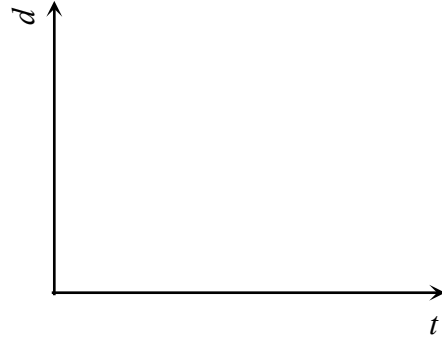
We will now create graphs of distance-time functions to match descriptions of movements. The graphs should show a person's distance from a motion detector sensor over time.

3. Sketch the distance-time graphs for the following scenarios.

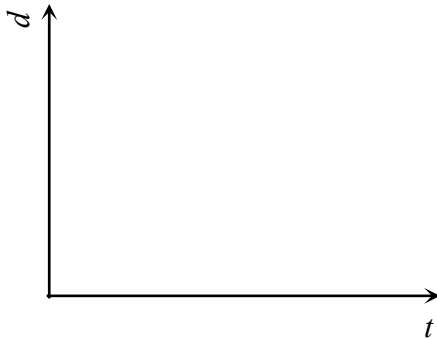
- a. Stand one meter from the sensor, walk at a constant (steady) slow pace away from the sensor.



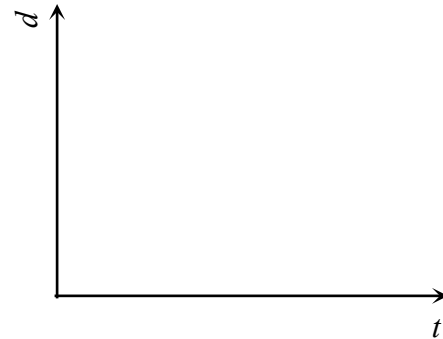
- b. Stand one meter from the sensor, walk away from the sensor changing your pace from slow to fast.



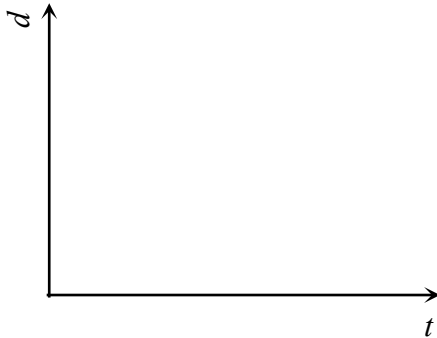
- c. Stand one meter from the sensor, and as you walk away, change your pace from fast to slow.



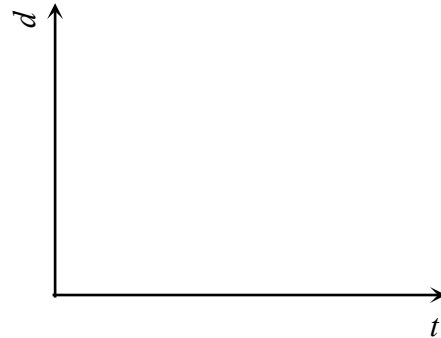
- d. Stand five meters from the sensor and walk toward the sensor at a constant rate.



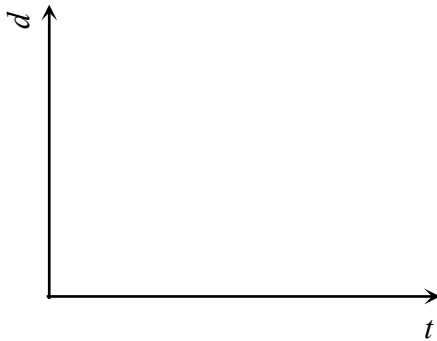
- e. Stand five meters from the sensor and walk towards the sensor slowly at first, then speed up.



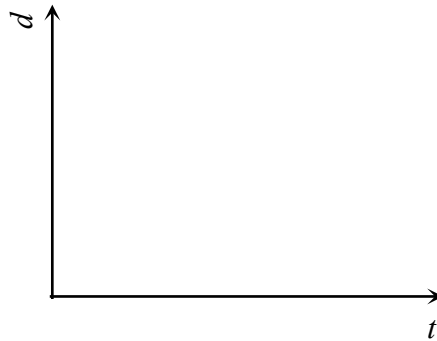
- f. Stand five meters from the sensor and walk toward the sensor quickly at first, then slow down.



- g. Stand one meter from the sensor and stand still the whole time.



- h. Stand one meter from the sensor and stand still for 3 seconds, then walk away at a constant rate.



4. a. Describe any similarities among the graphs in 3a, 3b and 3c.

- b. What are differences between the graph 3a and the graphs 3b and 3c?

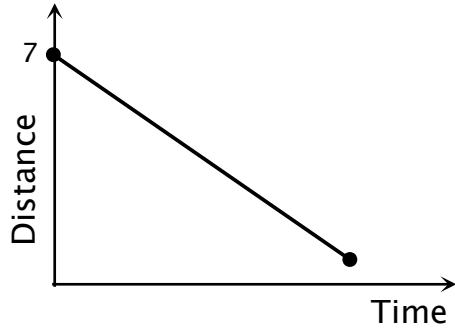
5. Which of the graphs in question (3) could be considered *linear* and which are *nonlinear*?

Linear:

Nonlinear:

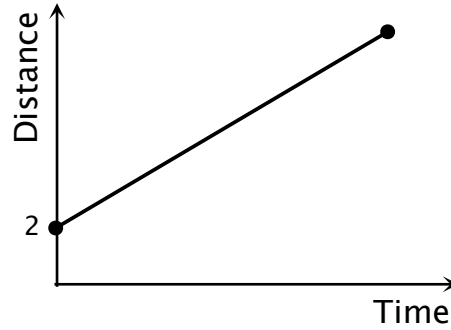
6. Describe a scenario of someone walking/running that could create the graphs below.

A



A:

B



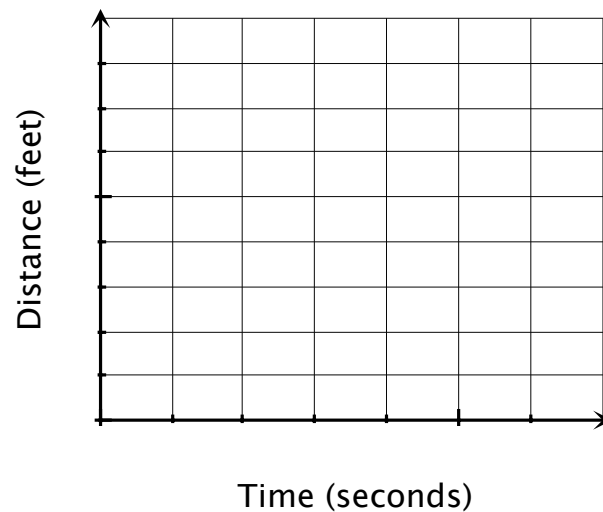
B:

7. Describe a motion that creates a linear function.

8. Describe a motion that creates a non-linear function.

9. The following table has values collected from measuring a person who attempted to walk at a constant rate. Use the data to determine whether or not the person was successful. Support your answer with a graph.

<b>Time (# of seconds)</b>	<b>Distance (# of feet)</b>
1	3.0
2	4.1
3	5.1
4	6.2
5	7.1
6	8.1



## What is Slope?

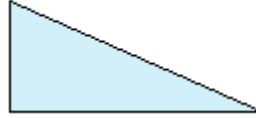
What is slope? If you have ever walked up or down a hill, then you have already experienced a real life example of slope. Keeping this fact in mind, by definition, the slope is the measure of the *steepness of a line*. In math, **slope is defined from left to right**.

There are four types of slope you can encounter. A slope can be **positive**, **negative**, **zero**, or **undefined**.



### Positive slope:

If you go from left to right and you go up, the line has a positive slope.



### Negative slope:

If you go from left to right and you go down, the line has a negative slope.



### Zero slope:

If you go from left to right and you don't go up or down, the line has a zero slope.

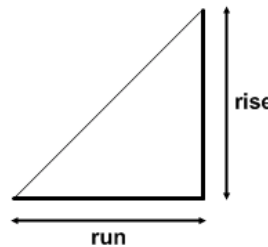


### Undefined slope:

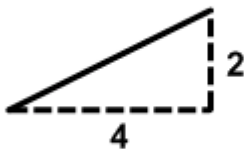
If you can only go up or you can only go down, the line has an undefined slope.

Here is one method of finding the slope of a line. Remember, **slope is a measure of how steep a line is**. That steepness can be measured with the following formula.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

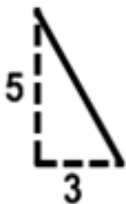


Let's illustrate with two examples:



For this situation, we see that the **rise is 2** and the **run is 4**.

So, the slope =  $\frac{2}{4}$  or  $\frac{1}{2}$  after simplification. Since  $\frac{1}{2}$  is positive, you are going uphill. Every time you go up 1 unit, you go across or horizontally to the right 2 units.



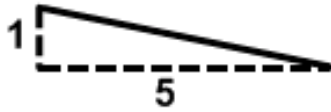
For this situation, we see that the **rise is -5** and the **run is 3**.

So, the slope =  $-\frac{5}{3}$ . Since  $-\frac{5}{3}$  is negative, you are going downhill.

Every time you go down 5 units, you go horizontally to the right 3 units.

1. Find the rise and the run for each solid line. Then state the slope of the solid line. Remember, slope is defined from left to right.

a.

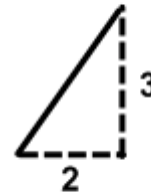


Rise = \_\_\_\_\_

Run = \_\_\_\_\_

Slope = \_\_\_\_\_

b.



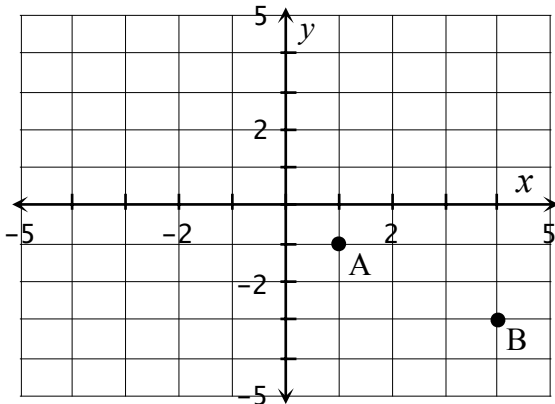
Rise = \_\_\_\_\_

Run = \_\_\_\_\_

Slope = \_\_\_\_\_

2. Starting at point  $A$  find the rise and run to get to point  $B$ . Then connect the points to make a solid line. Identify the rise, run, and slope for the line segment between each pair of points below.

a.

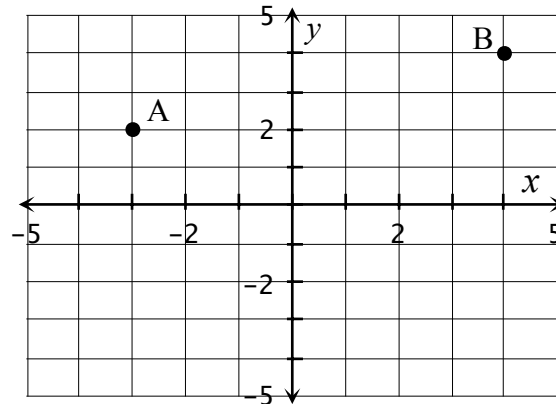


Rise = \_\_\_\_\_

Run = \_\_\_\_\_

Slope = \_\_\_\_\_

b.



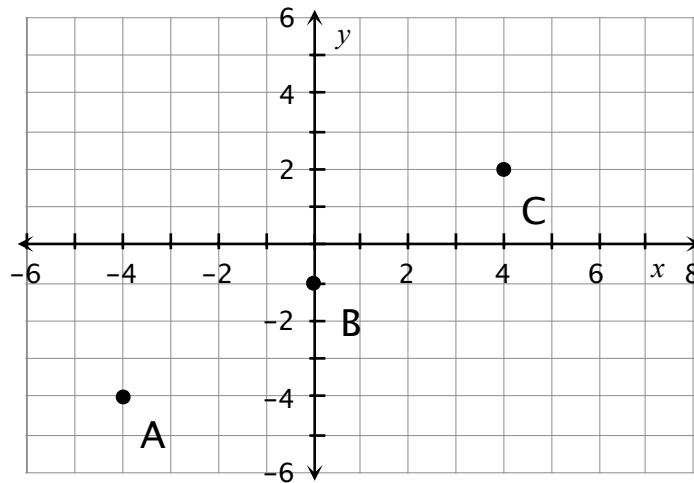
Rise = \_\_\_\_\_

Run = \_\_\_\_\_

Slope = \_\_\_\_\_



3. Use the coordinate plane below.



- Connect the points using a straightedge. Extend the line past points  $A$  and  $C$  and place arrows at each end.
- Find the slope between points  $A$  and  $B$ .

$$\text{Rise} = \underline{\hspace{2cm}} \quad \text{Run} = \underline{\hspace{2cm}} \quad \text{Slope} = \underline{\hspace{2cm}}$$

- Find the slope between points  $B$  and  $C$ .

$$\text{Rise} = \underline{\hspace{2cm}} \quad \text{Run} = \underline{\hspace{2cm}} \quad \text{Slope} = \underline{\hspace{2cm}}$$

- Find the slope between points  $A$  and  $C$ .

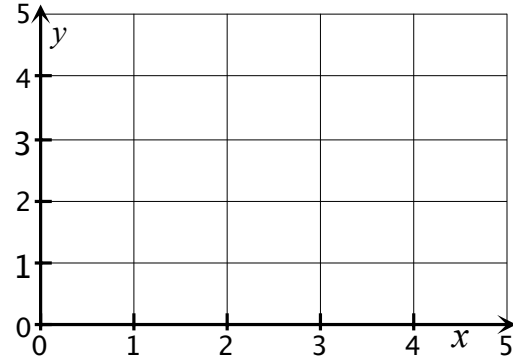
$$\text{Rise} = \underline{\hspace{2cm}} \quad \text{Run} = \underline{\hspace{2cm}} \quad \text{Slope} = \underline{\hspace{2cm}}$$

- What can you conclude about the slope of this line looking at your results in parts b thru d?
- Starting at point  $C$  find a fourth point which would belong to the same line. Label your fourth point  $D$  and explain how you arrived at it using what you know about slope.

4. Now, let's see how to find the slope when we don't know the rise and the run. If we graph the slope on the coordinate system, we will be able to derive another formula for slope using the  $x$  and  $y$  values of the coordinates.

- a. Let's put a line with a slope of  $\frac{1}{2}$  on the coordinate system.

- Begin by plotting the point  $(1, 3)$  and labeling it point  $A$ .
- From point  $A$  do the rise and run for the slope that is  $1/2$ . Plot this second point, and label it point  $B$ .
- Connect the points using a straight edge and name the coordinates of point  $A$  and point  $B$ .
- Extend the line past points  $A$  and  $B$  and place arrows at each end.



- b. Write the ordered pair for the points:  $A$  (\_\_\_\_,\_\_\_\_)  $B$  (\_\_\_\_,\_\_\_\_)

- c. The two coordinates for points  $A$  and  $B$  can be used to get the slope of  $\frac{1}{2}$ .

*Let us find the difference in the y-coordinates:*

Since we cannot call both coordinates  $y$ , we can call one  $y_1$  and call the other  $y_2$ .

Let  $y_1$  represent the  $y$ -coordinate of point  $A$ . Therefore,  $y_1 =$  \_\_\_\_\_

Let  $y_2$  represent the  $y$ -coordinate of point  $B$ . Therefore,  $y_2 =$  \_\_\_\_\_

Now subtract  $y_2 - y_1 =$  \_\_\_\_\_

The difference in the  $y$ -coordinates can be expressed as  $y_2 - y_1$ . This is the **RISE**.

*Let us find the difference in the x-coordinates:*

Since we cannot call both coordinates  $x$ , we can call one  $x_1$  and call the other  $x_2$ .

Let  $x_1$  represent the  $x$ -coordinate of point  $A$ . Therefore,  $x_1 =$  \_\_\_\_\_

Let  $x_2$  represent the  $x$ -coordinate of point  $B$ . Therefore,  $x_2 =$  \_\_\_\_\_

Now subtract  $x_2 - x_1 =$  \_\_\_\_\_

The difference in the  $x$ -coordinates can be expressed as  $x_2 - x_1$ . This is the **RUN**.

The **formula for the slope** between the two points A and B can be found by using the  $x$  and  $y$  coordinates of the two points. Call the ordered pair for point A  $(x_1, y_1)$  and the ordered pair for point B  $(x_2, y_2)$ .

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

5. Use the formula above to find the slope of the line passing through the given points. Show your work.

a. (1, 5) & (2, 9)

$$y_1 = \underline{\hspace{2cm}} \quad y_2 = \underline{\hspace{2cm}}$$

$$x_1 = \underline{\hspace{2cm}} \quad x_2 = \underline{\hspace{2cm}}$$

b. (2, 4) & (1, 1)

$$y_1 = \underline{\hspace{2cm}} \quad y_2 = \underline{\hspace{2cm}}$$

$$x_1 = \underline{\hspace{2cm}} \quad x_2 = \underline{\hspace{2cm}}$$

c. (4, 0) & (8, -2)

$$y_1 = \underline{\hspace{2cm}} \quad y_2 = \underline{\hspace{2cm}}$$

$$x_1 = \underline{\hspace{2cm}} \quad x_2 = \underline{\hspace{2cm}}$$

d. (-8, 6) & (3, 4)

$$y_1 = \underline{\hspace{2cm}} \quad y_2 = \underline{\hspace{2cm}}$$

$$x_1 = \underline{\hspace{2cm}} \quad x_2 = \underline{\hspace{2cm}}$$

e.  $(-3, -5)$  &  $(-1, -2)$

f.  $(0, 7)$  &  $(5, 0)$

$y_1 = \underline{\hspace{2cm}}$   $y_2 = \underline{\hspace{2cm}}$

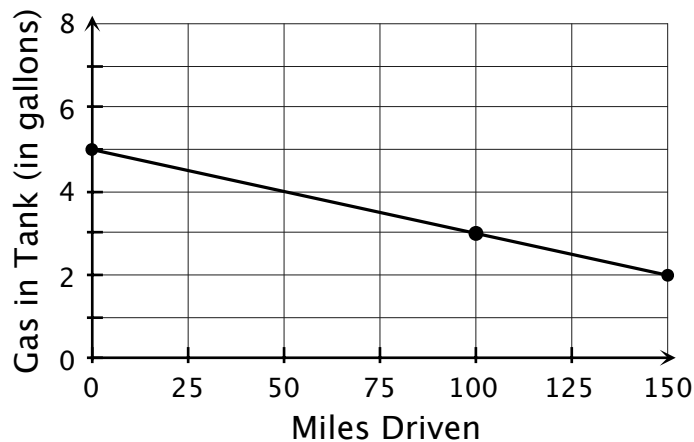
$y_1 = \underline{\hspace{2cm}}$   $y_2 = \underline{\hspace{2cm}}$

$x_1 = \underline{\hspace{2cm}}$   $x_2 = \underline{\hspace{2cm}}$

$x_1 = \underline{\hspace{2cm}}$   $x_2 = \underline{\hspace{2cm}}$

Slope is a measure of **steepness** and **direction**. Slope describes a **rate of change**.

6. Todd had 5 gallons of gasoline in his motorbike. After driving 100 miles, he had 3 gallons of gasoline left. The graph below shows Todd's situation.



- a. What are the coordinates of two points that you could use to find the slope of the line?

$$A(\underline{\hspace{1cm}}, \underline{\hspace{1cm}}), B(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

- b. What is the slope of the line? Write in fraction form and use the units of measure you find on the  $y$  and  $x$  axes.

- c. Write the slope as a *unit rate* that will be in gallons per mile.

A **rate** is a ratio that compares two units of measure.

An example of a rate in fraction form is  $\frac{170 \text{ dollars}}{20 \text{ hours}}$ . Slopes are rates.

You can rename rates like you rename fractions. In this example divide the numerator and denominator by 10, to obtain an equivalent rate of  $\frac{17 \text{ dollars}}{2 \text{ hours}}$ .

Divide the numerator and denominator by 2 to obtain  $\frac{17 \text{ dollars}}{1 \text{ hour}}$ . This is a **unit rate**, because 1 is in the denominator.

Writing the fraction in decimal form gives  $\frac{8.5 \text{ dollars}}{1 \text{ hour}}$ . In every day language, we say “\$8.50 per hour is the rate of pay.” This is also a **unit rate**.

One way to obtain a **unit rate** is to rewrite the fraction so the denominator is 1. You can also think of renaming the fraction to decimal form.

7. Sam and Kim went on a hike. The graph at the right shows their situation.

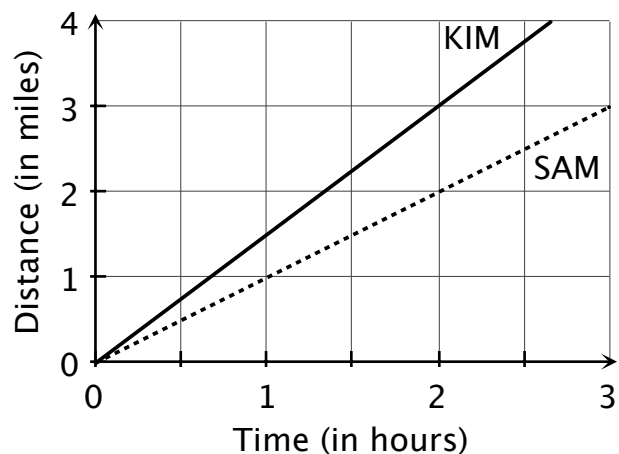
a. Find the slope of Kim’s hike.  
(Always include units of measure.)

b. Write Kim’s slope as a unit rate.

c. Find the slope of Sam’s hike.

d. Write Sam’s slope as a unit rate.

e. Who is hiking at a faster speed, Kim or Sam? Explain how you know by looking at the graph and by using the numbers for slope that you obtained above.



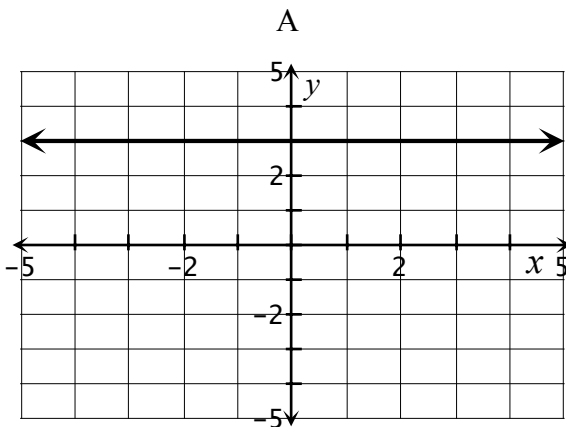
## Horizontal and Vertical Lines

*Warm Up:* Simplify: a.  $\frac{3-3}{6} =$       b.  $\frac{3}{6-6} =$       c.  $\frac{3-3}{6-6} =$

State a conclusion:

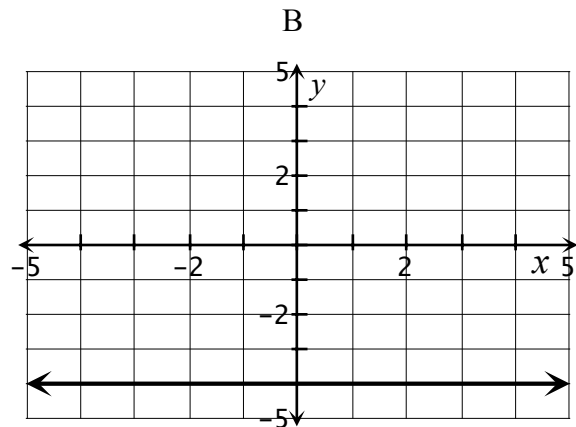
- If zero is in the numerator of a fraction, but not in the denominator, the fraction equals \_\_\_\_.
- If zero is in the denominator of a fraction, the fraction is \_\_\_\_\_

1. Tell whether or not the graphs below display a function. Calculate the slope ( $m$ ) of each line. You may either find the rise and run directly from the graphs or use the slope formula to get your answers. Write your answers as a fraction and then simplify the fraction if possible.



Function: yes or no?

Slope of Line A



Function: yes or no?

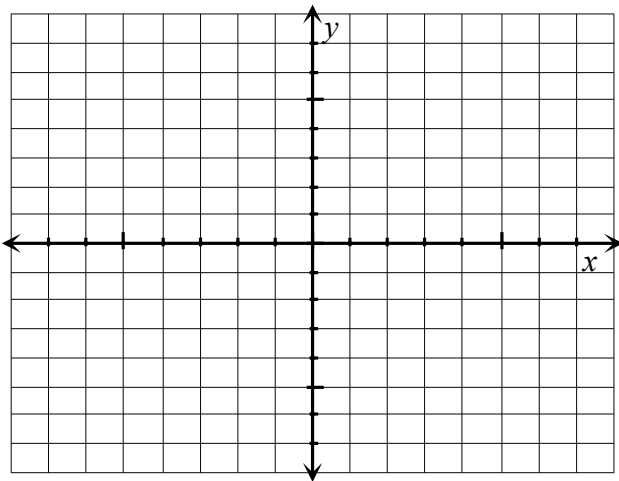
Slope of Line B

State a conclusion: The slope of a **horizontal line** equals \_\_\_\_\_.

2. Complete a table for each function below and then plot the points from the table on the following coordinate plane. Using a ruler, connect the points on each coordinate plane.

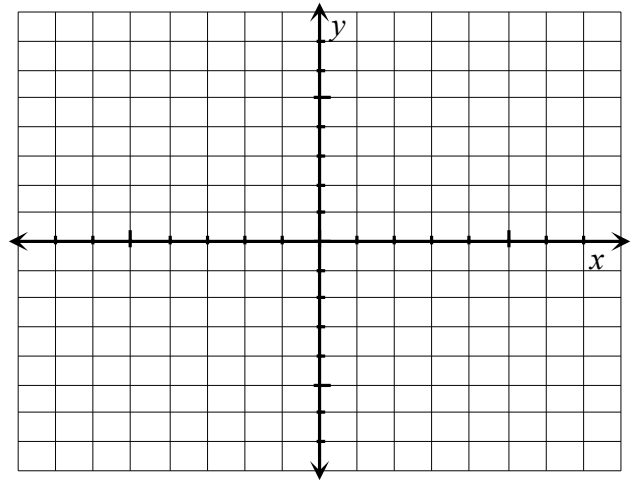
a.  $y = 5$  is the same as  $y = 0x + 5$

$x$	-2	-1	0	1	2
$y$					



b.  $y = -4$  is the same as  $y = 0x - 4$

$x$	-5	-2	0	2	5
$y$					



*State a conclusion:* An equation of the form  $y = \underline{\hspace{2cm}}$  will be a **horizontal line**.

3. Which of the following equations will give a graph that is a horizontal line? (Circle all that apply.)

a.  $y = \frac{-2}{3}x + 1$

b.  $y = 17$

c.  $y = x$

d.  $y = 0x + -1$

e.  $y = 0$

f.  $x = 5$

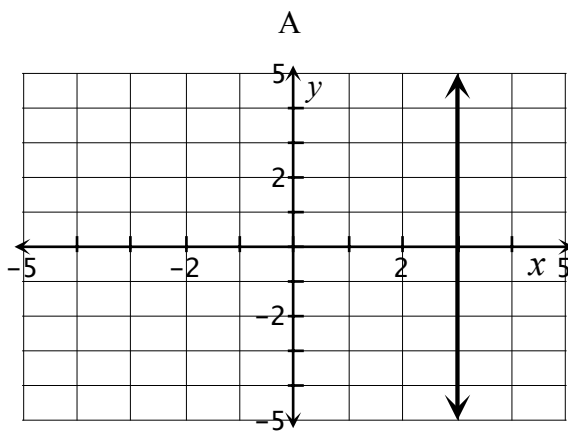
4. The slope formula is:

5. Find the slope between the two points using the slope formula.

a. (1,-3) and (-5,-3)

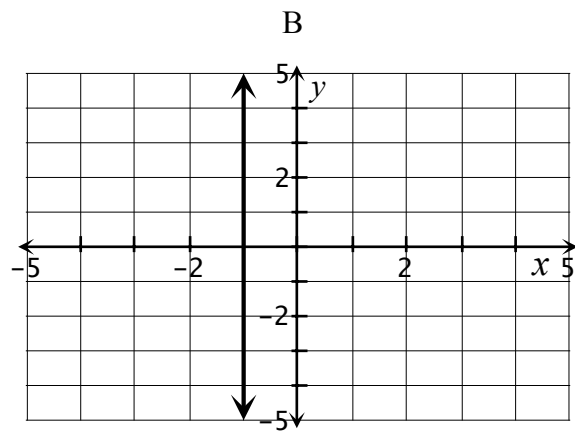
b. (-4,4) and (5,4)

6. Without using the slope formula, how can you tell if the slope of a line between two points will be zero just by looking at the two points?
7. Tell whether or not the graph displays a function. Calculate the slope ( $m$ ) of each line. You may either find the rise and run directly from the graph or use the slope formula to get your answers. Write your answer as a fraction and then put the fraction in simplest form.  
*Hint:* Pick 2 easy points from each line to work with.



Function: yes or no?

Slope of Line A



Function: yes or no?

Slope of Line B

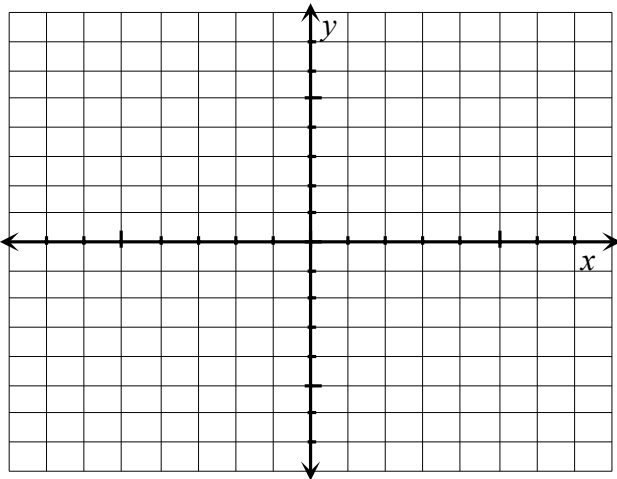
State a conclusion: The slope of a **vertical line** is \_\_\_\_\_.



8. Complete a table for each equation below and plot the points from the table on the following coordinate plane. Using a ruler, connect the points on each coordinate plane.

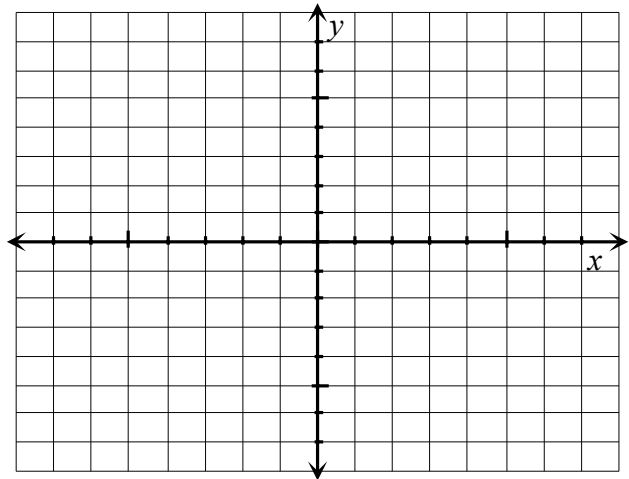
a.  $x = 5$  is the same as  $x + 0y = 5$

$x$					
$y$	-2	-1	0	1	2



b.  $x = -4$  is the same as  $x + 0y = -4$

$x$					
$y$	-5	-2	0	2	5



State a conclusion: An equation of the form  $x = \underline{\hspace{2cm}}$  will be a **vertical line**.

9. Which of the following equations will give a graph that is a vertical line?  
(Circle all that apply.)

a.  $x = 50$

b.  $y = 7$

c.  $y = 3x + 50$

d.  $x = 1$

e.  $x = 0$

f.  $x + y = 50$

10. Find the slope between the two points using the slope formula.

a. (1,3) and (1,5)

b. (-4,4) and (-4,7)

11. Without using the slope formula, how can you tell if the slope of a line between two points will be undefined just by looking at the two points?

## Effects of Changing Parameters

In this activity, you will learn how the parameters (numbers)  $m$  and  $b$  affect a linear function in the form  $y = mx + b$ . The form  $y = mx + b$  is known as **slope-intercept** form.

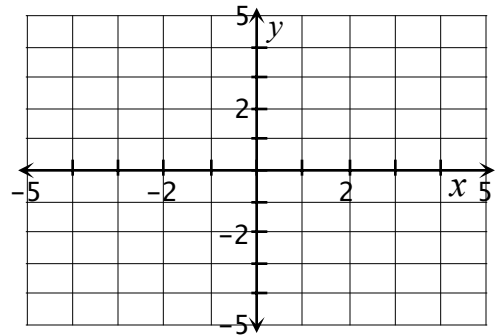
### Instructions:

We will use our graphing calculators to explore linear functions. First, we need a good window. The window controls the range of  $x$  and  $y$  values displayed on the graphing calculator. We will use a window where the  $x$ -axis will go from negative five to five, and the  $y$ -axis will go from negative five to five. To do this:

- Turn the calculator **ON**.
- Press the **WINDOW** button.
- In the **WINDOW** menu, set **Xmin = -5**, **Xmax = 5**, **Xscl = 1**, **Ymin = -5**, **Ymax = 5**, **Yscl = 1**
- Enter the function:  $y = 1x + 0$  into the graphing calculator. To do this:
  - Press the **Y=** button.
  - Enter your equation into **Y1=**. For the  $x$ -variable, the button is the **X,T,θ,n** button.
- Graph the function. To do this, press the **GRAPH** button.

1. Sketch the graph of  $y = 1x + 0$ .

- What is the slope?
- What is the value of  $b$  in the equation?
- What is the  $y$ -intercept?



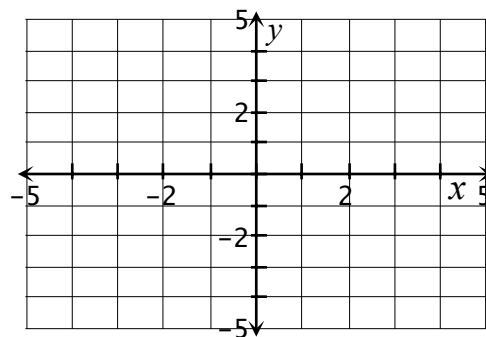
2. Graph  $y = 2x + 0$  in the calculator and sketch the graph.

a. What changed from the graph in question (1)?

b. What is the slope?

c. What is the value of  $b$  in the equation?

d. What is the  $y$ -intercept?



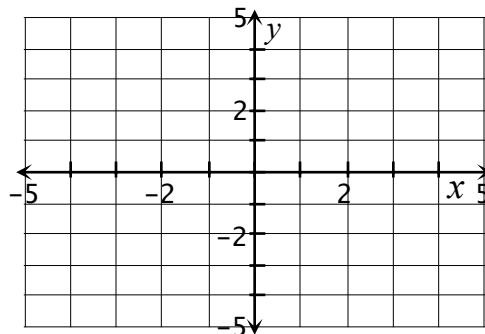
3. Graph  $y = \frac{1}{2}x + 0$  in the calculator and sketch the graph.

a. What changed from the graph in question (1)?

b. What is the slope?

c. What is the value of  $b$  in the equation?

d. What is the  $y$ -intercept?



4. If  $m$  is a positive number, what happens to the graph of a linear function as  $m$  increases?
5. If  $m$  is a positive number, what happens to the graph of a linear function when  $m$  decreases but remains positive?

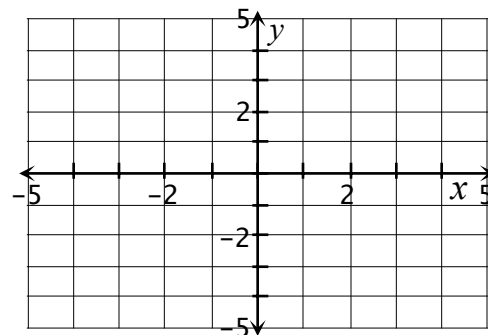
6. Graph  $y = -1x + 0$  in the calculator and sketch the graph.

a. What changed from the graph in question (1)?

b. What is the slope?

c. What is the value of  $b$  in the equation?

d. What is the  $y$ -intercept?



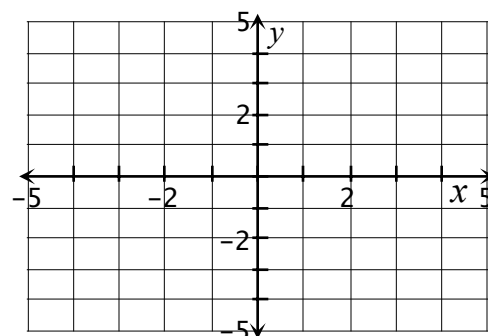
7. Graph  $y = -2x + 0$  in the calculator and sketch the graph.

a. What changed from the graph in question (6)?

b. What is the slope?

c. What is the value of  $b$  in the equation?

d. What is the  $y$ -intercept?



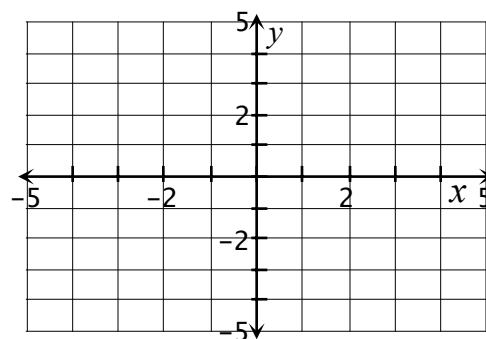
8. Graph  $y = -\frac{1}{2}x + 0$  in the calculator and sketch the graph.

a. What changed from the graph in question (6)?

b. What is the slope?

c. What is the value of  $b$  in the equation?

d. What is the  $y$ -intercept?



9. What happens to the graph of a linear function when  $m$  is negative?

10. When  $m$  is negative, describe how you can change  $m$  to make the line steeper, and how you can change  $m$  to make the line flatter.

11. What point do all of the lines you have graphed have in common?

12. Does changing  $m$  have any effect on the  $y$ -intercept of the graph?

13. Predict what the graph will look like when  $m=0$ .

14. Test your prediction by graphing a line with  $m=0$  on your calculator. Was your prediction correct? If not, what kind of line did you get?

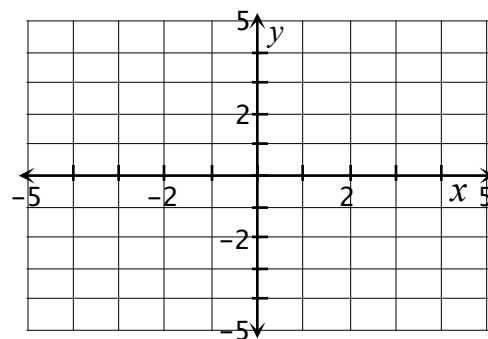
15. In your own words, describe the value of  $m$ 's overall effect on the graph of a line.

16. Graph  $y = 1x + 0$  in the calculator and sketch the graph.

a. What is the slope?

b. What is the value of  $b$  in the equation?

c. What is the  $y$ -intercept?



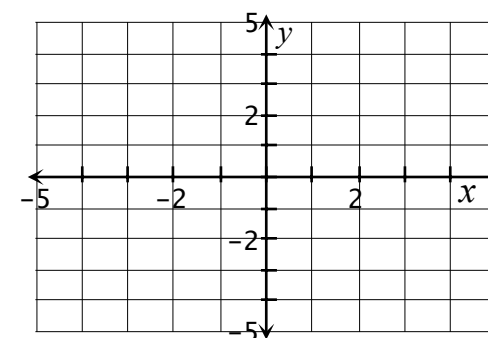
17. Graph  $y = 1x + 2$  in the calculator and sketch the graph.

a. What changed from the graph in question (16)?

b. What is the slope?

c. What is the value of  $b$  in the equation?

d. What is the  $y$ -intercept?



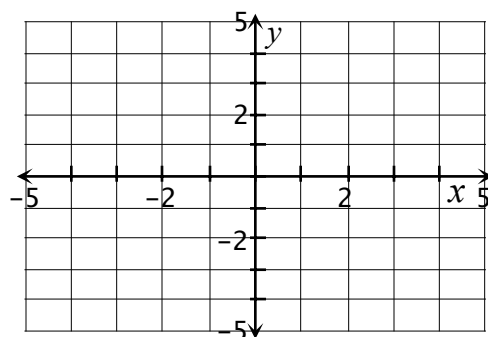
18. Graph  $y = 1x + 4$  in the calculator and sketch the graph.

a. What changed from the graph in question (16)?

b. What is the slope?

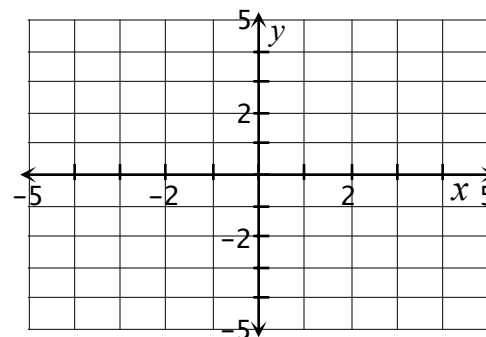
c. What is the value of  $b$  in the equation?

d. What is the  $y$ -intercept?



19. Graph  $y = 1x - 2$  in the calculator and sketch the graph.

- What changed from the graph in question (16)?
- What is the slope?
- What is the value of  $b$  in the equation?
- What is the  $y$ -intercept?



20. If  $b$  has a negative value then the  $y$ -intercept is (above, below) the  $x$ -axis. Circle one answer.

21. If  $b$  has a positive value then the  $y$ -intercept is (above, below) the  $x$ -axis. Circle one answer.

22. What is the  $y$ -intercept of the equation  $y = 2x + 4$ ?

23. What is the  $y$ -intercept of the equation  $y = x - 5$ ?

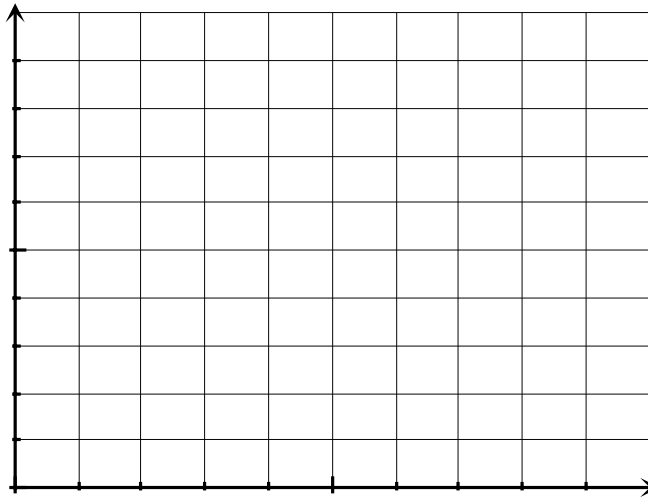
24. How does changing  $b$  in a linear function affect the graph? Be as specific as possible.

## Trends in Bottled Water Consumption

Here is a data table that shows the consumption of bottled water in the United States in the years 2000 and 2007 in billions of gallons. Let's assume that during this period consumption was a linear function of time.

<b>Year</b>	<b>Billions of Gallons</b>
2000	4.7
2007	8.8

1. Let  $x$  represent the number of years from 2000 and  $y$  represent the amount of water consumed in billions of gallons. Make a graph with  $x$  on the horizontal axis and  $y$  on the vertical axis by plotting the two points and using a ruler to draw a line between the two points (do not extend the line.)



2. Find the rate of change in water consumption per year using data for the years 2000 and 2007.
3. Use the rate of change from question 2 and the  $y$ -intercept of your graph to write a linear equation in slope-intercept form.



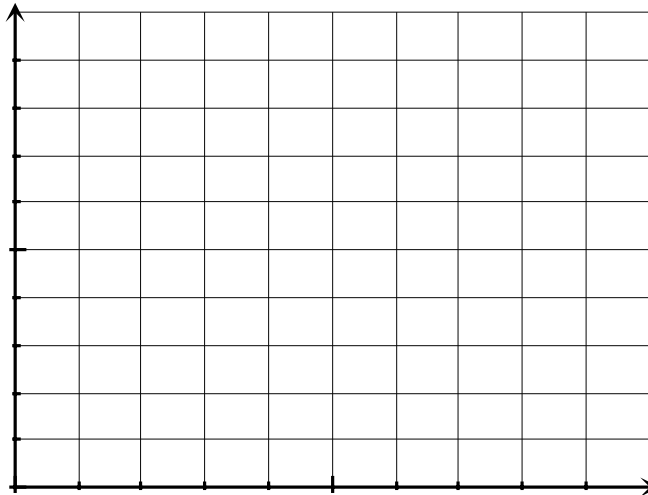
4. Use your equation from question 3 to determine the consumption of bottled water in 2004.
5. A more accurate figure for the consumption of bottled water in 2007 is 8757.4 million.
- Write this large number in standard decimal notation.
  - Assume that the average water bottle contains 24 ounces. Estimate the actual number of water bottles sold. Use the fact that one gallon contains 128 ounces.
  - The population of the United States in 2007 was about 300 million. On average how many water bottles were purchased by each person in the country?
  - In 2008, bottled water consumption decreased from about 8.8 billion gallons in 2007 to 8.7 billion in 2008. What are some possible reasons?

6. Here is a table that shows the consumption of bottled water in the United States from 2007 to 2009 in billions of gallons. As you can see, consumption continues to decline. So from 2007 to 2009 we will use a new linear function to model bottled water consumption.

**U.S. Bottled Water Consumption**

Year	Billions of Gallons
2007	8.80
2009	8.45

- a. As you did in question 1 above, let  $x$  represent the number of years from 2000 and  $y$  represent the amount of water consumed in billions of gallons. Make a graph with  $x$  on the horizontal axis and  $y$  on the vertical axis by plotting the two points and using a ruler to draw a line between the two points and extending the line to the right.



- b. Find the rate of change in water consumption per year using data for the years 2007 and 2009.

Unlike the equation for the water consumption in question 1, we are not able to find the  $y$ -intercept unless we do a little work. So how will we find an equation? Recall that the slope between any two points on a line is the same.

- c. If water consumption continues to decline at a steady rate, the slope of the line will continue to be \_\_\_\_\_.

- d. Use slope to count over to a third point on your line to the left of  $(7,8.8)$ , and state the coordinates.
- e. Then find the slope between the point you picked and the point  $(7,8.8)$ .

Check with a few classmates to see if you all are getting the same slope you found in question 6b.

- f. Now label an arbitrary point on your line  $(x,y)$ . Instead of using numbers in the second point, use the variables  $x$  and  $y$ , because there are many different values of  $x$  and  $y$  on the line.
- g. Use the slope formula to find the slope from this arbitrary point  $(x,y)$  to the point  $(7, 8.8)$  and you should still get the same slope. Why?

Here is the algebra:

$$\frac{y-8.8}{x-7} = m$$

Do a little algebra to transform the equation.

$$y - 8.8 = m(x - 7)$$

Multiply both sides of the equation by  $x - 7$  to bring the denominator up to the right side.

$$y - 8.8 = -0.175(x - 7)$$

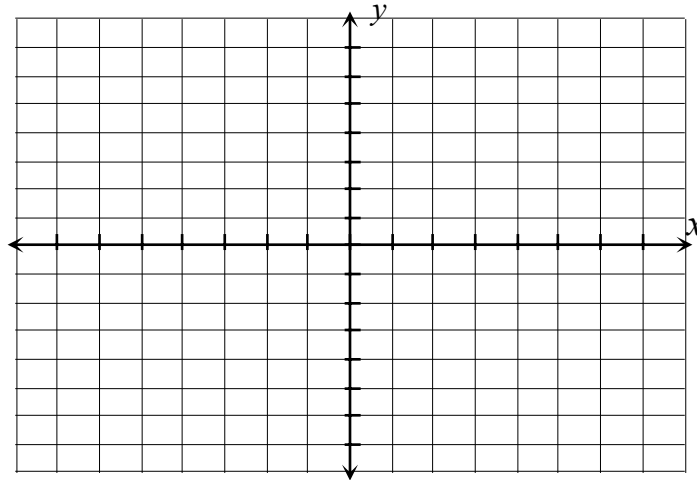
Fill in the value for slope and you will have an equation in Point-Slope form.

Point-slope form simply asserts that the slope between the fixed point  $(7, 8.8)$  on the line and any arbitrary point  $(x,y)$  is the same slope  $m$ ; in this case  $m = -0.175$ .

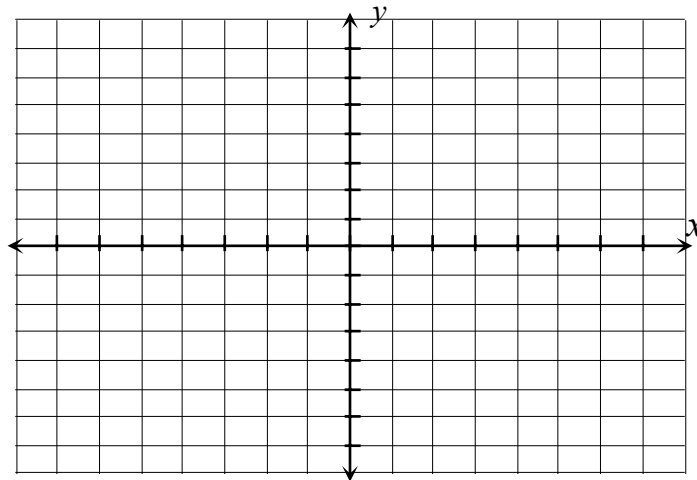
- h. To confirm that this equation contains the point  $(7, 8.8)$ , substitute  $x = 7$  and  $y = 8.8$  into the equation  $y - 8.8 = -0.175(x - 7)$ . Check to see if the result is a true statement.
- i. To confirm that this equation contains the point  $(9, 8.45)$ , substitute  $x = 9$  and  $y = 8.45$  into the equation  $y - 8.8 = -0.175(x - 7)$ . Check to see if the result is a true statement.
- j. Solve for  $y$  to transform the equation to slope-intercept  $(y = mx + b)$  form.

## Point-Slope Form of an Equation

1. Graph the equation  $y = \frac{3}{4}x + 4$  by starting at  $(0,4)$  and moving to another point on the line using the slope.



2. Now draw another graph of  $y = \frac{3}{4}x + 4$ . This time pick the point  $(-8, -2)$  which is a point on the line, and use slope to count up and right from that point to find other points on the graph. Do you end up with the same line as you did in part 1a above?



3. Notice that you can find points on a line or graph a line by starting at a point and moving according to the slope. Does it matter which point on the line is chosen to start with?

### Facts about Point-Slope Form

The point-slope form of a line is a special form that tells you the SLOPE of a line and one POINT on the line.

The point-slope formula is:  $y - y_1 = m(x - x_1)$

$m$  is the slope of the line; it will be a number

$x_1$  is the  $x$  coordinate of a particular point on the line, and it will be a number

$y_1$  is the  $y$  coordinate of a particular point on the line, and it will be a number

$x$  is the variable  $x$

$y$  is the variable  $y$

Point-slope form comes from the fact that the slope between any two points on a line is always the same. Use the slope formula between the specific fixed point  $(x_1, y_1)$  and any moveable point

$(x, y)$ : 
$$\frac{y - y_1}{x - x_1} = m$$

Multiply both sides of this equation by the denominator  $x - x_1$  to obtain:  $y - y_1 = m(x - x_1)$

When you substitute the values of the specific point  $(x_1, y_1)$  into the point slope formula, you will obtain a true statement “ $0=0$ ” which proves that  $(x_1, y_1)$  is a point on the line.

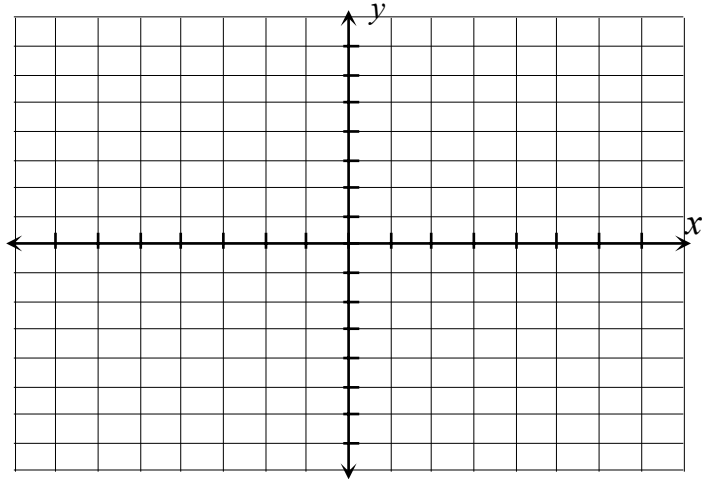
The slope-intercept form of a line is  $y = mx + b$ .  $m$  is the same value in both forms. Notice that it is the coefficient of the  $x$  variable.

**EXAMPLE:** For the equation  $y - 7 = \frac{4}{3}(x - 2)$ , the particular point is  $(2, 7)$ . The  $x$ -coordinate of this point is 2 and the  $y$ -coordinate of this point is 7. The line has slope of  $\frac{4}{3}$ .

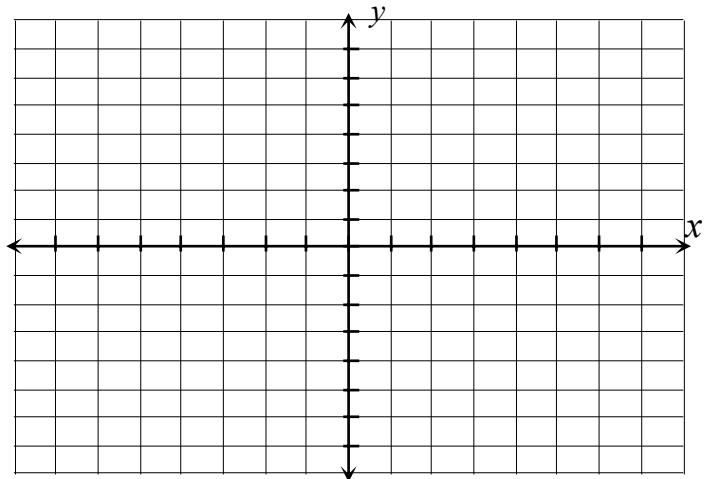
- Verify that  $(2, 7)$  is a solution to the equation  $y - 7 = \frac{4}{3}(x - 2)$  by evaluating the equation when  $x=2$  and  $y=7$ .

5. For each equation in point-slope form, identify the particular point and the slope. Then graph each equation. Test your point in the equation to be sure that the point makes the equation true.

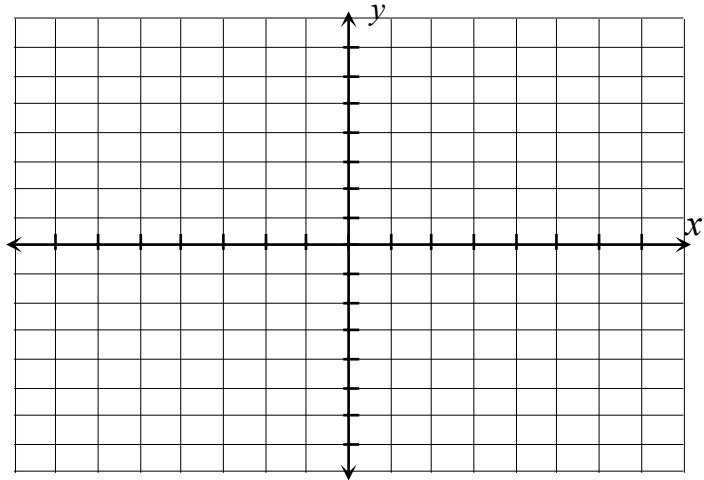
a.  $y - 3 = \frac{4}{5}(x - 1)$



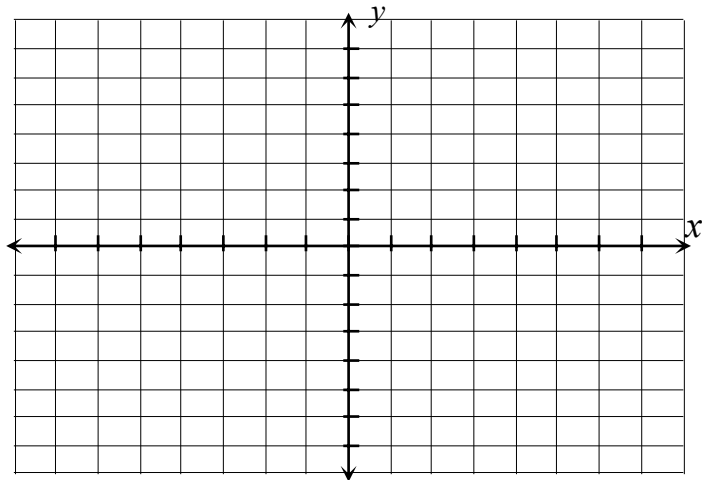
b.  $y - 8 = -\frac{5}{2}(x - 2)$



c.  $y - 2 = \frac{1}{3}(x - (-5))$



d.  $y + 6 = \frac{2}{3}(x + 2)$



6. Use the point and the slope to write an equation of the line in point-slope form.

$$y - y_1 = m(x - x_1)$$

a.  $(3,5), m = 2$

b.  $(2,6), m = \frac{-2}{7}$

c.  $(3,0)$ , parallel to the line  $y = 9x + 5$

d.  $(0,4)$ , perpendicular to the line  $y = \frac{-8}{5}x + 2$

e.  $(3,2), m = 0$

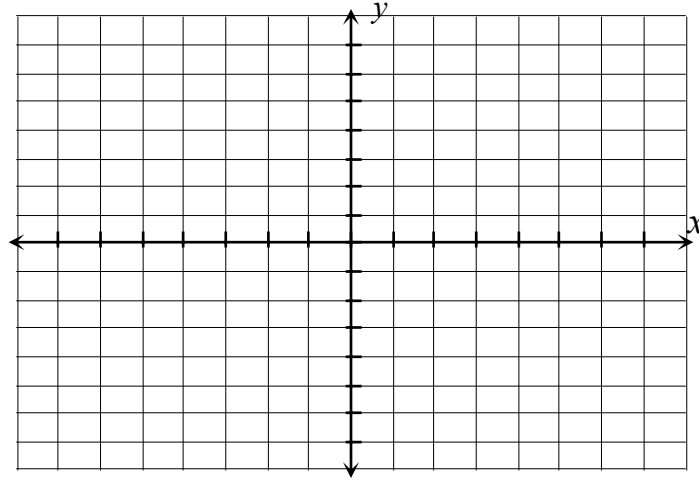
f.  $(-3,2), m = \frac{7}{3}$

g.  $(-5,-1), m=3$



7. Plot the two points  $(-3,7)$  and  $(1,-3)$ .

- Draw the line containing the two points.
- What is the slope between the two points?
- Write an equation in point-slope form:



8. Find an equation of the line between the given two points by first finding the slope, then finding the point-slope form of the equation.

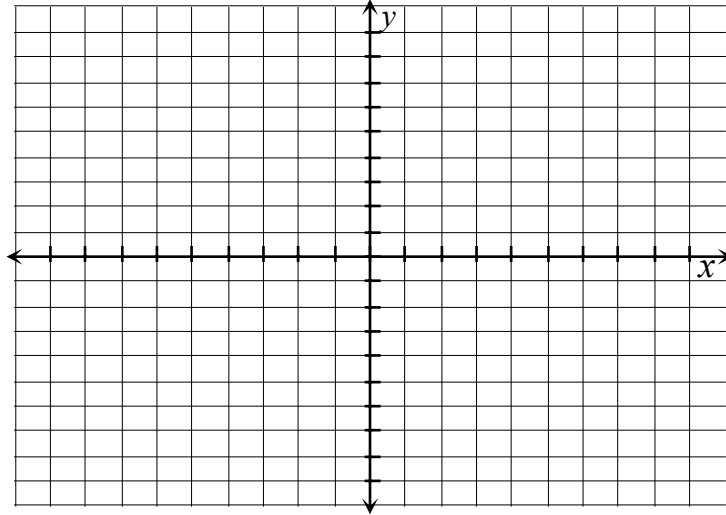
a. through points  $(5,8)$  and  $(-2, 7)$

b. through points  $(-2, -6)$  and  $(-7, 5)$

9. This February and March, the middle school students had their most successful food drive, topping last year's total by 57 items. They started the food drive on day 0 with 8 cans of fruit juice which had been donated too late to be included in the November food drive. Contributions poured in at a constant rate of 12 food items per day. By the time the drive was over, the cans covered the cafeteria stage.
- What is the dependent variable?
  - What is the independent variable?
  - Find the slope described in the situation.
  - Find a point described in the situation.
  - Write an equation of the line in point-slope form.
  - Write an equation of the line in slope-intercept form.
  - Use either equation to tell how many food items had been collected by the 10<sup>th</sup> day.
  - Use either equation to tell how many days it took to collect 488 items

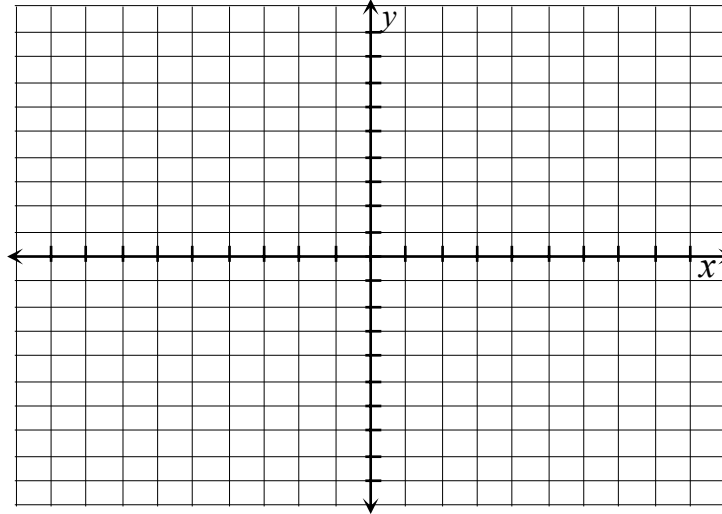
***Transforming a function from point-slope form into slope-intercept form***

10. a. Sketch the graph of the function  $y - 2 = 4(x - 3)$ .



- b. Transform the previous equation into slope intercept form by applying the distributive property on the right side and solving for  $y$ .
- c. What are the slope and  $y$ -intercept?
- d. Confirm that the equation in slope-intercept form gives the same graph as the equation in point-slope form.

11. a. Sketch the graph of the function  $y + 4 = \frac{3}{4}(x + 6)$ .



- b. Transform the previous equation into slope-intercept form by applying the distributive property on the right side and solving for  $y$ .
- c. What are the slope and  $y$ -intercept?
- d. Confirm that the equation in slope-intercept form gives the same graph as the equation in point-slope form.

## Finding and Using Linear Functions

To do these problems, choose among the three forms of linear equations we have studied:

Slope-Intercept Form:  $y = mx + b$

Standard Form:  $Ax + By = C$

Point-Slope Form:  $y - y_1 = m(x - x_1)$

### 1. Pedro's Parking Ticket

Pedro thought he could just run into the store for a minute, so he didn't put any money in the parking meter. He got a ticket for \$25 due one week later. The ticket said that it would be an additional \$13.00 for each day it was paid late.

a. Define the variables and write an equation that represents the total fine for any number of days late paying the ticket.

b. Use the equation to find the amount Pedro must pay if he is six days late.

c. When Pedro finally goes to the Town Clerk to pay the bill he has to pay \$155.00. How many days late was he?





#### 4. Writing Equations

Write equations in point-slope form, slope-intercept form, or standard form for the line that passes through each pair of points. Try to use at least two forms of the equation for each pair.

	Pair of Points	Point-slope form	Slope-intercept form	Standard form
a.	(5, 3) and (7, 9)			
b.	(0, 8) and (3, 0)			
c.	(5, -3) and (-3, -4)			
d.	(-1, 2) and (0, 6)			
e.	(4, 9) and (0, 9)			