

Module 2
Participant Guide

Focus on Content Standards

Section 5

Connecticut Core Standards for Mathematics



Grades 6–12

Systems of Professional Learning

Connecticut Core Standards Systems of Professional Learning

The material in this guide was developed by Public Consulting Group in collaboration with staff from the Connecticut State Department of Education and the RESC Alliance. The development team would like to specifically thank Ellen Cohn, Charlene Tate Nichols, and Jennifer Webb from the Connecticut State Department of Education; Leslie Abbatiello from ACES; and Robb Geier, Elizabeth O'Toole, and Cheryl Liebling from Public Consulting Group.

The Systems of Professional Learning project includes a series of professional learning experiences for Connecticut Core Standards District Coaches in English Language Arts, Mathematics, Humanities, Science, Technology, Engineering, Mathematics (STEM), and Student/Educator Support Staff (SESS).

Participants will have continued support for the implementation of the new standards through virtual networking opportunities and online resources to support the training of educators throughout the state of Connecticut.

Instrumental in the design and development of the Systems of Professional Learning materials from PCG were: Sharon DeCarlo, Debra Berlin, Jennifer McGregor, Judy Buck, Michelle Wade, Nora Kelley, Diane Stump, and Melissa Pierce.

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Section 5

Section 5: Supporting Change

Video Observation Sheets

Instructions: As you watch the videos look for the instructional shifts required by the CCS-Math Content Standards (focus, coherence, and the three aspects of rigor) and reviewed during Module 2 as well as students exhibiting the Standards for Mathematical Practice (focus of Module 1).

(The first video can be found here: <https://www.teachingchannel.org/videos/adding-integers-lesson-idea.>)

GRADE 7 VIDEO: WHAT’S YOUR SIGN: INTEGER ADDITION

Did you see evidence of the following and if so, what was the teacher doing and what were the students doing?

Element	Evidence
Development of Conceptual Understanding	
Development of Procedural Skill and Fluency	
Students Working through Tasks that Require the Application of Mathematics	
Use of Cognitively Rigorous Tasks	
Development of the Practice Standards	
Additional Instructional Strategies Used by the Teacher:	

GRADE 7 VIDEO: ZERO PAIRS, MANIPULATIVES, AND A REAL-WORLD SCENARIO

(The second video can be found here: <https://www.teachingchannel.org/videos/teaching-subtracting-integers.>)

Did you see evidence of the following and if so, what was the teacher doing and what were the students doing?

Element	Evidence
Development of Conceptual Understanding	
Development of Procedural Skill and Fluency	
Students Working through Tasks that Require the Application of Mathematics	
Use of Cognitively Rigorous Tasks	
Development of the Practice Standards	
Additional Instructional Strategies Used by the Teacher:	

A New Spin on Old Strategies

Instructions: Use the space below to make notes on important points/ideas that you want to bring back to teachers at your school.

Table 1: Math Journals

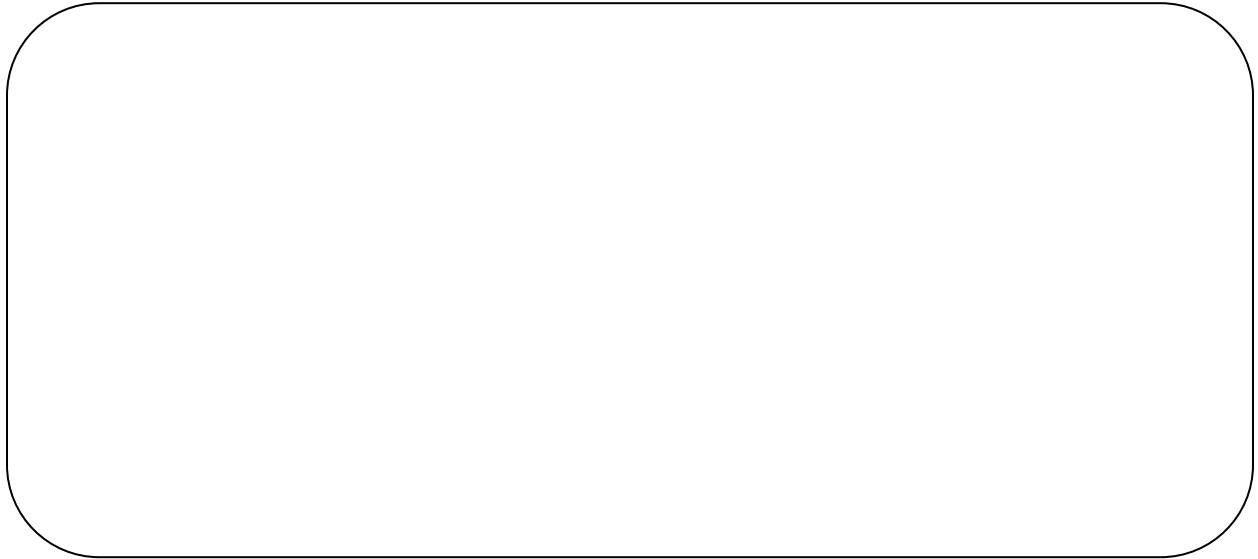


Table 2: Mathematical Language

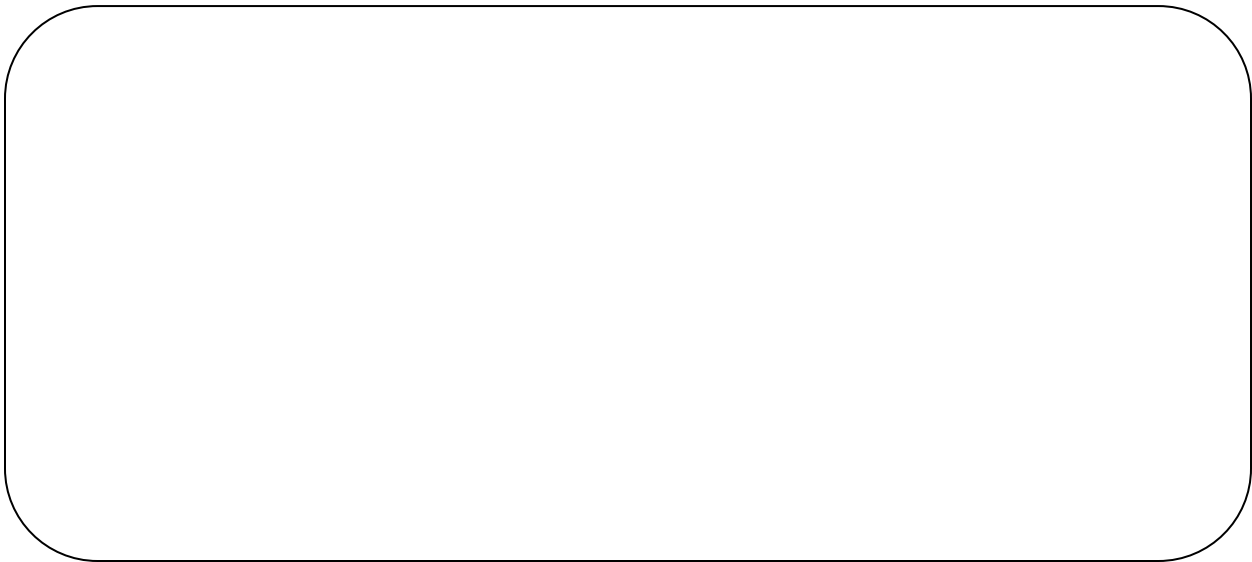


Table 3: Engagement Strategies

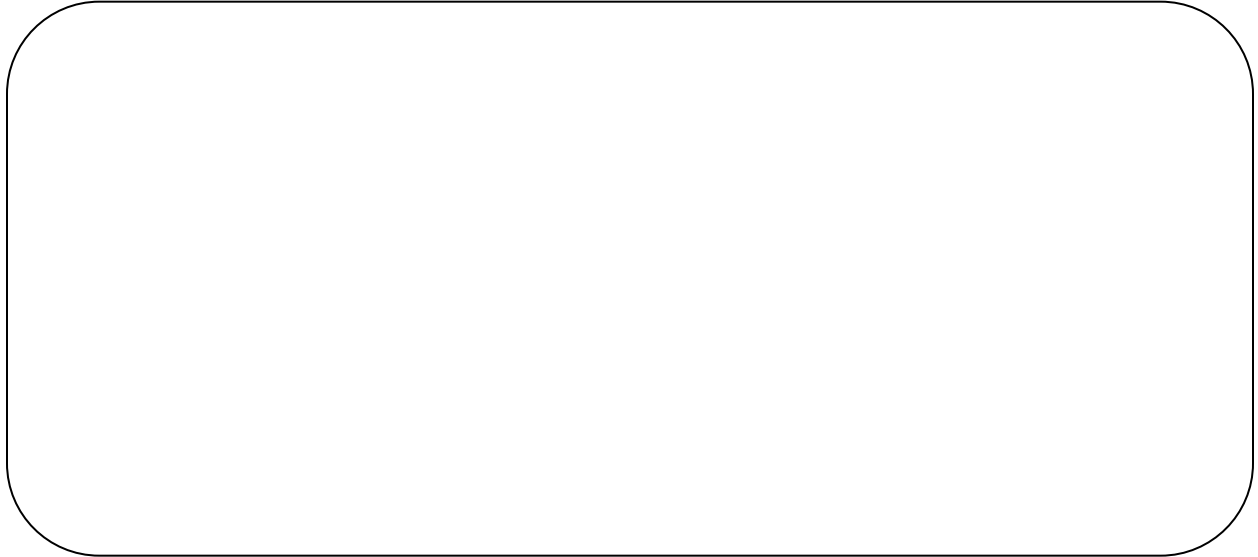


Table 4: Group Work & Decision Making



Group 1: Math Journals

Math journals have been a staple in many mathematics classrooms over the years. Journals have been used to record notes, to complete homework, to answer writing prompts, and so forth. Today, however, math journals are taking on a slightly different role in the Common Core classroom. As students work to deepen their understanding of the mathematics content, teachers are working to help students develop the Math Practices. A student's math journal, or problem solving notebook as they are known by many, are playing a key role in Content and Practice Standards development.

MATH JOURNAL FAQs

- **How can math journals be used to promote mathematical understanding and help meet the goals of the Common Core Standards?** Part of students developing deep understandings in mathematics is being able to articulate their mathematical thinking and understanding. Math journals can be used to provide students with the opportunity to practice organizing their thoughts, to test out solution strategies, to use mathematics vocabulary, to clarify and reflect on their learning, to solve rich math problems, and so forth. When students are presented with a rich problem they can record their solution, along with the strategy and thought processes used to arrive at the solution. Math journals can also be a place for students to write about their learning. They can answer questions about what they already know about a topic before a lesson or unit is started and/or what they know about the topic at the end of the lesson or unit. A combination of prompts and problems can be used to develop a clearer picture of what students understand and are able to do.
- **What can I expect from my students as I introduce math journals?** Initially students will need extra support and guidance when learning how to record their thinking. You can expect students, especially young students, to use drawings and pictures to help support their thinking. It is important to note here that the use of pictures and/or multiple representations can continue to support students thinking, and help them to make connections, even after students are able to better articulate their thinking in words, so their use should be strongly encouraged. Provide several models for students to examine, complete journal entries together in large and small groups, and make sure to stress the importance of developing complete solutions (solutions that include words, pictures, charts, or graphs, and an explanation of thinking) versus one 'right' way to solve the problem even though students may, at first, be looking for the 'right' way. Encourage students to develop solutions that make sense to them even if the person next to them is doing something different. This may be difficult for students at first, but over time students will become more comfortable with the idea of creating multiple solutions over time.
- **What makes a good journal prompt or problem?** Good journal prompts and problems are no different from other problems and/or prompts that students are asked to complete. These problems and/or prompts:

- Allow for multiple entry points into the mathematics and recording techniques. These multiple entry points build differentiation into the discussion and tasks.
- Provide the opportunity for students to learn by answering the questions and for the teacher to learn about the student by examining their responses.
- Have more than one solution or a variety of possible solution paths that range from simple to complex and require more than just remembering a fact or reproducing a skill.
- Provide opportunities for students to represent their mathematical ideas using models and written language.
- Provide opportunities for students to justify their reasoning and evaluate the reasoning of others.
- Have clear, concise directions.
- Provide opportunities for group work and discussions.
- **How do I get started?** First, decide what type of notebook you want students to use for their journal. It is suggested that students use a notebook that pages cannot easily be removed from and are durable so that pages stay in place throughout the year. Then, determine what information you want all students to have on the first few pages of their journal. It is a good idea to have either a rubric or general expectations for journal entries that students can use as guidelines when completing their work. For example:
 - Try new ideas.
 - Use pictures, words, and math symbols.
 - Tell what you did and why (or, explain your thinking).
 - Don't erase. (This is important because you want to see how students thinking has changed over the course of solving a problem. You may ask older students to write in pen.)
 - Put the date on every entry.
 - Write down your questions.
 - Check your work.
 - Don't give up!
- Determine how often you will send the journals home. Sending journals home once a month or so is a good way for parents to see a record of their students' work and growth over time.

ADDITIONAL TIPS:

- Revisit problems that were particularly challenging for students and challenge them to think of another way to solve the problem now that they know more.
- Have students go back to questions they wrote in their journal about things they were unsure of and see if they can answer them on their own at a later date.

- Have students reflect on earlier entries and tell how their thinking about a topic or concept has changed.
- Use students' journals to show mastery/non-mastery of concepts.
- Provide students' notebooks to the teacher at the next grade level whenever possible so that students' have the resource for the next school year.

RESOURCE: MATH TEACHING RESOURCES

- <http://www.k-5mathteachingresources.com/math-journals.html>

Group 2: Mathematical Language

CONCEPT CARD: MODIFIED FRAYER MODEL

Description

The Frayer Model was developed by Dorothy Frayer and her colleagues at the University of Wisconsin. This model is used to help students graphically represent new concepts and/or terms.

New Spin

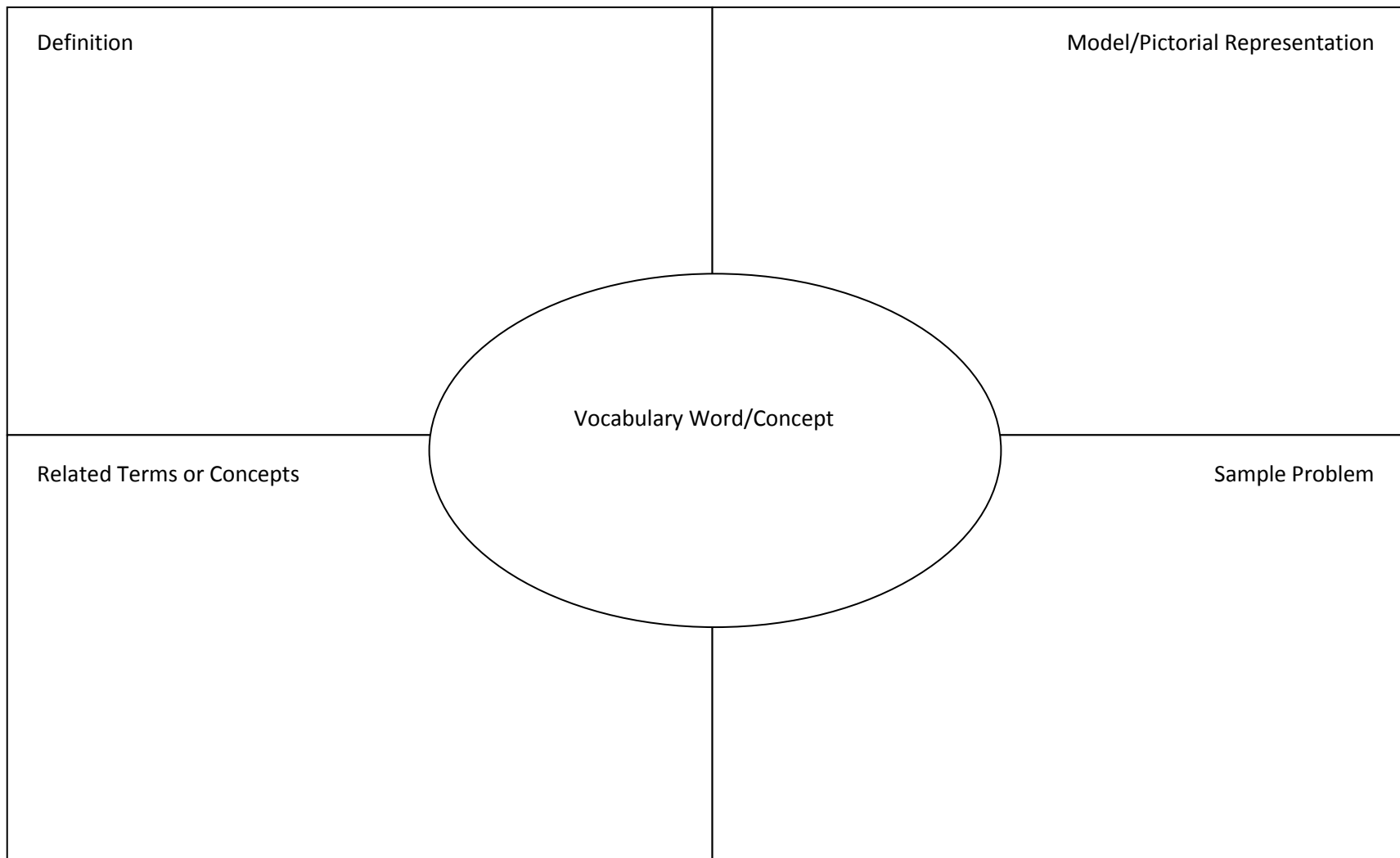
The traditional Frayer Model has students provide a definition, characteristics, examples, and non-examples of the term or concept. To help students make connections between terms and concepts and to learn through models and multiple representations, the *new spin* on the Frayer Model asks students to provide a definition, a model or pictorial representation, identify related terms and/or concepts, and to use the term or concept in a mathematics problem. Teachers may choose to customize the Frayer Model further by adding back in the non-examples when appropriate as well.

Promoting Student Learning

The modified Frayer Model promotes student learning by:

- Activating prior knowledge.
- Helps students clarify and communicate their understanding.
- Allows students to fill in the information with ideas, examples, etc. that make sense to them.
- Can be used as part of a Word Wall or in journals and used as reference for students as they work.
- Can be used as formative assessments.
- Can be completed at the beginning, during, or at the end of a lesson or unit.
- Should be shared as part of small and large group discussions so that students can add to what they have already written.

CONCEPT CARD: MODIFIED FRAYER MODEL



Group 3: Instructional Implementation Sequence: Attaining the CCSS Mathematical Practices Engagement Strategies

**Instructional Implementation Sequence:
 Attaining the CCSS Mathematical Practices
 Engagement Strategies**

SAMPLE

Strategy	Description	Practice	Degree	Matrix Code
Think pair-share	Pair-Share, or Think-Pair-Share, is a strategy easy to implement in any classroom at any grade level or subject.	<ul style="list-style-type: none"> Make sense of problems 	<ul style="list-style-type: none"> Explain their thought processes in solving a problem one way. 	1a I

- See copies of the complete **Instructional Implementation Sequence: Attaining the CCSS Mathematical Practices Engagement Strategies** that have been provided.
- This is also available at:
<http://www.mathleadership.com/sitebuildercontent/sitebuilderfiles/instructseqchart5.4ccss.pdf>
- Note: The last column of this table (the Matrix Code) links to another document by the authors entitled “Standards of Student Practice in Mathematics Proficiency Matrix. This matrix and further explanation of its purpose can be retrieved at:
<http://www.mathleadership.com/sitebuildercontent/sitebuilderfiles/standardssoftudentpracticeinmathematicsproficiencymatrix.pdf>.

Group 4: Group Work and Decision Making

Helping Students Work in Groups

Teaching mathematics through the CCS provides opportunities for students to work with challenging mathematics tasks in different collaborative configurations: individually, in pairs, in small groups, and as a whole class. Each of the work arrangements can enhance student learning by providing opportunities to discuss the mathematics, to see other approaches to a problem, and to personalize the understanding for each student. The following are suggestions for making time spent in each work configuration beneficial to both the teacher and the students.

Large Group/Whole Class Work

At the beginning of the lesson:

- Pose the problem to students and as a large group determine what the problem is asking and have students in their own words explain what the problem is about.
- Have students brainstorm and chart for everyone a list of possible methods and/or tools that may be useful in solving the problem based on prior problem solving experiences.

At the end of the lesson:

- Allow students to present their work and talk through how they solved the problem(s).
- Make sure that different strategies are presented and help students to see the mathematical connections between each strategy.
- Focus students attention on the strategies used, not on the students presenting the strategy.
- Have students summarize the lesson from their point of view and fill in any gaps and call attention to new vocabulary used if students do not do this specifically.

Tip: Create rituals/routines for what is expected during whole class instruction so that students are able to focus their thinking. For example, if during the end of lesson whole group time you want students to ask questions of the student presenters, provide a list of question starters to students that they can use as a scaffold for creating their question.

Small Group/Pair Work

Working in small groups and/or in pairs allows students to work on challenging problems in a safe environment. However, there must be a balance between group work and individual accountability for learning.

Create guidelines that students will use every time they work collaboratively, such as:

- Move into your groups quickly.

- Allow each person to present their initial ideas about the problem before starting to work the problem out together.
- Don't interrupt a peer's presentation.
- If you are unsure or confused about something that is said, ask for clarification.
- Assign a number to each person in the group and then role a number cube and have that person present the group's work to the rest of the class. Other members of the group are free to assist in the presentation but the person who's number is called is the main presenter.
- Ask questions of each group member during the presentation.
- Remember that mistakes are part of the foundation of learning. We learn from our mistakes in the real world and we can learn from our mathematical mistakes as well.

Helping Students Make Decisions

When working alone or in large or small groups, students will come to a point in the lesson where they will have to make a decision about what to do to get started or what to do next. This is not always an easy step to take, especially for young students. Help students learn how to make decisions by doing the following:

- **Use Agreement Statements.** When someone presents an idea students may choose to agree or disagree with a statement or to state that they need more information. In addition, they are asked to describe their thinking about why they agree, disagree, or are unsure. As a group students can describe what they can do to investigate the statement by testing their ideas, examining what is already known, or using other means of mathematical inquiry.
- **Use Agreement Circles.** *Agreement Circles* provide a kinesthetic way to activate thinking and engage students in discussing and defending their mathematical ideas. Students stand in a large circle as the teacher reads a statement. The students who agree with the statement step to the center of the circle. Those who disagree remain standing on the outside of the circle. Those in the inner circle face their peers still standing around the outside circle and then divide themselves into small groups of students who agree and disagree. The small groups then engage in discussion to defend their thinking. This is repeated with several rounds of statements relating to the same topic, each time with students starting by standing around the large circle.
- **When making generalizations make a determination of Always, Sometimes, or Never True.** *Always, Sometimes, or Never True* involves a set of statements that students examine and decide if they are always true, sometimes true, or never true. This strategy is useful in revealing whether students over-generalize or under-generalize a mathematical concept. In addition, they are asked to provide a justification for their answer.

- **Have students look back.** If unsure of what steps to take when solving a problem, have students look back at what they have learned over a given instructional period of time. Students recount specific examples of things they know now that they didn't know before and describe how they learned them. This strategy provides students with an opportunity to look back on and summarize their learning. Asking students "how they learned it" helps them think about their own learning and the different ways, as learners, they are able to integrate new mathematical understandings.

(Keely & Tobey, 2011)

Next Steps

Instructions: Review your work during Module 2 and generate a list of implementation steps you would like to do, think you can do, challenges you might face, and ways to work around the challenges. Be prepared to share with the group.

What do you think should be the next steps at your school to promote implementation of CCS-Math?

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What can teachers do now to promote implementation of CCS-Math?

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What are some expected challenges?

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How can you work around and through the challenges?

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Instructional Implementation Sequence:
Attaining the CCSS Mathematical Practices
Engagement Strategies

Strategy	Description	Practice	Degree	Matrix Code
Think pair-share	<p>Pair-Share, or Think-Pair-Share, is a strategy easy to implement in any classroom at any grade level or subject. This strategy does not require any other change in pedagogy or materials. For pair – share, teachers merely ask a question or assign a problem and allow students to think and work with a partner for one to three minutes before requesting an answer to the question or problem. In think – pair – share students are given a brief period of time to think independently before working with a partner. While effective in results, this strategy is a significant first step in engaging all students in classroom instructional activities.</p>	<ul style="list-style-type: none"> • Make sense of problems. • Critique the reasoning of others. 	<ul style="list-style-type: none"> • Explain their thought processes in solving a problem one way. • Understand and discuss other ideas and approaches. 	<p>1a I</p> <p>3b I</p>
Showing thinking in classrooms	<p>Teachers need to work toward higher degrees of student involvement in classroom activities. Once pair – share is incorporated into classroom routines, teachers need to incorporate additional strategies that promote “every pupil response” (EPR). EPR strategies include such responses as “thumbs up/thumbs down,” or use of individual white boards for noting answers. Students are also pressed to be more aware of their thinking and express their thinking in more detail. Students are routinely asked to share their thinking in mathematics classrooms. However, what is routinely accepted as thinking is actually process description. Students merely provide the steps they used to solve the problem, not their reasoning and thinking about how they knew which processes</p>	<ul style="list-style-type: none"> • Construct viable arguments. • Attend to precision. 	<ul style="list-style-type: none"> • Explain their thinking for the solution they found. • Communicate their reasoning and solution to others. 	<p>3a I</p> <p>6 I</p>

	to use. In order to reveal student thinking, more challenging, open-ended problems are needed.			
Questioning and wait time	As thinking is increased in mathematics classroom, better questioning and wait time are required. Teachers need to provide thought provoking questions to students, then allow the students time to think and work toward an answer.	<ul style="list-style-type: none"> • Make sense of problems. • Persevere in solving them. • Construct viable arguments. • Critique the reasoning of others. 	<ul style="list-style-type: none"> • Explain their thought processes in solving a problem in several ways. • Stay with a challenging problem for more than one attempt. • Explain their thinking with accurate vocabulary both their own thinking and thinking of others. • Explain other student’s solutions and identify strengths and weaknesses of the solution. 	<p>1a IN</p> <p>1b I</p> <p>3a IN</p> <p>3b IN</p>
Grouping and engaging problems	The strategy of “grouping and engaging problems” is a significant shift in pedagogy and materials. Students are given challenging problems to work, and allowed to work on the problem in a group of two, three, or four. Challenging mathematics problems take time, effort, reasoning, and thinking to solve.	<ul style="list-style-type: none"> • Make sense of problems. • Persevere in solving them. • Reason abstractly and quantitatively. 	<ul style="list-style-type: none"> • Discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways. • Try several approaches in finding a solution, and seek only hints if stuck. • Reason with models or pictorial representations to solve problems. 	<p>1a A</p> <p>1b IN</p> <p>2 I</p>

		<ul style="list-style-type: none"> • Reason abstractly and quantitatively. • Construct viable arguments. • Critique the reasoning of others. • Model with mathematics. • Use appropriate tools strategically. • Use appropriate tools strategically. • Look for and express regularity in repeated reasoning. 	<ul style="list-style-type: none"> • Translate situations into symbols for solving problems. • Justify and explain, with accurate language and vocabulary, why their solution is correct. • Compare and contrast various solution strategies and explain the reasoning of others. • Use models to represent and solve a problem, and translate the solution to mathematical symbols. • Use the appropriate tool to find a solution. • Select from a variety of tools the ones that can be used to solve a problem, and explain their reasoning for the selection. • Look for patterns, and use if, then reasoning strategies for obvious patterns. 	<p>2 IN</p> <p>3a A</p> <p>3b A</p> <p>4 I</p> <p>5 I</p> <p>5 IN</p> <p>8 I</p>

<p>Using questions and prompts with groups</p>	<p>Once students are provided with opportunities to solve challenging problems in groups, teachers need to increase their ability to ask supporting questions that encourage students to continue working, provide hints or cues without giving students the answers, and ask probing questions to better assess student thinking and current understanding.</p>	<ul style="list-style-type: none"> • Model with mathematics. • Look for and make use of structure. 	<ul style="list-style-type: none"> • Use models and symbols to represent and solve a problem, and accurately explain the solution representation. • Look for structure within mathematics to help them solve problems efficiently. 	<p>4 IN</p> <p>7 I</p>
<p>Allowing students to struggle</p>	<p>Students learn to persevere in solving challenging mathematics problems by being allowed to struggle with challenging problems. Students need to understand that mathematical problems do not usually have a quick, easy solution. Effective effort is a life-skill and should be learned interdependently and independently. Appropriate degree of difficulty is foremost on teachers' minds. If the problem is too easy, students do not need to struggle. If the problem is far too difficult, students are not capable of solving the problem. Teachers need to balance working in groups and working independently, and be able to quickly adjust grouping strategies as the need arises.</p>	<ul style="list-style-type: none"> • Persevere in solving them. • Model with mathematics. • Use appropriate tools strategically. • Attend to precision. • Look for and make 	<ul style="list-style-type: none"> • Struggle with various attempts over time, and learn from previous solution attempts. • Use a variety of models, symbolic representations, and technology tools to demonstrate a solution to a problem. • Combine various tools, including technology, explore and solve a problem as well as justify their tool selection and problem solution. • Incorporate appropriate vocabulary and symbols in communicating their reasoning and solution to others. • Compose and decompose number situations and 	<p>1b A</p> <p>4 A</p> <p>5 A</p> <p>6 IN</p> <p>7 IN</p>

		<p>use of structure.</p> <ul style="list-style-type: none"> • Look for and express regularity in repeated reasoning. 	<p>relationships through observed patterns in order to simplify solutions.</p> <ul style="list-style-type: none"> • Find and explain subtle patterns. 	8 IN
Encouraging reasoning	<p>Students need to be encouraged to carefully think about mathematics, and to understand their level of knowledge. They also need to be able to accurately communicate their thinking. Reasoning, in this context, is used to convey having students stretch their understanding and knowledge to solve challenging problems. Reasoning requires students to pull together patterns, connections, and understandings about the rules of mathematics, and then apply their insight into finding a solution to a difficult, challenging problem.</p>	<ul style="list-style-type: none"> • Reason abstractly and quantitatively. • Attend to precision. • Look for and make use of structure. • Look for and express regularity in repeated reasoning. 	<ul style="list-style-type: none"> • Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations. • Use appropriate symbols, vocabulary, and labeling to effectively communicate and exchange ideas. • See complex and complicated mathematical expressions as component parts. • Discover deep, underlying relationships, i.e. uncover a model or equation that unifies the various aspects of a problem such as discovering an underlying function. 	<p>2 A</p> <p>6 A</p> <p>7 A</p> <p>8 A</p>