Mathematics Instructional Cycle Guide

Concept (F.LE.1 and F.LE.5)

Created by Amanda A. Peterson, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP3: Construct viable arguments and critique the reasoning of others.

- Students explain and support their statements about linear and exponential functions.

MP4: Model with mathematics.

- Students create linear and exponential models and then create tables and graphs to represent them.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(page 2)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pages 3 - 8)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pages 9 - 13)**
- Supporting lesson materials **(pages 14 - 25)**
- Precursory research and review of standard **F.LE.1 and F.LE.5** and assessment items that illustrate the standard **(pages 26 - 31)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the ***Exponential verse Linear Functions Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **List needed lesson materials here**
  - Blocks
  - Crayons
  - Copies
  - Projector
  - iPad/Labtop/Smartboard
  - Glue or tape

TIME NEEDED

Exponential verse Linear Functions administration: **5 – 10 minutes**
Follow-Up Lesson Plan: **1 to 2 instructional blocks**
Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">Sally took a job at Pear© with a starting salary of $52,000. According to her contract, each following year Sally continues to work at the company, she is eligible for a raise equal to 4% of her previous year's salary.

Sally calculated her salary for the next few years and this is what she determined:

<table><tr><th>Year</th><th>Salary in dollars</th></tr><tr><td>0</td><td>$52,000</td></tr><tr><td>1</td><td>$54,080</td></tr><tr><td>2</td><td>$56,160</td></tr><tr><td>3</td><td>$58,240</td></tr><tr><td>4</td><td>$60,320</td></tr><tr><td>5</td><td>$62,400</td></tr></table>
Sally is incorrect with calculating out her salary for the next few years.

1. Find the correct salary for the years given.

2. Explain both models, Sally's and yours, that were used and their differences.</td><td>CT Core Standard:</td><td>F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>How do students distinguish between situations that can be modeled with linear functions and with exponential functions by proving that linear functions grown by equal differences over equal intervals, and that exponential functions grow by equal factors over open intervals.
Interpret the parameters in a linear and exponential function in terms of the context
Calculate and exponential model</td></tr>
</table>

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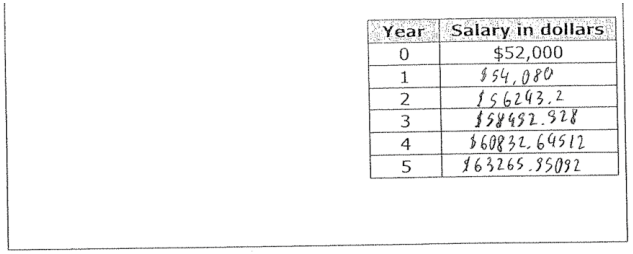
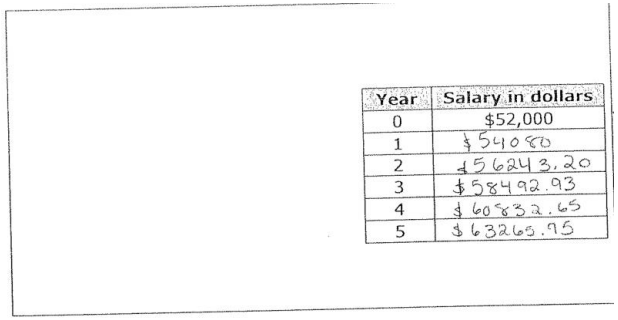
Step 2: Analyze and Interpret Student Work		
Student Response Guide		
Got It	**Developing**	**Getting Started**
What will a response include from a student who has demonstrated conceptual understanding and mastery?	What will a response include from a student who demonstrated some understanding and possibly some misunderstandings or undeveloped understanding?	What will a response include from a student who demonstrated minimal understanding and possibly misconceptions?



Year	Salary in dollars
0	$52,000
1	$54,080
2	$56243.2
3	$58492.928
4	$60832.64512
5	$63265.95092

2. Explain both models, Sally's and yours, that were used and their differences.

I Sally model, she added only the 4% of $52,000 for each year. But in my, I added the 4% of the Salary that Sally got from the previous year.



Year	Salary in dollars
0	$52,000
1	$54080
2	$56243.20
3	$58492.93
4	$60832.65
5	$63265.95

2. Explain both models, Sally's and yours, that were used and their differences.

I multiplied each time by 1.04 her highest number was $62,400 and mine was $63,265.95

Year	Salary in dollars
0	$52,000
1	54080
2	56243.20
3	58492.93
4	60832.65
5	63,265.95

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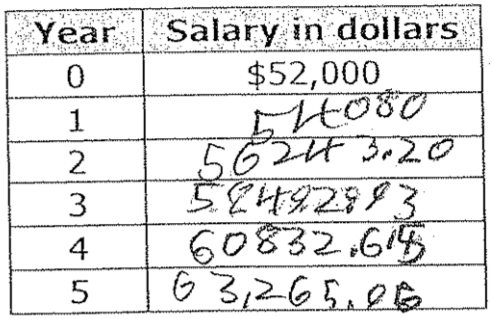
Getting Started

Student Response Example

What will a response include from a student who demonstrated minimal understanding and possibly misconceptions?



Year	Salary in dollars
0	$52,000
1	54080
2	56243.20
3	58492.93
4	60832.65
5	63,265.06

Indicators

- What possible indicators may be included in a student response who has demonstrated minimal understanding of the standard?
  - The student could only fill out the table.
  - There was no explanation or math shown.
- What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
  - The student understands how to find 4% and calculate a salary.
- What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
  - The student did not know how to write a function for their model or explain their calculations.
  - The student did not understand what was incorrect about the first model.
  - The student could not explain either of the models.

In the Moment Questions/Prompts

What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking?

- Can you explain how you completed the table?
- What kind of model is that and how do you know?
- What was wrong with Sally's model?

Closing the Loop (Interventions/Extensions)

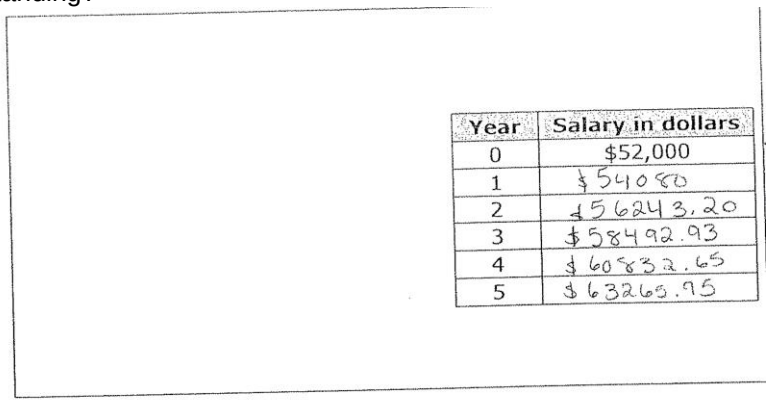
LZ video lesson links that may extend and deepen student understanding and procedural fluency

LZ2391: Identify linear functions by linking equal differences to equal intervals
https://learnzillion.com/student/lessons/2391
LZ2547: Graph an exponential function
https://learnzillion.com/lessons/2547-graph-an-exponential-function
LZ2590: Understand the rate of change in exponential growth functions
https://learnzillion.com/lessons/2590-understand-the-rate-of-change-in-exponential-growth-functions
LZ2654: Distinguish between linear and exponential functions by examining intervals
https://learnzillion.com/lessons/2654-distinguish-between-linear-and-exponential-functions-by-examining-intervals
LZ2693: Distinguish between linear and exponential functions using tables
https://learnzillion.com/lessons/2693-distinguish-between-linear-and-exponential-functions-using-tables
LZ2606: Write and graph an exponential function by examining a table
https://learnzillion.com/lessons/2606-write-and-graph-an-exponential-function-by-examining-a-table

Developing

Student Response Example

What will a response include from a student who demonstrated some understanding and possibly some misunderstandings or undeveloped understanding?



Year	Salary in dollars
0	$52,000
1	$54080
2	$56243.20
3	$58492.93
4	$60832.65
5	$63265.95

2. Explain both models, Sally's and yours, that were used and their differences.

I multiplied each time by 1.04 her highest number was $62,400 and mine was $63,265.95

Indicators

- What possible indicators may be included in a student response who has demonstrated some understanding of the standard?
  - The student can find the correct salary.
  - The student can explain the mathematics that they used to calculate the new salary.
- What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
  - The student understands how to find 4% and calculate a salary.
  - The student understands how to find the multiplier in a exponential function.
- What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
  - The student did not know how to write a function for their model.
  - The student did not understand what was incorrect about the first model.

In the Moment Questions/Prompts

What questions could you ask, or feedback could you provide in the moment to develop student understanding, create disequilibrium, or advance student thinking?

- Can you explain the initial model that Sally used?
- What kind of model did you create?
- Can you write the equation for the model you used?

Closing the Loop (Interventions/Extensions)

LZ video lesson links that may extend and deepen student understanding and procedural fluency

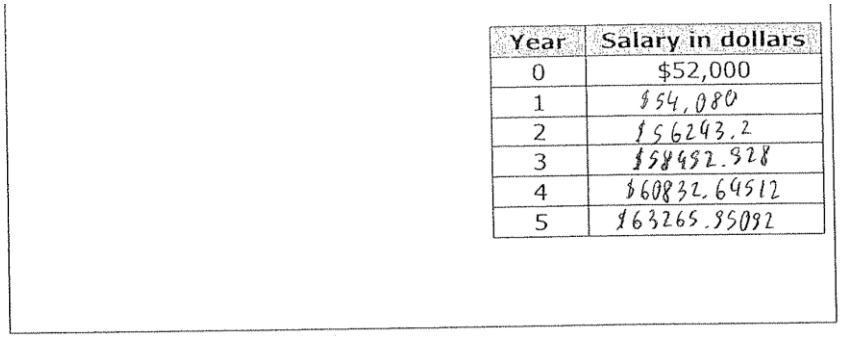
LZ2391: Identify linear functions by linking equal differences to equal intervals
https://learnzillion.com/student/lessons/2391
LZ2547: Graph an exponential function
https://learnzillion.com/lessons/2547-graph-an-exponential-function
LZ2590: Understand the rate of change in exponential growth functions
https://learnzillion.com/lessons/2590-understand-the-rate-of-change-in-exponential-growth-functions
LZ2654: Distinguish between linear and exponential functions by examining intervals

	https://learnzillion.com/lessons/2654-distinguish-between-linear-and-exponential-functions-by-examining-intervals LZ2693: Distinguish between linear and exponential functions using tables https://learnzillion.com/lessons/2693-distinguish-between-linear-and-exponential-functions-using-tables LZ2606: Write and graph an exponential function by examining a table https://learnzillion.com/lessons/2606-write-and-graph-an-exponential-function-by-examining-a-table

Got it

Student Response Example

What will a response include from a student who has demonstrated conceptual understanding and mastery?



Year	Salary in dollars
0	$52,000
1	$54,080
2	$56243.2
3	$58492.928
4	$60832.64512
5	$63265.35092

2. Explain both models, Sally's and yours, that were used and their differences.

I Sally model, she added only the 4% of $52,000 for each year. But in my, I added the 4% of the salary that Sally got from the previous year.

Indicators

- What indicators must be included in an exemplar student response
  - The student is able to calcite the correct salary.
  - The student is able to explain both models.
- What strategies, and representations will or will not be used? What understandings or procedural fluency does the student response reveal?
  - The student understands the differences between the two models.
  - The student understand that one model is linear and the other in exponential.
  - The student can explain their mathematical reasoning.
- What undeveloped understandings, misconceptions, and common mistakes may be revealed in the student response to this item?
  - The student did not use the precise mathematical vocabulary of linear and exponential.

Thinking

What questions could you ask, or feedback could you provide in the moment to extend or push student understanding, create disequilibrium, or advance student thinking?

- What kind of function is it when you add a constant amount?
- What kind of function is it when you multiple by a constant amount?
- Can you write the equation for the model that you created?

Closing the Loop (Interventions/Extensions)

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LZ2654: Distinguish between linear and exponential functions by examining intervals
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LZ2693: Distinguish between linear and exponential functions using tables
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	LZ2606: Write and graph an exponential function by examining a table https://learnzillion.com/lessons/2606-write-and-graph-an-exponential-function-by-examining-a-table LZ3162: Graph exponential growth functions https://learnzillion.com/lessons/3162-graph-exponential-growth-functions#quickcode-modal LZ3343: Solve for time in a continuously compounding function https://learnzillion.com/lessons/3343-solve-for-time-in-a-continuously-compounding-function#quickcode-modal

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Students will be able to distinguish between linear and exponential functions.
Content Standard(s):	F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions. F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
Targeted Practice Standard :	MP3: Construct viable arguments and critique the reasoning of others. Are students able to explain and support their statements about linear and exponential functions? MP4: Model with mathematics. Are students able to create tables and graphs to model linear and exponential models?

Mathematical Goals	Success Criteria
Distinguish between situations that can be modeled with linear functions and with exponential functions. Interpret the parameters in a linear or exponential function in terms of a context.	Create a linear and exponential model with blocks. Write a linear equation, $y = mx + b$ or exponential equation, $y = ab^x$, given a table, graph or description. Write linear or exponential equations to represent real world problems.

Launch (Probe and Build Background Knowledge)

Purpose: ***Activate and asses background knowledge about patterns and liner functions.***

Provide the students with graph paper. Project the following question.

"Which would you rather have, $1,000,000 or 1 cent that doubles in value every day for a month?"

- Discuss the statement with their group members.
- Come to a consensus about which method they would prefer.
- Make sure to justify your answer.
- Have the students present their findings to the class.

Visual representation of the problem can be found here titled, *Penny a Day*:
https://www.youtube.com/watch?v=Uka5owZdFDI

- Start the video and end it at 0:35 to introduce the topic.
- 0:35 – 1:21 to show a visual representation of the problem.
- 1:22 – 2:41 to show the explanation and answer.

Interdisciplinary video titled, *1967 Scrooge McDuck and Money*:
https://www.youtube.com/watch?v=BAFSrKqIKdc

- This video discusses money in general connecting the history and mathematics.

Instructional Task

Purpose: ***Introduce linear and exponential functions with a table, equation, graph and word problems.***

Engage (Setting Up the Task)

Project the note sheet *Exponential versus Linear Functions* (pages 14 – 17). For each example do the first two and have the students try the second two examples.

Guiding Questions:

- Example 1:
  - What are the patterns that you are noticing?
  - What is different about the last table then the other three?
- Example 2:
  - What are the similarities and differences that you notice about the equations?
- Example 3:
  - Can you always just look at the viewing window/image that you are given?
  - What does the rate of change tell you in an exponential function?
- Example 4:
  - List the key words for a linear and an exponential word problem.
  - When you write an equation for either linear of exponential you have to make sure you have what?

Explore (Solving the Task)

1. Provide students time to work on *Creating Exponential and Linear Models* activity (pages 18 – 21) and modeling blocks in groups of two. Circulate to observe, questions, and note students who are strategic candidate to share out responses. Possible questions/prompts to ask students engaged in task:

Focusing Questions	Probing Questions	Advancing Questions
• *What information do you know that can help you make sense of this problem?* • *Linear functions mean you _____ each time?* • *Exponential functions mean you _____ each time?* • *What is the formula for a linear function?* • *What is the formula for and exponential function?* • *How do you write a recursive rule?*	• *What is the initial value of every linear and exponential model?* • *Can you build the next two stages of your model?* • *What would the 6th stage be?*	• *How might you prove your solutions another way?* • *What would stage 10 be?* • *Can you create any other type of function?*

When the students are done either have the student hang up their work or project their work with a document camera.

2. Provide students time to work on *Exponential versus Linear Functions in the Real World* activity (pages 22 -24 with differentiation on page 24). Groups of two or three are good for this activity and every group needs scissors and glue. Circulate to observe, questions, and note students who are strategic candidate to share out responses. Possible questions/prompts to ask students engaged in task:

Focusing Questions	Probing Questions	Advancing Questions
• *What information do you know that can help you make sense of this problem?* • *Find the pattern in each of the tables.*	• *Find the pattern and y intercept in each of the tables.* • *How did/do you match the graphs quickly to the equations?*	• *How might you prove your solutions another way?* • *Can you write more then one situation to model the data?*

- *Linear functions mean you _____ each time?*
- *Exponential functions mean you _____ each time?*
- *What is the formula for a linear function?*
- *What is the formula for and exponential function?*
- *Can you explain how you wrote your real life problem?*
- *How can you check your work?*
- *Create your own word problem and have your partner write the equation. Make sure to check each others work.*

When the students are done either have the student hang up their work or project their work with a document camera.

Elaborate (Discuss Task and Related Mathematical Concepts)

Call the students back together to facilitate a task discussion. Project or post the following questions for students to consider as other share their work.

1. How is the approach the same or different as the approach we used?
2. Do you agree or disagree with this solution? Why?

Common Misunderstanding

Purpose: ***Address a common misunderstanding students often have about writing the equations for exponential functions.***

Remind the students about the equation of an exponential function, $y = a(b)^x$.

Project the following problem.

Amy wrote the following equation to model her table of value.

$y = 2(4)^x$

X	Y
0	4
1	8
2	16

Is Amy right? Explain your thinking.

- Give the students 1 minute to think about the problem.
- Instruct the students to write down the observations they made about Amy's problem. If they think the problem is wrong have the students create the equation they think it should be.
- Have the students turn and talk with a partner about their observations. Have each group fill out the chart below (pages 26 – 27).

Initial Value:	**Pattern:**	**Equation:**
Model:		

Real Life Application:

_____

_____

_____

- Have a member from each group come up to the board and hang up their responses.
- Facilitate a discussion to share the reasoning from students.

Checking for Understanding

Purpose: ***Provide students an opportunity to self – asses their own learning related to the success criteria by asking them these questions on the exit slip*** *(pages 28 – 29).*

Rate yourself on how comfortable you are with *identifying a linear function*.

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Rate yourself on how comfortable you are with *identifying an exponential function*.

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Rate yourself on how comfortable you are with *creating a linear or exponential model*.

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Rate yourself on how comfortable you are with *writing the equation for an exponential function.*

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Closure

Purpose: ***Pose the following question as an exit pass*** *(pages 28 – 29)* ***to elicit evidence of students' understanding of exponential verse linear functions*****.**

Solve the given problems:

X	Y
0	27
1	9
2	3
3	1/3

Exponential **or** Linear

Reason:

Equation:

X	Y
-1	6
0	7.5
1	9
2	10.5

Exponential **or** Linear

Reason:

Equation:

Extension Task

Purpose: ***Provide an extension task for those students who are ready to deepen their understanding of linear verse exponential models.***

Compound interest is found in loans, mortgages, credit cards and savings account. Compound interest is an exponential function. If you get a credit card that charges 21% interest every month, it will be an exponential function.

1. Write the exponential function if you started with a balance of 100.
2. Assuming that you do not pay your bill for 3 months, how much do you owe?
3. How much will you owe in a year?

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Exponential Functions vs. Linear Functions Note Sheet

The growth pattern of a **linear function** is a constant rate added or subtracted each time.

The growth pattern of an **exponential function** is a constant rate multiplied of divided each time.

Example 1: Tables

X	Y
0	3
1	9
2	27
3	81

Exponential **or** Linear

Reason:

Equation:

X	Y
0	-4
1	-6
2	-8
3	-10

Exponential **or** Linear

Reason:

Equation:

X	Y
0	10
1	5
2	2.5
3	1.25

Exponential **or** Linear

Reason:

Equation:

X	Y
0	19
2	38
4	57
6	76

Exponential **or** Linear

Reason:

Equation:

Example 2: Equations

$y = 2x - 6$

Exponential **or** Linear

Increasing **or** Decreasing

Initial Value:

Pattern:

$y = 3(2)^x$

Exponential **or** Linear

Increasing **or** Decreasing

Initial Value:

Pattern:

$y = 4(1.05)^x$

Exponential **or** Linear

Increasing **or** Decreasing

Initial Value:

Pattern:

$3x + 4y = 16$

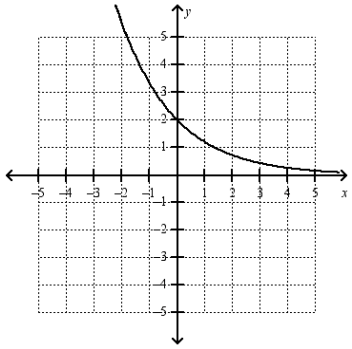
Exponential **or** Linear

Increasing **or** Decreasing

Initial Value:

Pattern:

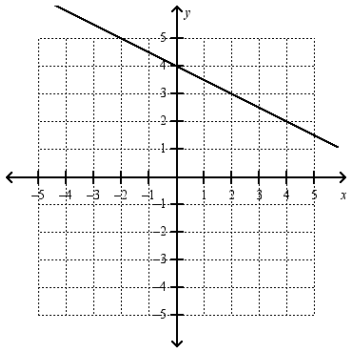
Example 3: Graphs



Exponential **or** Linear

Increasing **or** Decreasing

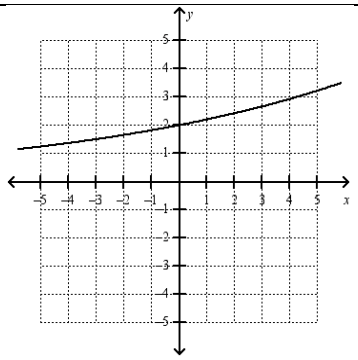
Y Intercept: _____



Exponential **or** Linear

Increasing **or** Decreasing

Y Intercept: _____



Exponential **or** Linear

Increasing **or** Decreasing

Y Intercept: _____

Example 4: Word Problems

1. Charles borrows $200 from his friend and each month he pays him half of the remaining balance.

Exponential **or** Linear	Increasing **or** Decreasing

2. My pet iguana was 20 cm long when I got him. Then each month his length was 8% longer than the month before.

Exponential **or** Linear	Increasing **or** Decreasing

3. Sue's scarf was only 8 inches long. Her grandmother took it and each day she knitted 75% of its original length.

Exponential **or** Linear	Increasing **or** Decreasing

Answer Key:

Exponential Functions vs. Linear Functions Note Sheet

The growth pattern of a **linear function** is a constant rate added or subtracted each time.

The growth pattern of an **exponential function** is a constant rate multiplied of divided each time.

Example 1: Tables

X	Y
0	3
1	9
2	27
3	81

Exponential **or** Linear

Reason:
You are multiplying by 3 each time.

Equation:
$y = 3(3)^x$

X	Y
0	-4
1	-6
2	-8
3	-10

Exponential **or** Linear

Reason:
You are by subtracting by 2 each time.

Equation:
$y = -2x - 4$

X	Y
0	10
1	5
2	2.5
3	1.25

Exponential **or** Linear

Reason:
You are dividing by 2 (multiplying by ½) each time.

Equation:
$y = 10(0.5)^x$
$y = 10(1/2)^x$

X	Y
0	19
2	38
4	57
6	76

Exponential **or** Linear

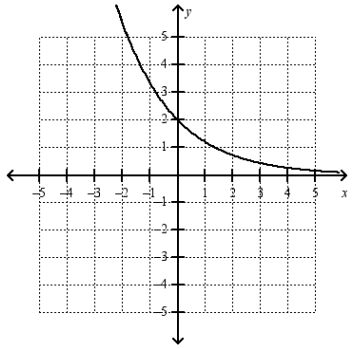
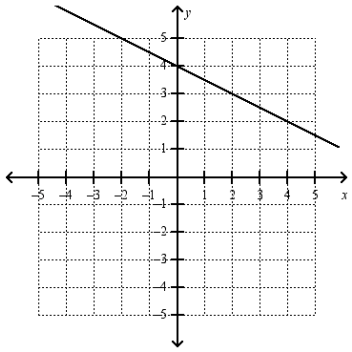
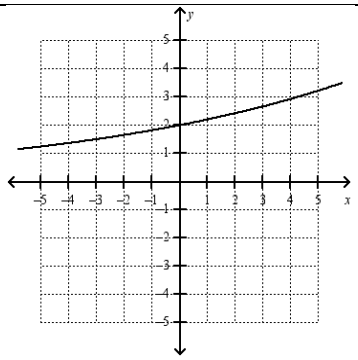
Reason:
You are adding each time.

Equation:
$y = 9.5x + 19$
$y= (19/2)x + 19$

Example 2: Equations

$y = 2x - 6$	$y = 3(2)^x$	$y = 4(1.05)^x$	$3x + 4y = 16$
Exponential **or** Linear	Exponential **or** Linear	Exponential **or** Linear	Exponential **or** Linear
Increasing **or** Decreasing	Increasing **or** Decreasing	Increasing **or** Decreasing	Increasing **or** Decreasing
Initial Value: *Start at -6*	Initial Value: *Start at 3*	Initial Value: *Start at 4*	Initial Value: *Start at 4*
Pattern: *Add 2*	Pattern: *Multiply by 2*	Pattern: *Multiply by 1.05* *Increase by 5%*	Pattern: *Subtract -3/4*

Example 3: Graphs

	Exponential **or** Linear Increasing **or** Decreasing Y Intercept: *(0, 2)*
	Exponential **or** Linear Increasing **or** Decreasing Y Intercept: *(0, 4)*
	Exponential **or** Linear Increasing **or** Decreasing Y Intercept: *(0, 2)* *Note the rate of change is not constant between (-6, 2), (0,2) and (4,3).*

Example 4: Word Problems

1. Charles borrows $200 from his friend and each month he pays him half of the remaining balance.

Exponential **or** Linear Increasing **or** Decreasing

2. My pet iguana was 20 cm long when I got him. Then each month his length was 8% longer than the month before.

Exponential **or** Linear Increasing **or** Decreasing

3. Sue's scarf was only 8 inches long. Her grandmother took it and each day she knitted 75% of its **original** length.

Exponential **or** Linear Increasing **or** Decreasing

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Name: _____

Creating Exponential and Linear Models

1. With the blocks create a **linear** model.

Draw a picture of the model to the right.

Fill out the table of values based on your model.

X	Y
1	
2	
3	
4	

Linear Model

Write a *recursive* rule for the linear model you created.

Write an *explicit* rule for the linear model you created.

2. With the blocks create an **exponential** model.

Draw a picture of the model to the right.

Fill out the table of values based on your model.

X	Y
1	
2	
3	
4	

Exponential Model

Write a *recursive* rule for the exponential model you created.

Write an *explicit* rule for the exponential model you created.

Student Examples:

Creating Exponential and Linear Models

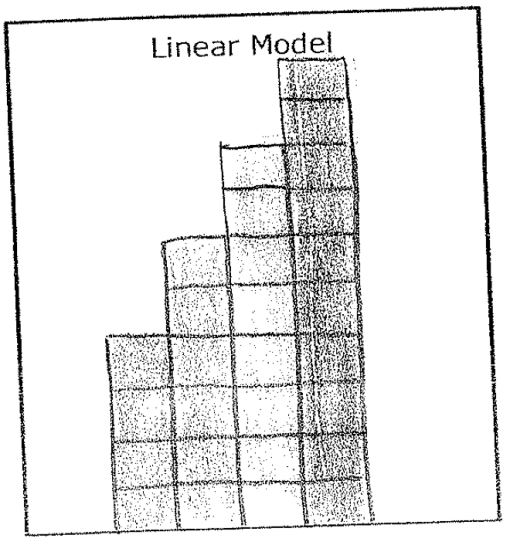
1. With the blocks create a **linear** model.

Draw a picture of the model to the right.

Fill out the table of values based on your model.

X	Y
1	4
2	6
3	8
4	10

Linear Model



Write a *recursive* rule for the linear model you created.

Start with 2 and add 2 each time

Write an *explicit* rule for the linear model you created.

$y = 2x + 2$

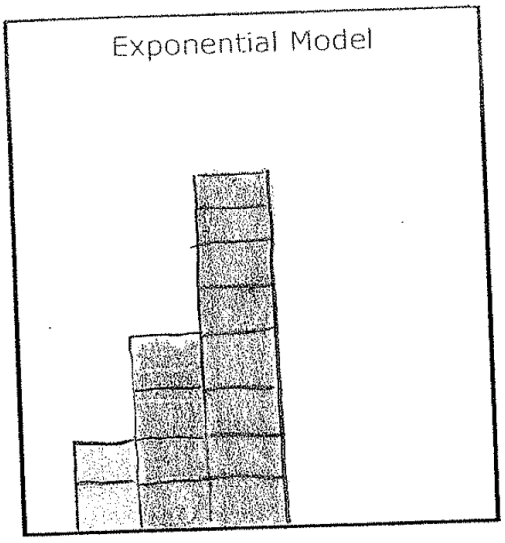
2. With the blocks create an **exponential** model.

Draw a picture of the model to the right.

Fill out the table of values based on your model.

X	Y
1	2
2	4
3	8
4	16

Exponential Model



Write a *recursive* rule for the exponential model you created.

Start with 1 and multiply by 2 each time

Write an *explicit* rule for the exponential model you created.

$y = 1(2)^x$

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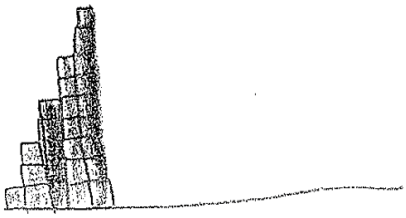
Creating Exponential and Linear Models

1. With the blocks create a **linear** model.

Draw a picture of the model to the right.

Fill out the table of values based on your model.

Linear Model



X	Y
1	3
2	5
3	7
4	9

Write a *recursive* rule for the linear model you created.

Start with 1 and add 2

Write an *explicit* rule for the linear model you created.

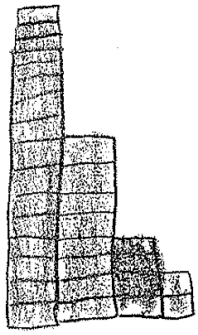
$y = 2x + 1$

2. With the blocks create an **exponential** model.

Draw a picture of the model to the right.

Fill out the table of values based on your model.

Exponential Model



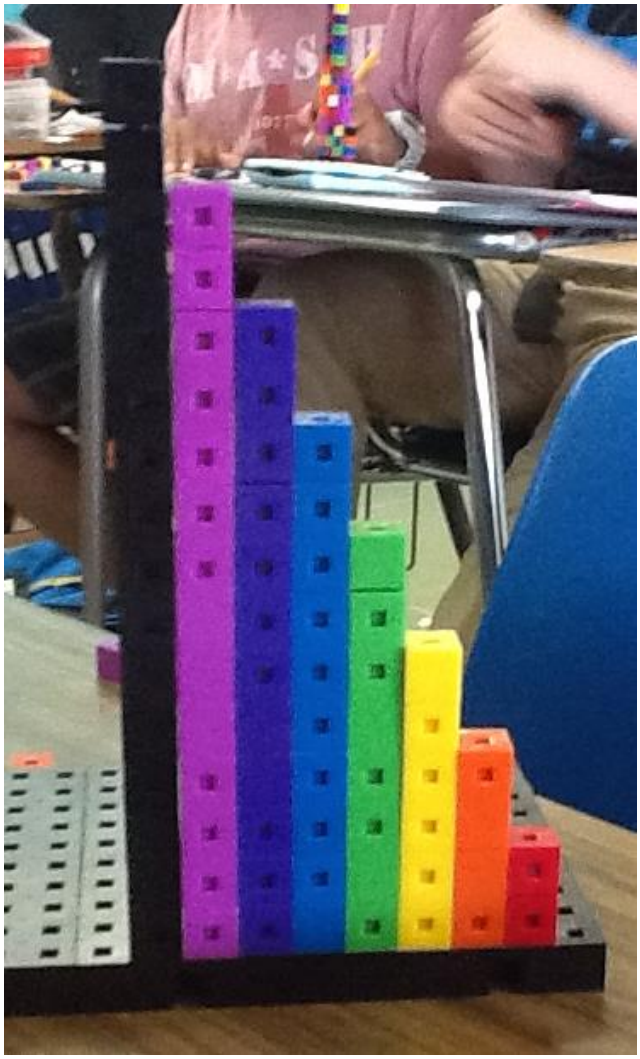
X	Y
1	16
2	8
3	4
4	2

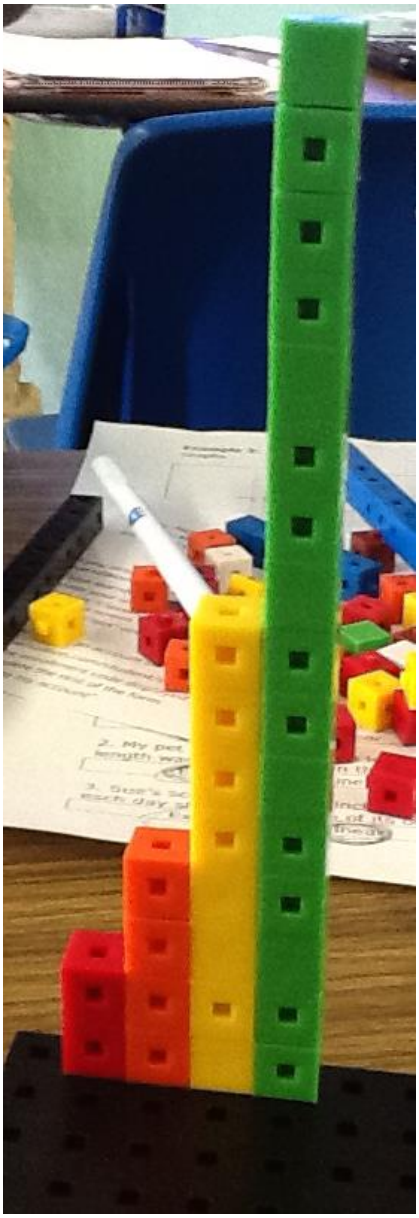
Write a *recursive* rule for the exponential model you created.

Start with 32 and multiply by .5

Write an *explicit* rule for the exponential model you created.

$y = 32(1/2)^x$





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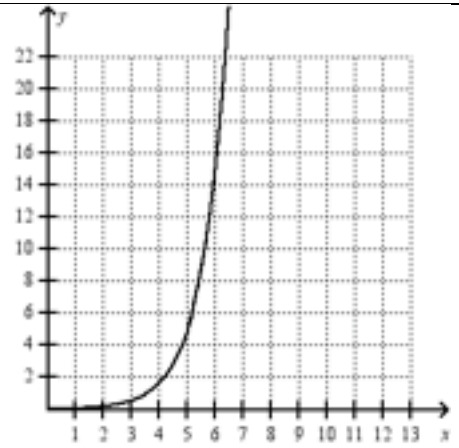
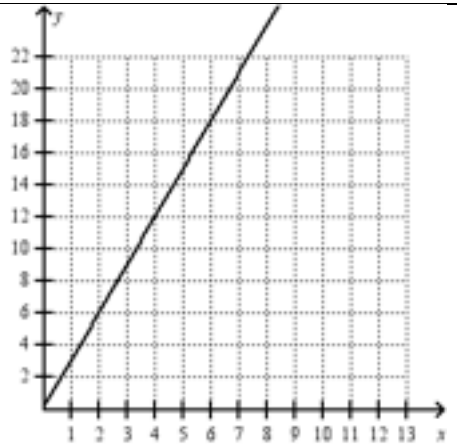
Exponential versus Linear Functions in the Real World

Directions: Cut out the graphs and tables below and paste or tape them to the equation they match on the next page. Once you have matched them create a real life problem to model the equation.

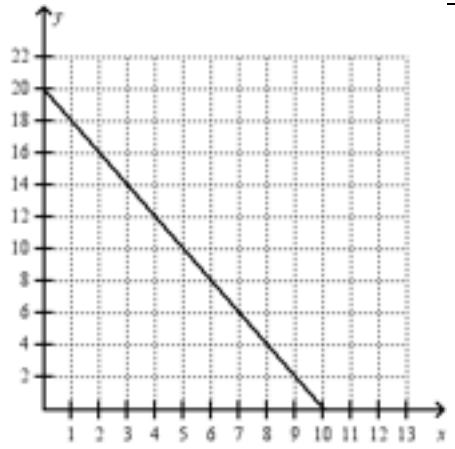
$y = 3x + 0.02$	$y = .02(3)^x$	$y = -2x + 20$	$y = 20(0.5)^x$

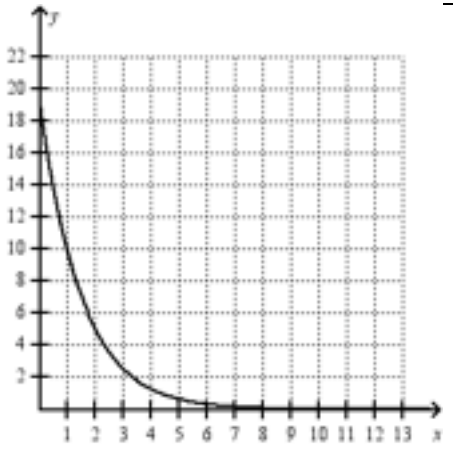




X	Y
0	0.02
2	6.02
4	12.02
6	18.02
8	24.02

X	Y
0	20
1	18
2	16
3	14
4	12





X	Y
0	0.02
4	1.62
8	131.22
12	10629
16	860934

X	Y
0	20
1	10
2	5
3	2.5
4	1.25

LearnZillion

Exponential versus Linear Functions in the Real World

$y = 3x + 0.02$	$y = .02(3)^x$	$y = -2x + 20$	$y = 20(0.5)^x$
Paste Graph Here	Paste Graph Here	Paste Graph Here	Paste Graph Here
Paste Table Here	Paste Table Here	Paste Table Here	Paste Table Here
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____

Differentiation:

Provide the struggling learners with two of the stories below. The student will then match them to the appropriate functions and have an example to look at to create the other functions.

Story 1: Anna gets paid $3 for walking the dog everyday. She currently has 2 cents in her piggybank.

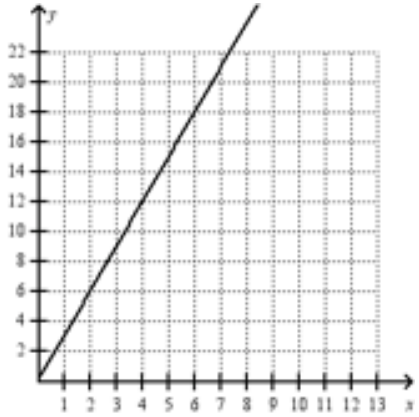
Story 2: Bill started with 2 cents and it triples everyday.

Story 3: Charlie has 20 dollars and spends 2 dollars on coffee every morning.

Story 4: Danielle has 20 dollars and losses 50% of that everyday because of the stock market.

Answer Key:

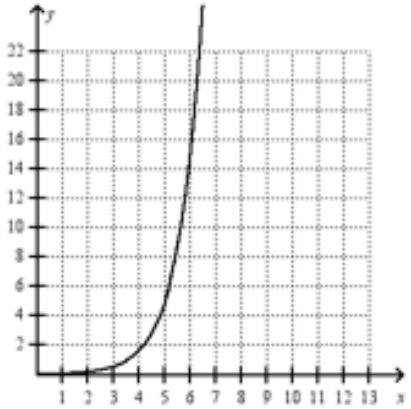
$y = 3x + 0.02$



X	Y
0	0.02
2	6.02
4	12.02
6	18.02
8	24.02

Anna gets paid $3 for walking the dog everyday. She currently has 2 cents in her piggybank.

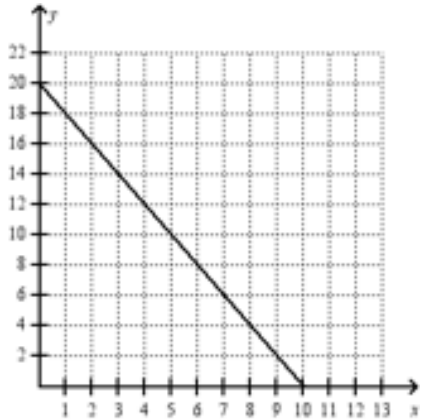
$y = .02(3)^x$



X	Y
0	0.02
4	1.62
8	131.22
12	10629
16	860934

Bill started with 2 cents and it triples everyday.

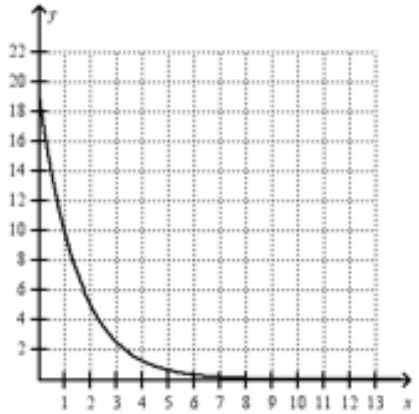
$y = -2x + 20$



X	Y
0	20
1	18
2	16
3	14
4	12

Charlie has 20 dollars and spends 2 dollars on coffee every morning.

$y = 20(0.5)^x$



X	Y
0	20
1	10
2	5
3	2.5
4	1.25

Danielle has 20 dollars and losses 50% of that everyday because of the stock market.

Exponential Functions vs. Linear Functions
Common Misunderstanding Worksheet and Chart

Amy wrote the following equation to model her table of value.

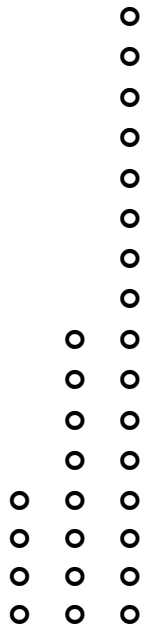
$y = 2(4)^x$

X	Y
0	4
1	8
2	16

Is Amy right? Explain your thinking.

Initial Value:	**Pattern:**	**Equation:**
Model:		
Real Life Application:		

Answer Key:

Initial Value: *4*	**Pattern:** *2*	**Equation:** $y = 4(2)^x$
Model: 		
Real Life Application: *Amy started with four stickers and they doubled every day because she does all of her schoolwork.*		

Exit Pass
Exponential Functions vs. Linear Functions
Date: _____

What did you learn in class today?

_____

_____

_____

Rate yourself on how comfortable you are with *identifying a linear function*.

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Rate yourself on how comfortable you are with *identifying an exponential function*.

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Rate yourself on how comfortable you are with *creating a linear or exponential model*.

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Rate yourself on how comfortable you are with *writing the equation for an exponential function.*

1 Lost	2 Need more help.	3 Good with it.	4 I could teach it.

Solve the given problems:

X	Y
0	27
1	9
2	3

X	Y
-1	6
0	7.5
1	9

3	1/3

Exponential **or**
Linear

Reason:

Equation:

2	10.5

Exponential **or**
Linear

Reason:

Equation:

Answer Key:

X	Y
0	27
1	9
2	3
3	1/3

Exponential **or**
Linear

Reason:
Dividing by 3 each time.

Equation:
$y = 27(1/3)^x$

X	Y
-1	6
0	7.5
1	9
2	10.5

Exponential **or**
Linear

Reason:
Add 1.5(3/2) each time

Equation:
$y = 1.5x + 7.5$
$y = (3/2)x + 7.5$

Research and review of standard	
Content Standard(s):	**Standard(s) for Mathematical Practice:**
Construct and compare linear, quadratic, and exponential models and solve problems. F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions. F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.	MP3: Construct viable arguments and critique the reasoning of others. MP4: Model with mathematics.
Smarter Balanced Claim	***Smarter Balanced Item***

Claim 4: Modeling and Data Analysis
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Mr. Miller starts working for a technology company this year. His salary the first year is $40,000. According to the company's employee handbook, each following year Mr. Miller works at the company, he is eligible for a raise equal to 2–5% of his previous year's salary.

Mr. Miller calculates the range of his raise on his first year's salary. He adds that amount as his raise for each following year. Mr. Miller thinks that:

- in his second year working at the company, he would be earning a salary between $40,800 and $42,000, and
- in his third year, he would be earning a salary between $41,600 and $44,000.

Part A

1. Based on this reasoning, what salary range would Mr. Miller expect to earn in his **tenth** year at the company?

2. Mr. Miller's reasoning is incorrect. Show with diagrams, equations, expressions, or words why his reasoning is incorrect.

Part B

Create a table of values to compare the expected salary increases for an employee with a starting salary of $100,000 based on Mr. Miller's incorrect reasoning and the more reasonable expected salary increases. List these ranges in separate columns of the table up to the employee's sixth year at this company.

CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)

Look at the Progressions documents, Learning Trajectories, LZ lesson library, unpacked standards documents from states, NCTM Essential Understandings Series, NCTM articles, and other professional resources. You'll find links to great resources on your PLC Platform.

Conceptual Understanding and Knowledge

- Distinguish between situations that can be modeled with linear functions and with exponential functions.
- Interpret the parameters in a linear or exponential function in terms of a context.
- Interpret the structure of an expression.
- Interpret expression for functions in terms of the situations they model.

Procedural Skills

- Write a linear equation, $y = mx + b$.
- Write an exponential equation, $y = ab^x$.

Representational

- Write equations to represent real world problems.
- Model exponential growth and decay using equations, graphs and tables.

	Social knowledge • Know a range of values. • Know how to find a percent. • Understand that function notation is f(x) and is read "f of x". • Understand that b^x is read b to the *x* power. • Understand that the variable in an exponential function is the exponent.

Standards Progression **Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*		
Pre-Requisites Standards	**Co-Requisite Standards**	**Future Standards**
8.EE.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. 8.F.3: Interpret the equation *y = mx + b* as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	N-RN.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	F-LE.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10 or *e*; evaluate the logarithm using technology. F-BF.5(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- Students unfamiliar with range of values will not understand how to create the table of values in Part B.

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students struggle to distinguish between linear and exponential functions, graphically and by a table.
- Students have difficulty verifying linearity versus exponentially from a graph based on the viewing window.
- Students do not understand how to write the equation for a linear or exponential function.

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- Students may not expand the graph to see it in its entirety and misinterpret the function.