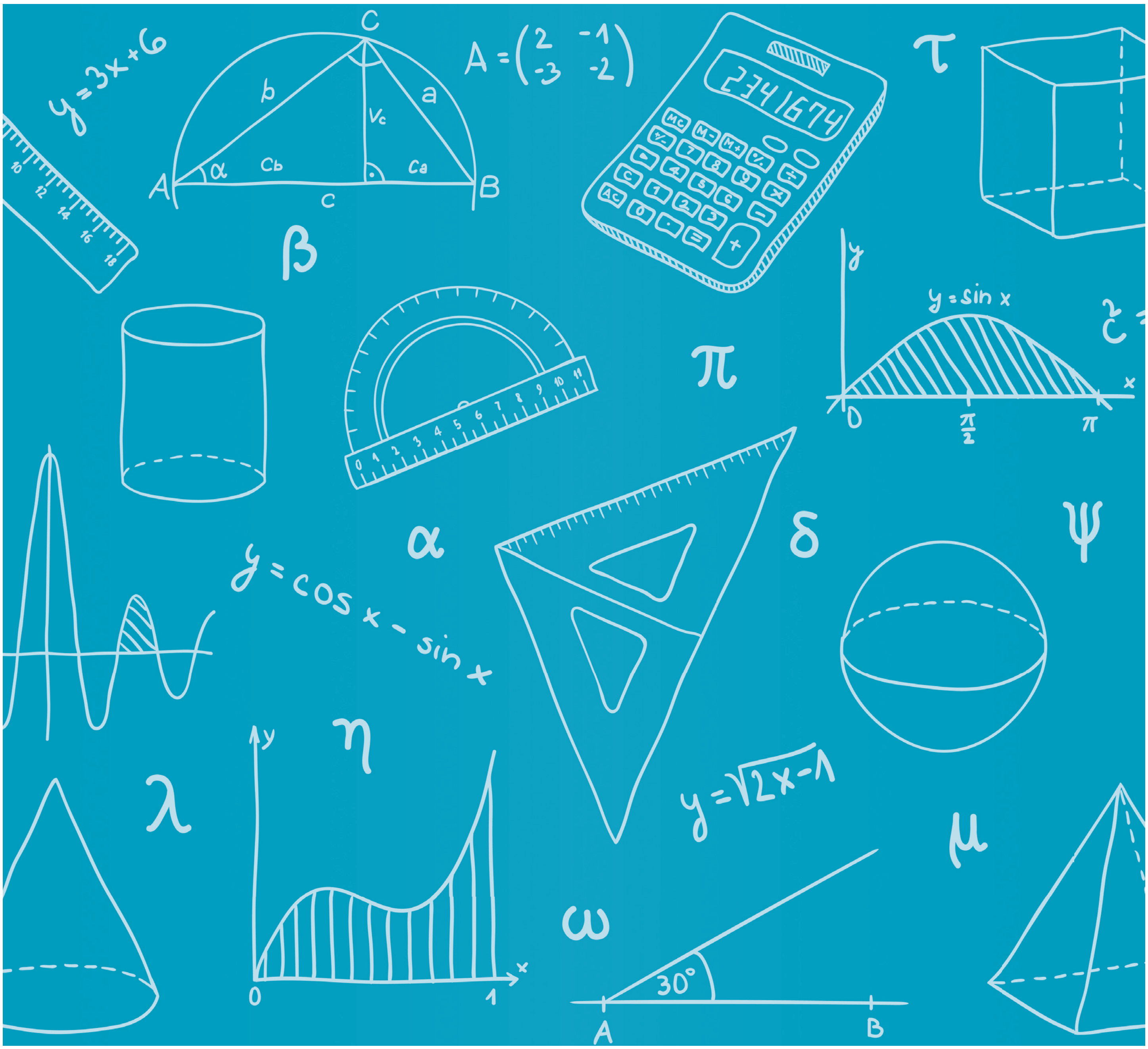
GRADE
6

LESSONS FOR LEARNING

FOR THE COMMON CORE STATE STANDARDS IN MATHEMATICS



PUBLIC SCHOOLS OF NORTH CAROLINA
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NC DEPARTMENT OF PUBLIC INSTRUCTION

June St. Clair Atkinson, Ed.D., State Superintendent
301 N. Wilmington Street :: Raleigh, North Carolina 27601-2825

M0910

Sixth Grade – Standards

1. **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems** – Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.
2. **Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers** – Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
3. **Writing, interpreting, and using expressions and equations** – Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.
4. **Developing understanding of statistical thinking** – Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

MATHEMATICAL PRACTICES

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

RATIOS AND PROPORTIONAL RELATIONSHIPS

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (Note: Expectations for unit rates in this grade are limited to non-complex fractions.)

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

THE NUMBER SYSTEM

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

6.NS.3 Fluently add, subtract, multiply, & divide multi-digit decimals using the standard algorithm for each operation.

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, & that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3° C > –7° C to express the fact that –3° C is warmer than –7° C.*

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.*

d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

EXPRESSIONS AND EQUATIONS

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* $5 - y$.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.*

6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$.

6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*

GEOMETRY

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

STATISTICS AND PROBABILITY

Develop understanding of statistical variability.

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Bake Sale Brownies

Common Core Standards:

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Note: 6.RP.3c is not addressed with this task.

Additional/Supporting Standard: 6.EE.9

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.

Student Outcomes:

- I can use knowledge of multiplication and division to scale recipes to make different numbers of servings
- I can identify quantitative relationships between scaled recipes
- I can represent relationships between quantities by plotting points on the coordinate grid

Materials:

- Task Guide and Extension Task handouts
- Cards (see blackline masters)
- Envelopes (1 per group)

Advance Preparation:

<u>Before the Lesson:</u>

- Make copies of Task Guide, Extension Task handouts, and cards (one set for each group of 3-4 students) on copy paper
- Cut cards apart and place into envelopes (mixing up the order of the cards before placing into the envelopes)
- Determine student groupings

Directions:

Students will work in groups to match the details of recipes that have been scaled up and down into larger and smaller batches, calculate missing quantities in those recipes, and determine quantitative relationships between the scaled recipes. They will then create a graph comparing the number of brownies made to the amount of sugar used.

- Divide students into groups of 3-4
- Explain the task (as detailed on the Task Guide handout).
- Have students group the cards for the basic recipe: 12 brownies – cost: $2.35 – ingredient list begins with ½ cup butter. Groups should then continue the process of grouping cards and showing connections between recipes (with arrow cards) on their own.
- Be sure that groups realize that the aim of the task is not to finish first, but to ensure that each member of the group understands what they are doing and agrees with the placement of each card and arrow. If someone disagrees or doesn't understand, the others shouldn't move on until they have reached agreement and understanding.
- Monitor student progress, posing some of the questions (below) to individual groups as they work. Note misunderstandings and common struggles to discuss later.
- Assign the Extension Task if/when appropriate.
- Wrap up the lesson with a class discussion of some of the questions below.

Questions to Pose:

- Which costs aren't exact multiples (or factors) of each other? Why?
- Are there any arrows that you believe cannot be placed? Why or why not?
- Where could you place some arrows differently, yet still have correct placement?
- Is there a number of brownies which acts as a basis for the other calculations? (answers can vary; require justification for the answer)
- How many ways can you scale the information for 12 brownies to 18 brownies? (some examples: divide in half, then add to the original recipe; divide in half then multiply by 3; multiply by 1½; divide by 2 and then multiply by 3)
- Which arrows have the same meaning? Explain why they are the same.
- Which arrows have mathematically opposite meanings? Explain why they are opposite.
- Can you find other connections to put on the blank arrows?
- Looking at your graph, how could you determine how many cups of sugar would be required to make 30 brownies?
- On the graph, which variable is the independent variable? Which is the dependent variable? How do you know?

Possible Misconceptions/Suggestions:

- Students who lack cooking experience may not realize that 1 tablespoon = 3 teaspoons, a conversion necessary to determine some ingredient amounts.
- Students may not understand how the amount of cocoa was determined for the batch of 6 brownies. 1/3 cup = 5 1/3 tablespoon, or 5 tablespoons and 1 teaspoon = 16 teaspoons. Half of 16 teaspoons = 8 teaspoons = 2 tablespoons + 2 teaspoons. The task does not require students to make this conversion, but understanding this will help students working on the extension task understand why the recipe 30 brownies calls for ¾ cup + 1 tablespoon + 1 teaspoon unsweetened cocoa powder.

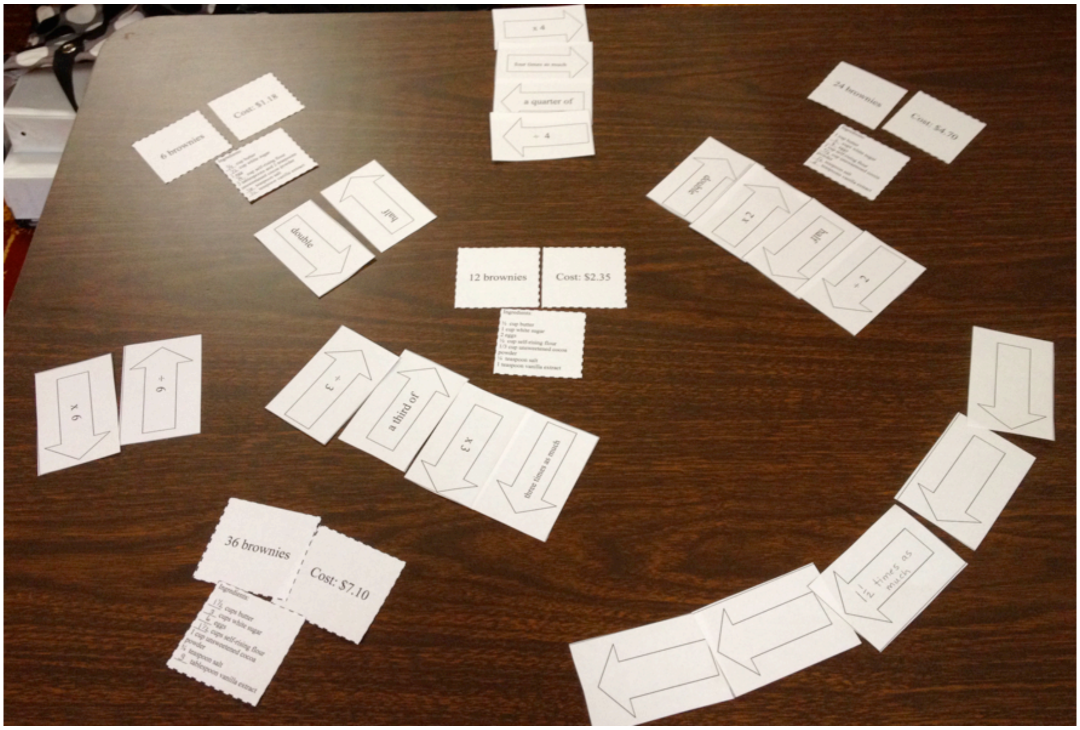
Special Notes:

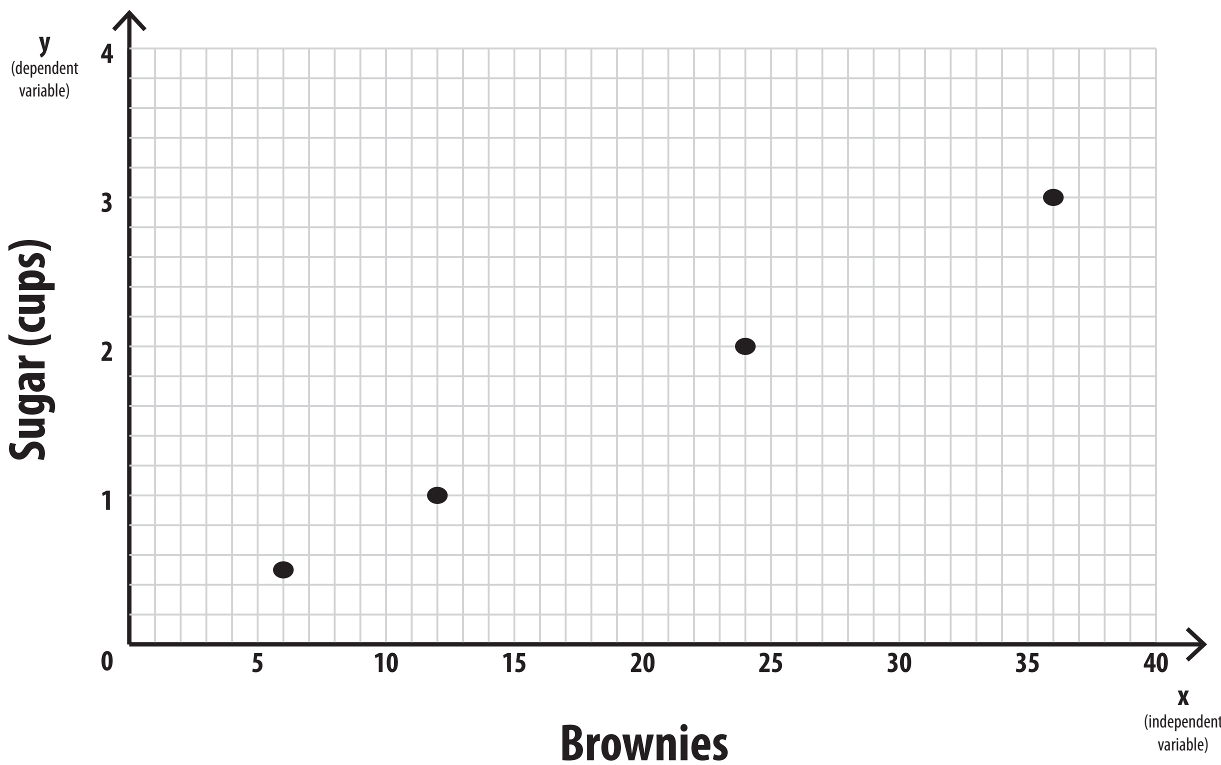
Optional additional extension: As a group, decide how much to charge for each brownie. You need to cover the cost of ingredients, and you want to make a decent profit for the charity, but you don't want to charge so much that nobody will buy the brownies.

Solutions:

Size of batch	Cost	Ingredients
6 brownies	$1.18	1/4 cup butter 1/2 cup white sugar 1 egg 1/4 cup self-rising flour 2 tablespoons and 2 teaspoons unsweetened cocoa powder 1/8 teaspoon salt 1/2 teaspoon vanilla extract
12 brownies	$2.35	1/2 cup butter 1 cup white sugar 2 eggs 1/2 cup self-rising flour 1/3 cup unsweetened cocoa powder 1/4 teaspoon salt 1 teaspoon vanilla extract
24 brownies	$4.70	1 cup butter 2 cups white sugar 4 eggs 1 cup self-rising flour 2/3 cup unsweetened cocoa powder 1/2 teaspoon salt 2 teaspoons vanilla extract
36 brownies	$7.05	1½ cups butter 3 cups white sugar 6 eggs 1½ cups self-rising flour 1 cup unsweetened cocoa powder 3/4 teaspoon salt 1 tablespoon vanilla extract
18 brownies	$3.53	3/4 cup butter 1½ cups white sugar 3 eggs 3/4 cup self-rising flour 1/2 cup unsweetened cocoa powder 1/4 teaspoon salt 1½ teaspoons vanilla extract
30 brownies	$5.86	1¼ cups butter 2½ cups white sugar 5 eggs 1¼ cups self-rising flour 3/4 cup + 1 tablespoon + 1 teaspoon unsweetened cocoa powder 1/2 teaspoon salt 2½ teaspoons vanilla extract

Below is a sample of possible arrangements of cards. Note that there are many possible arrangements. The teacher should closely monitor student work to be sure that the relationships identified are accurate.





Adapted from http://nrich.maths.org/7781

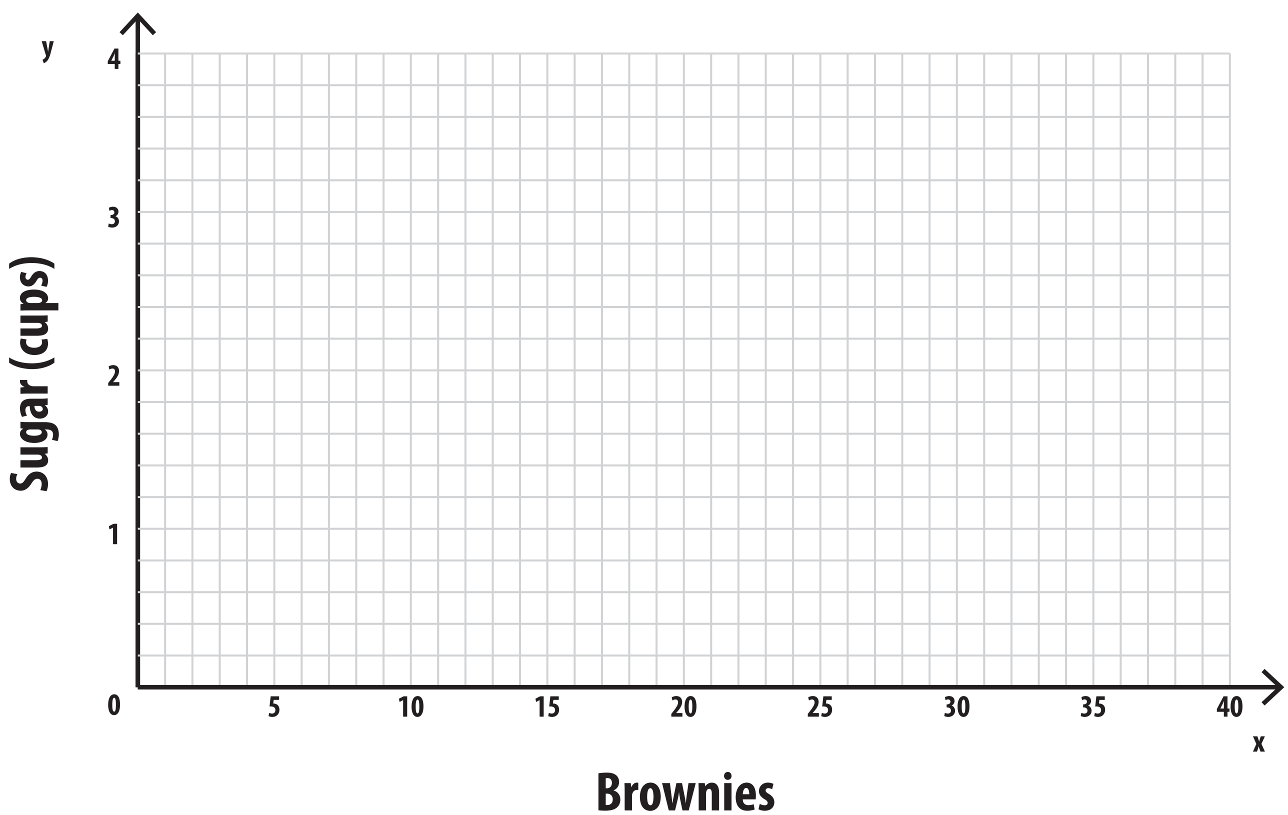
Bake Sale Brownies

The Caring Club wants to raise money for charity, so they decide to sell brownies during an upcoming basketball game. Each club member will make brownies to sell, and they all want to make the same kind of brownies. The recipe they have will make a batch of 12 brownies. Some club members say they can't make that many, and others want to make more than one batch.

Your task is to match the number of brownies made, the cost of ingredients, and the recipe for each size batch (and fill in the missing ingredient amounts). Once you've done this, work together to discover relationships between the recipes.

- Open the envelope and take out the cards.
- Sort the cards into four categories: size of batch, cost, ingredients, and arrows.
- Set the stack of arrows to the side until you've matched the other cards.
- Match the size of batch, cost, and ingredients cards into groups – in each group you should have one size of batch card, one cost card, and one ingredients card.
- Put each group of cards separately on the table.
- Now use the arrow cards to show the relationships between the different groups. You may use the blank arrows to write in your own relationships between cards.

Create a graph comparing the number of brownies made to the amount of sugar used.



Optional Extension Task:

When you're satisfied you've correctly sorted all of the cards, filled in the missing ingredient amounts, and found the connections between the cards, determine the missing information for these batches:

Size of batch	Cost	Ingredients
18 brownies	$_______	______ cup butter ______ cups white sugar ______ eggs ______ cup self-rising flour ______ cup unsweetened cocoa powder ______ teaspoon salt ______ teaspoons vanilla extract
30 brownies	$_______	______ cup butter ______ cups white sugar ______ eggs ______ cup self-rising flour 3/4 cup + 1 tablespoon +1 teaspoon unsweetened cocoa powder ______ teaspoon salt ______ teaspoons vanilla extract

The recipe for 30 brownies calls for ¾ cup + 1 tablespoon + 1 teaspoon unsweetened cocoa powder. Explain how this amount was determined.

6 brownies	12 brownies	24 brownies	36 brownies
Cost: $1.18	Cost: $2.35	Cost: $4.70	Cost: $7.10
Ingredients: ____ cup butter ____ cup white sugar 1 egg ____ cup self-rising flour 2 tablespoons and 2 teaspoons unsweetened cocoa powder ____ teaspoon salt ____ teaspoon vanilla extract	Ingredients: ½ cup butter 1 cup white sugar 2 eggs ½ cup self-rising flour 1/3 cup unsweetened cocoa powder ¼ teaspoon salt 1 teaspoon vanilla extract	Ingredients: 1 cup butter ____ cups white sugar ____ eggs 1 cup self-rising flour ____ cup unsweetened cocoa powder ____ teaspoon salt ____ teaspoons vanilla extract	Ingredients: ____ cups butter ____ cups white sugar ____ eggs ____ cups self-rising flour 1 cup unsweetened cocoa powder ¾ teaspoon salt ____ tablespoon vanilla extract

x 2	x 3	x 4
÷ 2	÷ 3	÷ 4
four times as much	four times as much	

double	double	three times as much
half	half	three times as much
x 6	÷ 6	

		a third of
half	half	a third of
a quarter of	a quarter of	

Paper Clip Comparisons

Common Core Standard:

Understand ratio concepts and use ratio reasoning to Solve Problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Note: 6.RP.3c is not addressed by this task.

Additional/Supporting Standard(s):

6.RP.1, 6.RP.2; (Optional extension addresses 6.EE.6 and 6.EE.9)

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.

Student Outcomes:

- I can represent equivalent ratios as a ratio table, as a graph, and on a double number line.
- I can analyze ratio tables to identify quantitative patterns.
- I can calculate unit rates by analyzing patterns in data.

Materials:

- Jumbo and standard size paper clips (15 jumbo and 20 standard paper clips for every pair of students)
- Handout, one for each pair of students
- Classroom items to measure (a variety of lengths, from 1 and 12 jumbo paper clips)
- Chart paper (optional)

Advance Preparation:

- Students should have had prior experience creating ratio tables and graphing data from ratio tables.
- Question #3 on the handout asks students to represent their data on a double number line. If students do not have prior experience with double number lines, use this opportunity to introduce the concept by completing that question as a whole group.
- Decide which students to pair together for the task.

- Prepare a table for use at the beginning of the task (either on the board or on chart paper), set up this way:

Object	Length in jumbo paper clips	Length in standard paper clips

- Make copies of handout (one handout for each pair of students).
- You may choose to gather items for students to measure, or you can allow students to find their own classrooms objects to measure.

Directions:

Students will measure classroom objects with standard and jumbo paper clips. They will then represent the data using ratio tables, graphs, and double number lines, and analyze the data to identify the proportional relationship between the measurements.

1. Assign students a partner to work with.
2. Give each pair of students 15 jumbo paper clips and 20 standard paper clips. Ask students to make two paper clip chains, one for each size of paper clip. One partner will measure objects with the jumbo paper clip chain; the other partner will measure the same objects with the standard size paper clip chain.
3. Allow time for student pairs to measure 2 or 3 objects (using both paper clip chains) before moving on to the rest of the lesson. Students should record their measurements on the table you've prepared on the board or on chart paper.
4. Pose the key question (at the top of the handout) and direct students to work through the handout tasks with their partners.
5. Monitor students as they work through the tasks on the handout. For the first task, they should lay the chains side-by-side with ends matching to observe and record equivalent lengths.
6. Leave 10-15 minutes at the end of the class period for a whole group discussion of the tasks. In addition to discussing the questions listed in the "After" section below, ask students to use the measurements gathered at the beginning of the lesson to help explain their logic. Some mistakes in initial measurements may be identified during the discussion.

Questions to Pose:

<u>Before:</u>

- What tools do we usually use to measure length? (ruler, measuring tape, yard stick)
- What are some standard units for measurement? (inches, feet, centimeters, meters)
- Name some conversions between those standard units. (1 foot=12 inches, 100 cm = 1 m, 3 feet=1 yard)

During:

- Looking at your table of data, how many times larger is the standard clip number than the jumbo clip number (when measuring the same item)?

After:

- Looking at your table of data, how many times larger is the standard clip number than the jumbo clip number (when measuring the same item)? Does the answer to that question depend on which item you measured?
- What patterns did you see in the table? In the graph? In the double-number line?
- How can you use the patterns you found to determine the length of an object in standard paper clips if you know its length in jumbo paper clips?
- What are some types of measurement conversions that might be more practical than jumbo and standard paper clips? (possible answers include feet and inches, yards and feet, or centimeters and millimeters)

Possible Misconceptions/Suggestions:

Misconception	Suggestion
If students measure items that don't exactly match the length of the paper clip chain, they may not know what to do.	Students may choose to estimate the fractional part of the paperclip lengths or they may choose to round to the nearest whole number of paperclips. These inconsistencies can be addressed during the whole group discussion after students have completed the tasks.
Graphs may not look the same, depending on which axis is used for each type of paper clip.	Explain that in some situations, such as this one, there is not a variable that must be considered the independent variable. Therefore, either representation is appropriate.

Special Notes:

- The task can be extended by repeating the same exercises but using other standard-size units; for example, they may compare paper clips to snap cubes or snap cubes to Cuisenaire rods. Alternately, students could complete the same exercise using two different conventional units of measurement, such as inches and feet, feet and yards, or inches and centimeters. This would provide good practice for creating double number lines.
- If time allows, students may measure larger classroom objects by joining their paper clip chains together.
- If you wish to extend the task to show connections to the Expressions and Equations domains, you can ask students to write an equation to express the length of an object in standard paper clips in terms of the length in jumbo paper clips ($s=1.5j$, where s=the length in standard clips and j=the length in jumbo clips) or to write an equation to express the length of an object in jumbo paper clips in terms of the length in standard paper clips ($j=2/3s$).

Solutions:

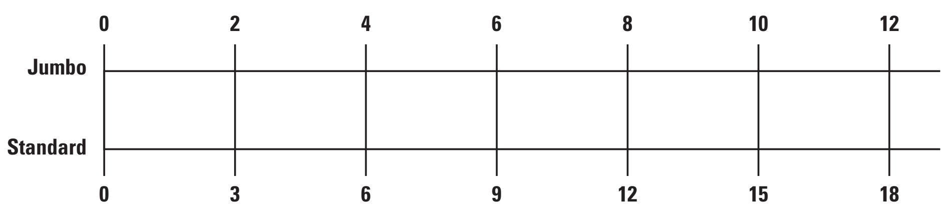
As they work through the task, students should discover that 3 standard paper clips have the same length as 2 jumbo paper clips, which means the ratio of standard : jumbo clips is 3:2. As a unit ratio this would be 1½:1. Multiplying the number of jumbo paper clips by 1½ gives the length in standard clips. The ratio for jumbo : standard clips is 2:3, making the unit rate 1:2/3.

1.

Number of jumbo paper clips	Number of standard paper clips
0	0
2	3
4	6
6	9
8	12
10	15
12	18

2. Graphs will vary depending on which size paper clip is represented on the x and y axes.

3.



4. Answers will vary. The jumbo clips go up by 2 every time; the standard clips go up by 3 every time; for every 2 jumbo clips there are 3 standard clips; multiply the jumbo clip number by 1 ½ to get the standard clip number.
5. Answers will vary. The points make a straight line.
6. Answers will vary. The numbers match the numbers in the ratio table. The jumbo clips go up by 2 every time; the standard clips go up by 3 every time; for every 2 jumbo clips there are 3 standard clips; multiply the jumbo clip number by 1 ½ to get the standard clip number.
7. Answers will vary. Encourage meaningful connections between the numbers and the graphical representations. Note that their descriptions will depend on how they set up their graph (which size paper clip is on the x and y axes).
8. Answers will vary. Multiply the jumbo number by 1.5 or 1½; multiply the jumbo number by 3 and divide by 2; divide the jumbo number by 2 and multiply by 3; use the line from the graph.
9. Answers will vary depending on the object measured.

Adapted from Partners for Mathematics Learning, 2009

Paper Clip Comparisons

Mario has a chain of jumbo paper clips, and his friend Luigi has a chain of standard size paper clips. Their teacher has asked them to measure classroom objects using their paper clip chains, but unfortunately Luigi lost his paper clip chain! If Mario measures the length of an object using his jumbo paper clip chain, how can Luigi determine the length in standard paper clips?

1. Complete the table of values by building chains of jumbo paper clips and standard paper clips.

Number of jumbo paper clips	**Number of standard paper clips**
0	
2	
4	
6	
8	
10	
12	

2. Make a coordinate graph of your data.

Adapted from Partners for Mathematics Learning, 2009

3. Represent the data (from #1) on a double number line.

4. Describe a pattern you see in the table.

5. Describe a pattern you see in the graph.

6. Describe a pattern you see in the double number line.

7. How do the patterns you found in the table and in the double number line show up on the graph?

8. Explain how can you use the patterns you found to determine the length of an object in standard paper clips if you know its length in jumbo paper clips.

9. Use a jumbo paper clip chain to measure an item in the classroom that is longer than 8 jumbo clips. Determine the item's length in standard paper clips using your strategy from #8.

Object	Length in jumbo paper clips	Length in standard paper clips

Now check your answer with a standard paper clip chain. Was the length you calculated (above) accurate?

I’ll Race you?

Common Core Standard:

Apply and extend previous understanding of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Additional/Supporting Standard(s):

MCC6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation

Standards for Mathematical Practice:

4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Student Outcomes:

- I can write an exponential expression as repeated multiplication
- I can evaluate an exponential expression with whole number exponents

Materials:

- *I’ll Race You* gameboard for each student
- Two different colors of dice for each pair of students
- Scrap paper for computation

Advance Preparation:

- Discuss with the class how to identify the base and exponent of an exponential expression
- Copy enough gameboards for students to have several to play with or laminate the gameboards and allow students to use erasable markers
- Be sure you have enough colored dice to give each pair of students one of each color

Directions:

1. Place students in pairs to play the game.
2. Give each pair of students two dice of two different colors
3. Instruct students to choose one color for the base and another color for the exponent
4. One student rolls the dice and identifies the base and the exponent value, writes it on his gameboard and passes the dice to his opponent
5. Once a student has rolled the dice, they complete the remainder of the row for that roll.
6. Once a row is completed, they roll the dice again. Students do not have to wait for their opponent to roll.
7. The first student to complete the table says STOP and the students exchange gameboards and check each other’s work.
8. If the winner is correct, students can begin another game. If not, the incorrect row is erased and the game continues.

Questions to Pose:

Before:

- What does the base of an exponential expression tell us?
- What does the exponent tell us?
- How can we write an exponential expression as repeated multiplication?

During:

- Exponents are part of what operation?

After:

- What was the role of the base in evaluating the exponential function?
- What was the role of the exponent in evaluating the exponential function?

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Students multiply the base by the exponent instead of doing repeated multiplication	Review the role of the base and the exponent Circulate the room and watch for this error to occur

Special Notes:

This activity can be extended to larger numbers by adding more dice. A third color can be added to the base dice to represent tens and another representing ones. The third color could also be used as a digit for tenths place to have practice with decimals. Fraction dice can be used as the base for this activity. Once students have had experience with the game board it can be used as an extension activity or for those that finish other tasks early.

Solutions: N/A

Adapted from Georgia Department of Education

Base	Exponent	Exponential Form	Standard Form	Value

Block Part-y

Common Core Standard:

Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Additional/Supporting Standard(s): n/a

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

Student Outcomes:

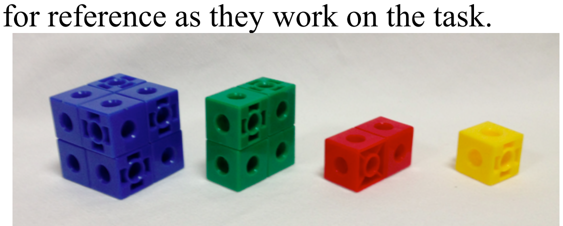
- I can find the volume of a right rectangular prism with fractional edge lengths.
- I can apply my knowledge of the formula $V = l\ w$ h to determine possible fractional dimensions of right rectangular prisms when given the volume.

Materials:

- Handouts
- Snap cubes
- Colored pencils

Advance Preparation:

- Students should have prior experience finding the volume of right rectangular prisms with whole number edge lengths. They should also be able to multiply mixed numbers.
- Make copies of handouts, one for each student.
- Students will need blue, green, red, and yellow colored pencils. It would be ideal to have one set for each student, but one set for every 2-3 students would be manageable.
- Prepare snap cubes for demonstration, as shown on the following photo. It is important to use the same colors as shown below (blue to demonstrate the unit cube, green for the ½ cube, red for the ¼ cube, yellow for the 1/8 cube). It is not necessary for each student or group to have a set, but it would be helpful to have several demonstration sets so that students can use them for reference as they work on the task.



blue green red yellow

Directions:

1. Spend a few minutes discussing the *Before* questions (below). Depending on your students, you may also choose to review how to multiply mixed numbers.
2. Explain that until now, they have only found the volume of right rectangular prisms that have whole number edge lengths. Now they will learn to find the volume when the edge lengths include fractions.
3. Demonstrate the concept of fractional cube dimensions with the snap cubes you prepared ahead of time. First show the blue cube, and explain that the blue cube *represents* a cube with dimensions of 1 by 1 by 1. Ask students to visualize the cube cut in half, and then show the green ½ cube. Ask students for the dimensions of the ½ cube. (1 by 1 by ½)
4. Again, ask students to visualize the green ½ cube cut in half, and then show the red ¼ cube. Ask what the dimensions would be. (1 by ½ by ½)
5. Finally, ask students to visualize the red ¼ cube cut in half, and then show the yellow 1/8 cube. Ask what the dimensions would be. (½ by ½ by ½)
6. Place these demonstration shapes in a location where students may access them (if needed) as they work through the task.
7. Distribute handouts.
8. Monitor students' progress as they work, anticipating the misconceptions listed in the section below.
9. If some students finish early, keep them engaged in the task by asking them to find more examples for the last problem or by giving them a different target volume to use for the same task. For extra challenge, give students a mixed number as the target volume.
10. It is important to stop individual work early enough to allow at least 10 minutes of follow-up discussion. It is okay if some students have not answered the last question, as long as they participate in the group discussion.
11. As a group, discuss the last question of the task as well as the *After* question below. Make a group list of all of the examples students came up with, looking for connections between the examples.

Questions to Pose:

<u>Before:</u>

- What words do we use to describe the dimensions of a right rectangular prism? (length, width, height)
- How do you find the volume of a right rectangular prism? (V = l•w•h)
- How does the formula for the area relate to the formula for volume? (A = l • w; multiply the area of the base by the height to find volume)

<u>During:</u>

- Will every face of the ½ (or ¼ or 1/8) cubes have the same shape and size? (½ and ¼ cubes' faces will not all be the same; 1/8 cubes' faces will be the same)
- How do you know this (*as you point to the face in question)* is the face of a (unit/half/fourth/eighth) cube?

<u>After:</u>

- Challenge question: Is it possible to have a rectangular prism with fractional dimensions for length, width, and height, but a whole number volume?

Possible Misconceptions/Suggestions:

Misconception	Suggestion
Only counting the cubes shown in the figure.	Build a rectangular prism with snap cubes and have the student look at it from a similar perspective as the figures are drawn. Discuss which cubes can and cannot be seen.
Thinking that if there are ¼ or 1/8 cubes in the figure, the dimensions of the figure will include the fractions ¼ or 1/8.	Use the unit, 1/2, 1/4 , and 1/8 cubes to demonstration cubes to talk about dimensions of each.
General confusion about whether the face seen in the figure is a (unit / ½ / ¼ / 1/8) cube.	Use the unit, 1/2, 1/4 , and 1/8 cubes to demonstration cubes to examine the characteristics of each.

Special Notes:

It is likely that students will make mistakes in their coloring. For this reason, it is important that they use colored pencils rather than crayons.

Solutions:

The unit cube has a volume of 1 cubic unit.

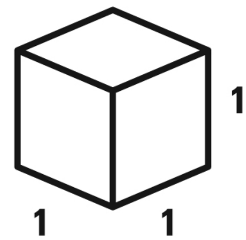
Figure 1:
Dimensions: 2½ by 2 by 3
How many uncut (unit) cubes are in the figure? 12
How many ½ cubes are in the figure? 6
Alternate solution for finding volume will vary, but may include putting the ½ cubes together to form whole cubes.
Volume = 15 cubic units

Figure 2:
Dimensions: 3½ by 4 by 2 ½
How many uncut (unit) cubes are in the figure? 24
How many ½ cubes are in the figure? 20
How many ¼ cubes are in the figure? 4
Alternate solution for finding volume will vary, but may include putting the twenty ½ cubes together to form 10 unit cubes, and putting the four ¼ cubes together to form 1 unit cube.
Volume = 35 cubic units

Figure 3:
Dimensions: 2½ by 3½ by 4½
How many uncut (unit) cubes are in the figure? 24
How many ½ cubes are in the figure? 26
How many ¼ cubes are in the figure? 9
Alternate solution for finding volume will vary, but may include putting the twenty-six ½ cubes together to form 13 unit cubes, putting the nine ¼ cubes together to form 2¼ unit cubes, plus the one 1/8 cube at the corner.
Volume = 39 3/8 cubic units
Rectangular prisms with a volume of 24: *Answers will vary.*

Block Part-y

The cube shown to the right represents the unit cube.
It has the dimensions 1 by 1 by 1.



Color the faces of the unit cube *blue*.

What is the volume of the cube?

1 x 1 x 1 = ________ cubic unit

The rectangular prism in *Figure 1* is made up of some unit cubes as well as other cubes that have been cut in half.

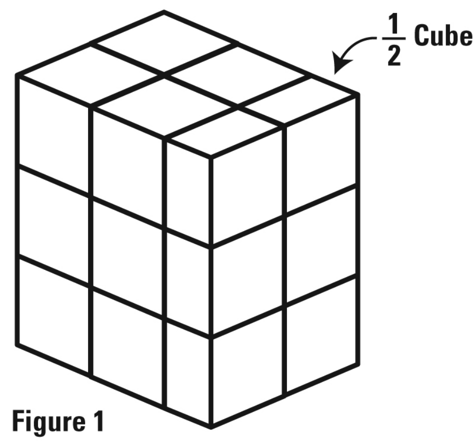


Figure 1

What are the dimensions of Figure 1?

2½ by ________ by ________

Color the faces of the unit cubes *blue*.
Color the faces of the ½ cubes *green*.

How many uncut (unit) cubes are in the figure? _______

How many ½ cubes are in the figure? ________

Without using the formula for finding volume, explain how you could find the volume of the prism.

Now show how to find the volume using the formula.

The rectangular prism in *Figure 2* is made up of some unit cubes, some ½ cubes and some ¼ cubes.

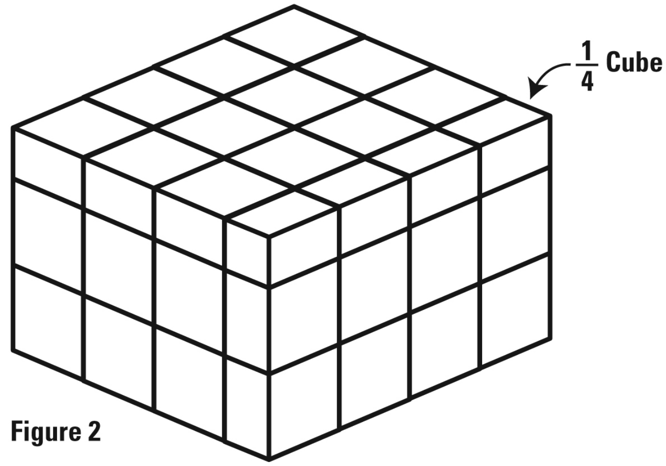


Figure 2

What are the dimensions of Figure 2?

________ by ________ by 2½

Color the faces of the unit cubes *blue*.
Color the faces of the ½ cubes *green*.
Color the faces of the ¼ cubes *red*.

How many unit cubes are in the figure? _______

How many ½ cubes are in the figure? _________

How many ¼ cubes are in the figure? _________

Without using the formula for finding volume, explain how you could find the volume of the prism.

Now show how to find the volume using the formula.

The rectangular prism in *Figure 3* is made up of unit cubes, ½ cubes, ¼ cubes and 1/8 cubes.

What are the dimensions of Figure 3?

________ by ________ by 4½

Color the faces of the unit cubes *blue*.
Color the faces of the ½ cubes *green*.
Color the faces of the ¼ cubes *red*.
Color the faces of the 1/8 cube *yellow*.

How many unit cubes are in the figure? _______

How many ½ cubes are in the figure? _________

How many ¼ cubes are in the figure? _________

How many 1/8 cubes are in the figure? _________

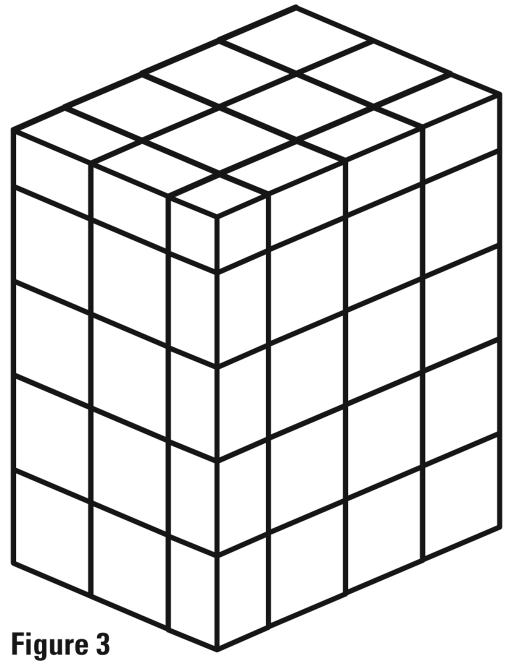


Figure 3

Without using the formula for finding volume, explain how you could find the volume of the prism.

Now show how to find the volume using the formula.

A rectangular prism with the dimensions of 2 by 3 by 4 has a volume of 24. Name at least 3 other rectangular prisms (length, width, and height) *with at least one fractional dimension* that have a volume of 24.

How MAD are you?
A Deeper look at Mean Absolute Deviation

Common Core Standard:

Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute value deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Additional/Supporting Standard(s):

Develop understanding of statistical variability.

6.SP.2, 6.SP.3, 6.SP.4

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.

Student Outcomes:

- I can use measures of center and measures of variability to summarize data sets in context.
- I can determine measures of center and variability of a data set and use the measures to draw conclusions.
- I can connect the measures of center and variability to the shape of the data distribution within the given context.

Materials:

- 1 copy of blank line plot (blackline provided) for each group
- A copy of all eight distribution sets on cards so that each group has an envelope that contains
  all 8 cards
- Small magnets (optional)
- markers
- calculators

Advance Preparation:

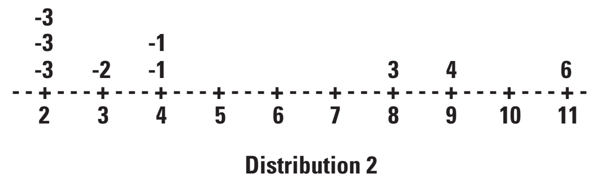
- Make a copy of a blank line plot on 8.5" x 11" paper, one per group.
- Copy the eight distribution plots on card stock, cut apart, and put all eight in an envelope (one envelope needed per group).
- Students need an understanding of mean and knowledge of how to compute it.
- Students need experiences with creating and reading line plots.

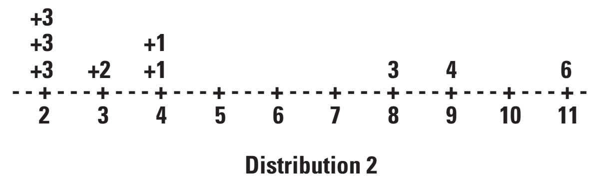
Directions:

1. Ask the students, "How MAD are you?" Using the Fist to Five method, have the students show you their knowledge level of the mathematical term MAD by holding fingers up in front of the them, (using a range of 0 for no knowledge to 5 for master knowledge). Nearly all if not everyone in the class will most likely show you a fist. How MAD are you? Get it… ? ;-)
2. Divide class into small groups so that each group contains 2-4 students.
3. Ask the groups to create a distribution of nine data points on your number line that would yield a mean of 5. Encourage groups to be creative with their answer and to check their answer for accuracy. (Use markers for these plots as they will be displayed for the class.)
4. Gather information from the class as to how they decided upon their 9 data points that averaged to 5. Have groups share in detail.
5. Collect the line plots from each group and post on the board in front of the class (assuming the white board is magnetic, small magnets are very efficient). Have the class look at the different distributions displayed. Ask, "What might the data represent?" (see Questions to Pose below for examples) Choose one of the suggestions from the class to use for the data contextual situation. Discuss the limitations of only knowing the mean of the data, instead of the actual data pieces. Points to consider:
    - The mean is a single value describing a data set but tells you little about the actual data. The data could be tightly clustered together or spread far apart and still have the same mean.
    - A common example of how the mean can be a misleading value to represent a data set is using a company's average salary. A company can advertise that their average salary is $60,000 but in fact, most of the employees make less than $30,000 but the owner makes $100,000 which misrepresents the data set. In other words, outliers can lead to misrepresentation.
    - Thus, this is why it is helpful to have a value that will tell us more about the spread or variability of a data set.
6. Now provide each group with an envelope that contains 8 line plots (blacklines provided). Each plot represents a data set of nine points and the mean of all sets is 5.
7. As groups look at the data sets, ask them to determine which data set seems to differ the least from the mean. They should be able to explain why they chose this data set.
8. Now ask them to determine which data set seems to differ the most from the mean. Again, allow them to explain.
9. Instruct students to work within their group to put all of the data sets in order from "Differs Least" from the mean to "Differs Most" from the mean. Discuss how they determined this list.
10. As a class, decide on the "best" order for all of the data sets varying least from the mean to greatest from the mean.

11. Teacher can now explain that one way to describe how far a value is from the mean is called the "deviation" from the mean. Deviation = Value-Mean
12. Next have the students focus specifically on Distribution #2. Since the sum of all deviations from the mean equals zero, look at the *distance* each value is away from zero (the absolute value of each individual data piece). As a class, work together to determine each value's *distance* away from zero.

$$Distance\ from\ the\ mean = deviation\ from\ mean|$$





13. When asked, "on average" how different the data values are away from the mean, you can use the Mean Absolute Deviation to find this. Total up the distances away from the mean, and then find the "average" of these by dividing by the total number of values in the distribution.

$$MAD = \frac{Total\ Distance\ (\textbf{total}\ of\ all\ values\ from\ the\ mean)}{number\ of\ values\ (sample\ size)}$$

For Distribution #2 above, the MAD = 26/9 or 2.8… Guide the class discussion back to the contextual situation for the data sets. Given a data set with a MAD of 2.8…, what can we conclude about the set in terms of the context?

- A small MAD means that the values do not vary much from the mean.
- A large MAD means that the values vary greatly from the mean.

14. Instruct the students to find the MAD of Distribution 4. First find the distances each value is from the mean. Total the distances, then divide by 9 (the number of values).
15. Instruct students to find the MAD of Distribution 6.
16. Draw the following table on the board and have students determine the MAD for the rest of the data sets.

Distribution	**MAD**
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	

17. Re-Order your distributions from the smallest MAD to the largest MAD.
18. Students will now use the MADs for all the data sets to summarize what we now know about the 8 data sets.
19. Close the lesson by having the students show you their level of understanding of MAD. Use the "Fist to Five" strategy just as you did to begin the lesson.

Questions to Pose:

Before:

- Ask the students, "How MAD are you?" Using the Fist to Five method, have the students show you their knowledge level of the mathematical term MAD by holding fingers up in front of the them, (using a range of 0 for no knowledge to 5 for master knowledge). Nearly all if not everyone in the class will most likely show you a fist.

During:

- Given the distribution sets that all represent nine points of data with an average of 5, what are some possible contexts for the sets? For example, "The Number of People in a Family", "The Amount of Allowance per Week", etc.

After:

- How could we rearrange the nine points in our data sets to decrease the MAD?
- How could we arrange the nine points in our data sets to increase the MAD?
- Close the lesson by formatively assessing using the same question as in the beginning, "How MAD are you?"

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Some students mistakenly assume that absolute value means opposite. They might say that since $\|-4\| = 4$, then $\|4\| = -4$.	Emphasize that the absolute value represents the distance of units from zero, regardless of direction.

Special Notes:

As an extension or more practice computing MAD, have students take the data sets groups created and displayed on line plots to determine the MAD of each set. Then order the groups data sets from the least variability to the most variability.

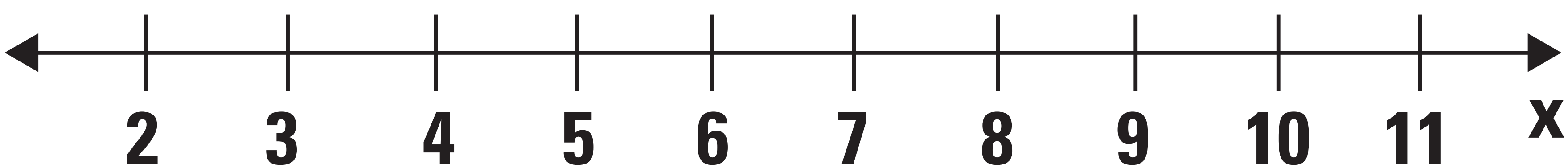
Solutions:

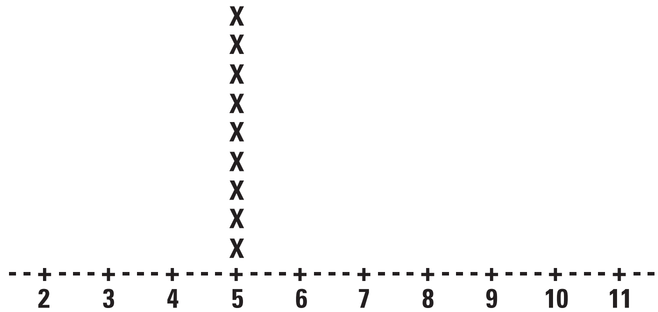
The MAD of each of the eight Distribution Sets:

Set 1: MAD = 0	Set 2: MAD = 2.8…	Set 3: MAD = 2.4…	Set 4: MAD = 1.7…
Set 5: MAD = 2.4…	Set 6: MAD = 2.6…	Set 7: MAD = 2.2…	Set 8: MAD = 4

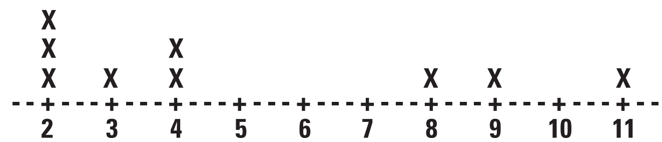
Adapted from: Kader, Gary D. "Means and MADs." *Mathematics Teaching in the Middle School* 4.6 (1999): 398-403.

How MAD are you?

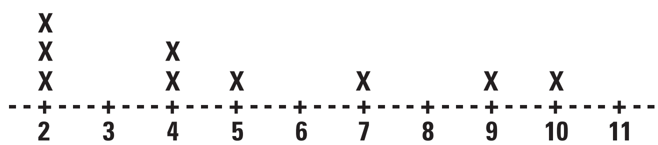




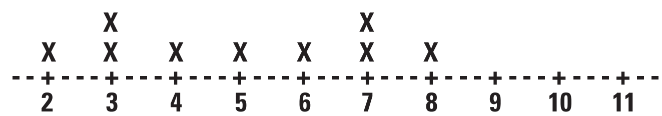
Distribution 1



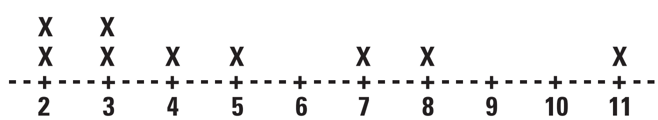
Distribution 2



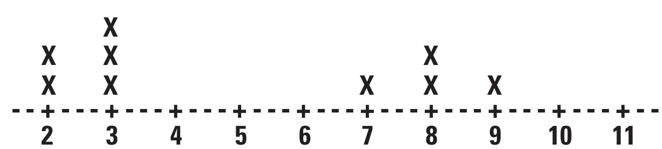
Distribution 3



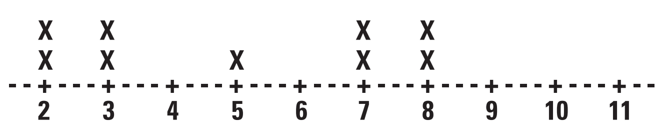
Distribution 4



Distribution 5



Distribution 6



Distribution 7

Distribution 8

Shakespeare vs. Rowling

Common Core Standard:

Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Additional/Supporting Standard(s):

6.SP.2, 6.SP.3, 6.SP.4
Develop understanding of statistical variability.
Summarize and describe distributions.

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

Student Outcomes:

- I can collect data and display that data on tally charts, histograms, and box plots.
- I can quantitatively analyze data to determine measures of center and variability.
- I can use measures of center and variability of data sets to compare data sets.

Materials:

- Handouts
- Dice (one for every 2 students)

Advance Preparation:

- For this task, students should have prior experience collecting data; creating histograms, box plots; and calculating Mean Absolute Deviation.
- Make one copy of the handouts for each pair of students.
- Decide which students to pair.

Directions:

Students collect data on length of words, using excerpts from Harry Potter and the Chamber of Secrets and Macbeth. They will analyze the data using histograms, mean, MAD, and box plots, and will use their analysis to reason about the data.

1. Before beginning the task, pose the questions listed below in the "Before" section. These questions lead into the key question posed by the task.
2. Pair students with a partner and distribute handouts.
3. Monitor students as they work through the questions, making note of student work and thinking you would like to discuss in the follow-up discussion.
4. If some pairs finish their work before others, put pairs of students together to compare their work while they wait for the group discussion.
5. Discuss the task as a group, using the handout questions as a guide.

Questions to Pose:

Before:

- Raise your hand if you've ever read a book by J.K. Rowling.
- Raise your hand if you've ever read a play by Shakespeare.
- Why do you think so many more kids have read Harry Potter books than Shakespeare plays? (Shakespeare is too hard, written too long ago, etc.)

During:

- As you monitor student progress, use questioning to scaffold students who may need more guidance or to help students realize their mistakes or misunderstandings.

After:

- Why did the task require groups to use different starting points for counting word lengths?
- What does the MAD tell you about the word length of the selections?
- How did the box plots help you decide which sample had longer words?
- Which excerpt has more variability in its word length? (Require specific examples as students explain their reasoning.)
- Based on the data you collected and analyzed, do you agree there has been a "dumbing down" of America's youth over the past few decades?
- Did you rely more on the MADs or the box plots to answer question of whether youth have been "dumbed down"? Explain your reasoning.
- Do you believe the comparisons are a reasonable way to decide whether there has been a "dumbing down" of youth? Why or why not? What other methods would you suggest?

Possible Misconceptions/Suggestions:

Misconception	Suggestion
When using tally charts or histograms to determine the mean, students mistakenly use the number of times a data value occurs rather than using the individual data values themselves.	Watch student work closely and use questioning techniques to guide them to the correct procedure.
Students may question the length of some of Shakespeare's words, especially ones that have apostrophes, such as *marshall'st* and/or words that are not currently used in the English language.	You may wish to address this possible confusion as a whole group before students begin working on the task. Count the letters, not apostrophes.

Special Notes:

The mean, MAD, and box plots will vary based on which words the students used in their samples.

Solutions:

Solutions will vary depending on which words the students used for their samples.

Adapted from Georgia Department of Education

Shakespeare vs. Rowling

Many people feel there has been a "dumbing down" of America's youth over the past few decades. To investigate this claim, we will compare excerpts two pieces of literature: Shakespeare's Macbeth and J.K. Rowling's Harry Potter and the Chamber of Secrets.

Is there a difference in the length of the words used in a Shakespeare play compared to a Harry Potter book? Today you will sample words from both pieces of literature to determine which author used longer words.

Follow the steps below to collect data on word lengths from each passage. You will use this data later to determine which piece of literature uses longer words.

- Roll a dice to determine which line to start with. For example, if you roll a 1, begin your count on the *first* line of the Shakespeare excerpt. If you roll a 6, start with the *sixth* line.
- Count the number of letters in each word and use the tally chart below to record the results. Regardless of your starting location, you should count the letters from 50 consecutive words (total) from the passage.

Excerpt from Shakespeare's Macbeth:

Is this a dagger which I see before me,
The handle toward my hand? Come, let me clutch thee.
I have thee not, and yet I see thee still.
Art thou not, fatal vision, sensible
To feeling as to sight? or art thou but
A dagger of the mind, a false creation,
Proceeding from the heat-oppressed brain?
I see thee yet, in form as palpable
As this which now I draw.
Thou marshall'st me the way that I was going;
And such an instrument I was to use.
Mine eyes are made the fools o' the other senses,
Or else worth all the rest; I see thee still,
And on thy blade and dudgeon gouts of blood,
Which was not so before. There's no such thing:

Length of Word	Count	Length of Word	Count
1 letter		7 letters	
2 letters		8 letters	
3 letters		9 letters	
4 letters		10 letters	
5 letters		11 letters	
6 letters		12 letters	

- Follow the same procedures to collect data from this Harry Potter excerpt.

Excerpt from Harry Potter and the Chamber of Secrets:

October arrived, spreading a damp chill over the grounds and into the castle. Madam Pomfrey, the nurse, was kept busy by a sudden spate of colds among the staff and students. Her Pepperup potion worked instantly, though it left the drinker smoking at the ears for several hours afterward. Ginny Weasley, who had been looking pale, was bullied into taking some by Percy. The steam pouring from under her vivid hair gave the impression that her whole head was on fire.

Raindrops the size of bullets thundered on the castle windows for days on end; the lake rose, the flower beds turned into muddy streams, and Hagrid's pumpkins swelled to the size of garden sheds. Oliver Wood's enthusiasm for regular training sessions, however, was not dampened, which was why Harry was to be found, late one stormy Saturday afternoon a few days before Halloween, returning to Gryffindor Tower, drenched to the skin and splattered with mud.

Length of Word	Count	Length of Word	Count
1 letter		7 letters	
2 letters		8 letters	
3 letters		9 letters	
4 letters		10 letters	
5 letters		11 letters	
6 letters		12 letters	

Using the data from your tally charts, create histograms showing the word lengths from each excerpt:

Histogram of word lengths in the Macbeth sample

Histogram of word lengths in the Harry Potter sample

Based on the histograms, predict which book's sample will have a greater mean, and explain your reasoning.

Check your prediction by finding the mean of the word lengths from each book.

Macbeth mean = ___________________ **Harry Potter mean= ___________________**

Taking into account each book's mean and histogram, predict which book you think will have the smaller Mean Absolute Deviation (MAD), and explain your reasoning.

Now find the MAD for each book.

Macbeth MAD = ___________________ **Harry Potter MAD= ___________________**

Create box plots for the word counts you found for <u>Macbeth</u> and <u>Harry Potter.</u>

Macbeth:

Harry Potter:

Based on the data you collected and analyzed, do you agree there has been a "dumbing down" of America's youth over the past few decades? Explain your logic and support your answer with numerical data you collected and analyzed.

Do you believe the comparisons are a reasonable way to decide whether there has been a "dumbing down" of youth? Why or why not? What other methods would you suggest?

Adapted from Georgia Department of Education