7.4 Open Channel Flow

7.4.1 General

Design analysis of both natural and artificial channels proceeds according to the basic principles of open channel flow (see Chow, 1970; Henderson, 1966). The basic principles of fluid mechanics (continuity, momentum, and energy) can be applied to open channel flow with the additional complication that the position of the free surface is usually one of the unknown variables. The determination of this unknown is one of the principle problems of open channel flow analysis and it depends on quantification of the flow resistance. Natural channels display a much wider range of roughness values than artificial channels.

7.4.2 Specific Energy

Specific energy, E, is defined as the energy head relative to the channel bottom. If the channel is not too steep (slope less than 10%) and the streamlines are nearly straight and parallel (so that the hydrostatic assumption holds), the specific energy E becomes the sum of the depth and velocity head:

$$\mathbf{E = y + \alpha\,(V^2/2g)} \qquad \textbf{(7.1)}$$

Where: y = depth, m (ft)
α = velocity distribution coefficient (see equation 7.2)
V = mean velocity, m/s (ft/s)
g = gravitational acceleration, 9.81 m/s^2 (32.2 ft/s^2)

The velocity distribution coefficient is taken to have a value of one for turbulent flow in prismatic channels but may be significantly different than one in natural channels.

7.4.3 Velocity Distribution Coefficient

Due to the presence of a free surface and also due to friction along the channel boundary, the velocities in a channel are not uniformly distributed in the channel section. As a result of nonuniform distribution of velocities in a channel section, the velocity head of an open channel is usually greater than the average velocity head computed as $(Q/A_t)^2/2g$. A weighted average value of the velocity head is obtained by multiplying the average velocity head, above, by a velocity distribution coefficient, α, defined as:

$$\boldsymbol{\alpha = \sum_{i=1}^{n} (K_i^3/A_i^2)/(K_t^3/A_t^2)} \qquad \textbf{(7.2)}$$

Where: K_i = conveyance in subsection (see equation 7.8)
K_t = total conveyance in section (see equation 7.8)
A_i = cross-sectional area of subsection, m^2 (ft^2)
A_t = total cross-sectional area of section, m^2 (ft^2)
n = number of subsections

7.4.4 Total Energy Head

The total energy head is the specific energy head plus the elevation of the channel bottom with respect to some datum. The locus of the energy head from one cross section to the next defines the energy grade line. See Figure 7-1 for a plot of the specific energy diagram.

7.4.5 Steady and Unsteady Flow

A steady flow is one in which the discharge passing a given cross section is constant with respect to time. The maintenance of steady flow in any reach requires that the rates of inflow and outflow be constant and equal. When the discharge varies with time, the flow is unsteady.

7.4.6 Uniform Flow and Nonuniform Flow

A nonuniform flow is one in which the velocity and depth vary in the direction of motion, while they remain constant in uniform flow. Uniform flow can only occur in a prismatic channel, which is a channel of constant cross section, roughness and slope in the flow direction; however, nonuniform flow can occur either in a prismatic channel or in a natural channel with variable properties.

7.4.7 Gradually-Varied and Rapidly-Varied

A nonuniform flow in which the depth and velocity change gradually enough in the flow direction that vertical accelerations can be neglected, is referred to as a gradually-varied flow; otherwise, it is considered to be rapidly-varied.

7.4.8 Froude Number

The Froude number, Fr, is an important dimensionless parameter in open channel flow. It represents the ratio of inertial forces to gravitational forces and is defined by:

$$\mathbf{Fr = V/[(gd/\alpha)^{.5}]} \tag{7.3}$$

Where: α = velocity distribution coefficient
V = mean velocity = Q/A, m/s (ft/s)
g = acceleration of gravity, 9.81 m/s^2 (32.2 ft/s^2)
d = hydraulic depth = A/T, m (ft)
A = cross-sectional area of flow, m^2 (ft^2)
T = channel topwidth at the water surface, m (ft)
Q = total discharge, m^3/s (ft^3/s)

This expression for Froude number applies to any single section channel. For rectangular channels the hydraulic depth is equal to the flow depth.

7.4.9 Critical Flow

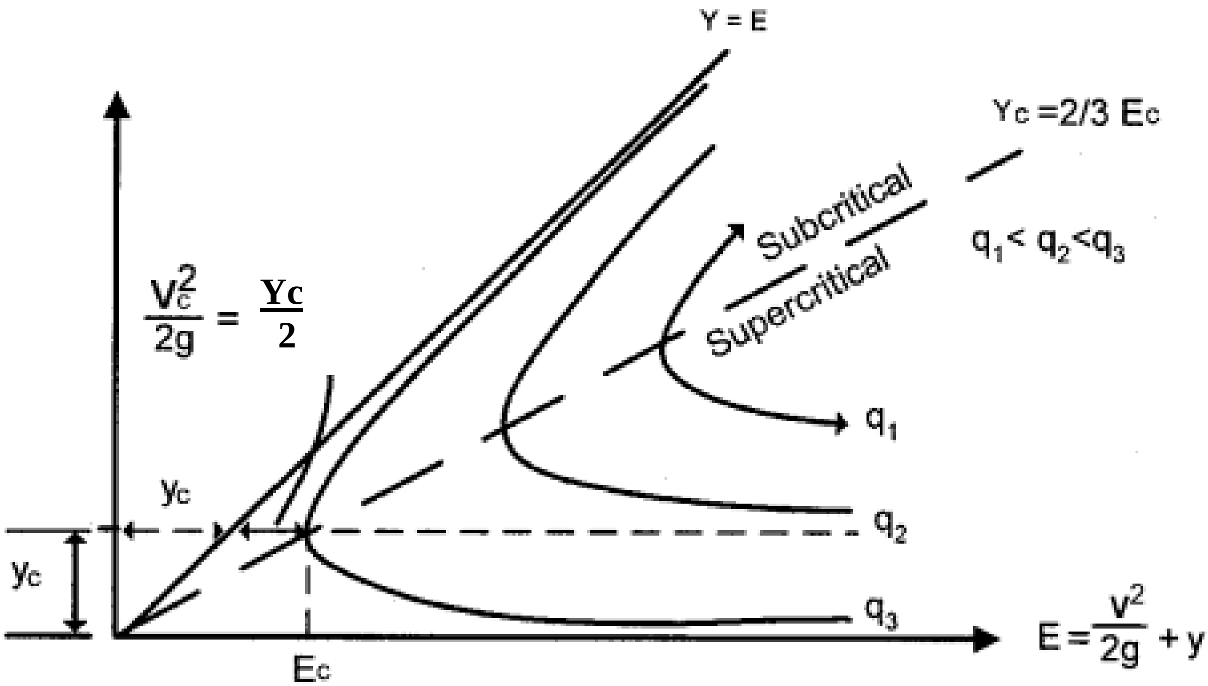
Critical flow occurs when the specific energy is a minimum. The variation of specific energy with depth at a constant discharge shows a minimum in the specific energy at a depth called critical depth at which the Froude number has a value of one. Critical depth is also the depth of maximum discharge when the specific energy is held constant. These relationships are illustrated in Figure 7-1. During critical flow the velocity head is equal to half the hydraulic depth. The general expression for flow at critical depth is:

$$\alpha Q^2/g = A^3/T \qquad \textbf{(7.4)}$$

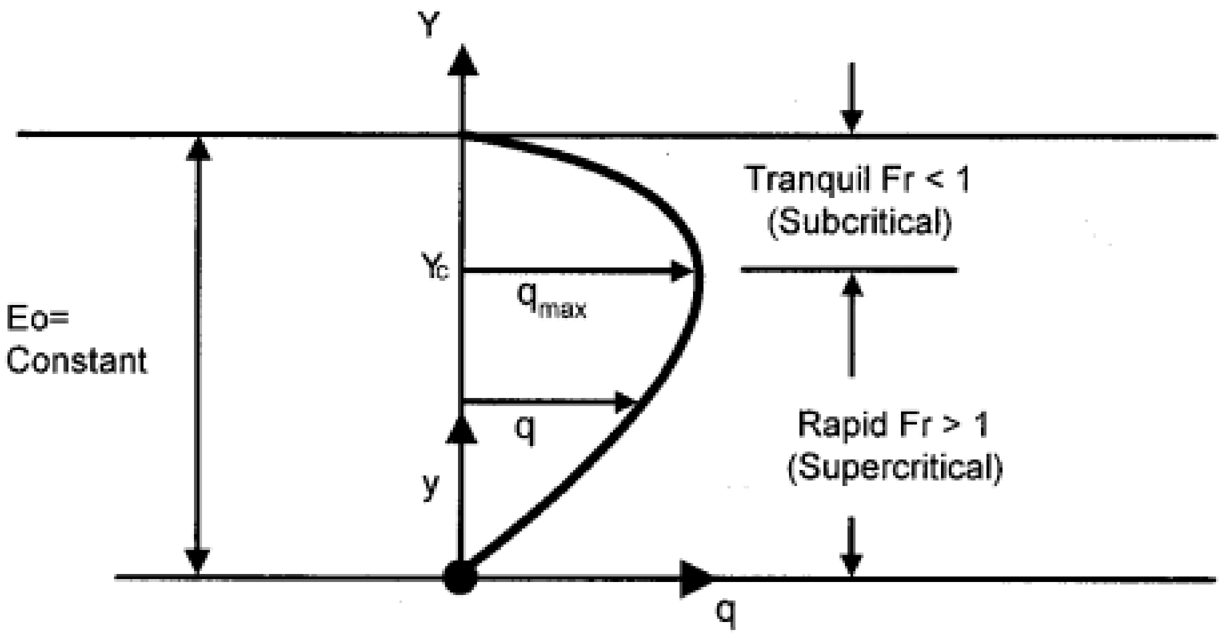
Where: α = velocity distribution coefficient
Q = total discharge, m^3/s (ft^3/s)
g = gravitational acceleration, 9.81 m/s^2 (32.2 ft/s^2)
A = cross-sectional area of flow, m^2 (ft^2)
T = channel topwidth at the water surface, m (ft)

When flow is at critical depth, equation 7.4 must be satisfied, no matter what the shape of the channel.

- Subcritical Flow - Depths greater than critical occur in subcritical flow and the Froude number is less than one. In this state of flow, small water surface disturbances can travel both upstream and downstream, and the control is always located downstream.
- Supercritical Flow - Depths less than critical depth occur in supercritical flow and the Froude number is greater than one. Small water surface disturbances are always swept downstream in supercritical flow, and the location of the flow control is always upstream.
- Hydraulic Jump - A hydraulic jump occurs as an abrupt transition from supercritical to subcritical flow in the flow direction. There are significant changes in depth and velocity in the jump, and energy is dissipated. For this reason, **the hydraulic jump is often employed to dissipate energy and control erosion at highway drainage structures. The location of an induced hydraulic jump in a natural channel shall <u>always</u> be at a point which is remote from any bridge crossing**.



(a) Specific Energy Diagram



(b) Discharge Diagram

Figure 7-1 Specific Energy And Discharge Diagram For Rectangular Channels

(Adopted From Highways In The River Environment)

7.4.10 Flow Classification

The classification of open channel flow can be summarized as follows:

- Steady Flow

1. Uniform Flow
2. Nonuniform Flow
   a. Gradually Varied Flow
   b. Rapidly Varied Flow

- Unsteady Flow

1. Unsteady Uniform Flow (rare)
2. Unsteady Nonuniform Flow
   a. Gradually Varied Unsteady Flow
   b. Rapidly Varied Unsteady Flow

The steady uniform flow case and the steady nonuniform flow case are the most fundamental types of flow treated in highway engineering hydraulics.

7.4.11 Equations

The following equations are those most commonly used to analyze open channel flow. The use of these equations in analyzing open channel hydraulics is discussed in Section 7.7.

- **Continuity Equation** - The continuity equation is the statement of conservation of mass in fluid mechanics. For the special case of one dimensional, steady flow of an incompressible fluid, it assumes the simple form:

$$\mathbf{Q = A_1V_1 = A_2V_2} \qquad \mathbf{(7.5)}$$

Where: Q = discharge, m^3/s (ft^3/s)
A = cross-sectional area of flow, m^2 (ft^2)
V = mean cross-sectional velocity, m/s (ft/s) (which is perpendicular to the cross section)

The subscripts 1 and 2 refer to successive cross sections along the flow path.

- **Manning's Equation** - For a given depth of flow in an open channel with a steady, uniform flow, the mean velocity, V, can be computed with Manning's equation:

$$\mathbf{V = (1/n)R^{2/3}S^{1/2}} \qquad \mathbf{(V = (1.49/n)R^{2/3}S^{1/2})} \qquad \mathbf{(7.6)}$$

Where: V = velocity, m/s (ft/s)
n = Manning's roughness coefficient
R = hydraulic radius = A/P, m (ft)
P = wetted perimeter, m (ft)

S = slope of the energy gradeline, m/m (ft/ft) (Note: For steady uniform flow, S = channel slope)

The selection of Manning's n is generally based on observation; however, considerable experience is essential in selecting appropriate n values. The range of n values for various types of channels and floodplains is given in Table 7-1.

The continuity equation can be combined with Manning's equation to obtain the steady, uniform flow discharge as:

$$\mathbf{Q = (1/n)AR^{2/3}S^{1/2} \qquad (Q = (1.49/n)AR^{2/3}S^{1/2})} \qquad \mathbf{(7.7)}$$

For a given channel geometry, slope and roughness, and a specified value of discharge Q, a unique value of depth occurs in steady, uniform flow. It is called normal depth and is computed from equation 7.7 by expressing the area and hydraulic radius in terms of depth. The resulting equation may require a trial and error solution. See Section 7.7.6 for a more detailed discussion of the computation of normal depth.

If the normal depth is greater than critical depth, the slope is classified as a mild slope, while on a steep slope, the normal depth is less than critical depth. Thus, uniform flow is subcritical on a mild slope and supercritical on a steep slope.

- **Conveyance** - In channel analysis, it is often convenient to group the channel properties in a single term called the channel conveyance K:

$$\mathbf{K = (1/n)AR^{2/3} \qquad (K = (1.49/n)AR^{2/3})} \qquad \mathbf{(7.8)}$$

and then equation 7.7 can be written as:

$$\mathbf{Q = KS^{1/2}} \qquad \mathbf{(7.9)}$$

The conveyance represents the carrying capacity of a stream cross section based upon its geometry and roughness characteristics alone and is independent of the streambed slope.

The concept of channel conveyance is useful when computing the distribution of overbank flood flows in the stream cross section and the flow distribution through the opening in a proposed stream crossing. It is also used to determine the velocity distribution coefficient, α (see equation 7.2)

- **Energy Equation** - The energy equation expresses conservation of energy in open channel flow expressed as energy per unit weight of fluid which has dimensions of length and is therefore called energy head. The energy head is composed of potential energy head (elevation head), pressure head, and kinetic energy head (velocity head). These energy heads are scalar quantities which give the total energy head at any cross section when added. Written between an upstream open channel cross section designated 1 and a downstream cross section designated 2, the energy equation is:

$$\mathbf{h_1 + \alpha_1(V_1^2/2g) = h_2 + \alpha_2(V^2_2/2g) + h_L} \qquad \textbf{(7.10)}$$

Where: h_1 and h_2 are the upstream and downstream stages, respectively, m (ft)
α = velocity distribution coefficient (equation 7.2)
V = mean velocity, m/s (ft/s)
h_L = head loss due to local cross-sectional changes (minor loss) as well as boundary resistance, m (ft)

The stage h is the sum of the elevation head z at the channel bottom and the pressure head, or depth of flow y, i.e., h=z+y. The terms in the energy equation are illustrated graphically in Figure 7-2. The energy equation states that the total energy head at an upstream cross section is equal to the energy head at a downstream section plus the intervening energy head loss. The energy equation can only be applied between two cross sections at which the streamlines are nearly straight and parallel so that vertical accelerations can be neglected.

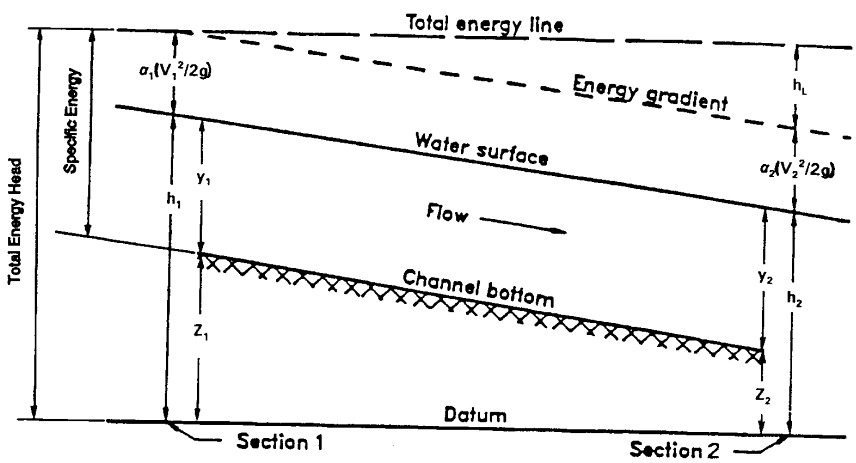


Figure 7-2 Terms In The Energy Equation

Source: FHWA, 1990

Table 7-1 Values of Roughness Coefficient n (Uniform Flow)

Type Of Channel And Description	Minimum	Normal	Maximum
EXCAVATED OR DREDGED			
a. Earth, straight and uniform			
1. Clean, recently completed	0.016	0.018	0.020
2. Clean, after weathering	0.018	0.022	0.025
3. Gravel, uniform section, clean	0.022	0.025	0.030
4. With short grass, few weeds	0.022	0.027	0.033
b. Earth, winding and sluggish			
1. No vegetation	0.023	0.025	0.030
2. Grass, some weeds	0.025	0.030	0.033
3. Dense weeds or aquatic plants in deep channels	0.030	0.035	0.040
4. Earth bottom and rubble sides	0.025	0.030	0.035
5. Stony bottom and weedy sides	0.025	0.035	0.045
6. Cobble bottom and clean sides	0.030	0.040	0.050
c. Dragline-excavated or dredged			
1. No vegetation	0.025	0.028	0.033
2. Light brush on banks	0.035	0.050	0.060
d. Rock cuts			
1. Smooth and uniform	0.025	0.035	0.040
2. Jagged and irregular	0.035	0.040	0.050
e. Channels not maintained, weeds and brush uncut			
1. Dense weeds, high as flow depth	0.050	0.080	0.120
2. Clean bottom, brush on sides	0.040	0.050	0.080
3. Same, highest stage of flow	0.045	0.070	0.110
4. Dense brush, high stage	0.080	0.100	0.140
NATURAL STREAMS			
1. Minor streams (top width at flood stage < 30 m)			
a. Streams on Plain			
1. Clean, straight, full stage, no rifts or deep pools	0.025	0.030	0.033
2. Same as above, but more stones/weeds	0.030	0.035	0.040
3. Clean, winding, some pools/shoals	0.033	0.040	0.045
4. Same as above, but some weeds/stones	0.035	0.045	0.050
5. Same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
6. Same as 4, but more stones	0.045	0.050	0.060
7. Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
8. Very weedy reaches, deep pools, or floodways with heavy stand of timber and underbrush	0.075	0.100	0.150

Table 7-1 Values of Roughness Coefficient n (Uniform Flow) (continued)

Type Of Channel and Description	Minimum	Normal	Maximum
b. Mountain streams, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages			
1. Bottom: gravels, cobbles and few boulders	0.030	0.040	0.050
2. Bottom: cobbles with large boulders	0.040	0.050	0.070
2. Flood Plains			
a. Pasture, no brush			
1. Short grass	0.025	0.030	0.035
2. High grass	0.030	0.035	0.050
b. Cultivated area			
1. No crop	0.020	0.030	0.040
2. Mature row crops	0.025	0.035	0.045
3. Mature field crops	0.030	0.040	0.050
c. Brush			
1. Scattered brush, heavy weeds	0.035	0.050	0.070
2. Light brush and trees in winter	0.035	0.050	0.060
3. Light brush and trees, in summer	0.040	0.060	0.080
4. Medium to dense brush, in winter	0.045	0.070	0.110
5. Medium to dense brush, in summer	0.070	0.100	0.160
d. Trees			
1. Dense Willows, summer, straight	0.110	0.150	0.200
2. Cleared land with tree stumps, no sprouts	0.030	0.040	0.050
3. Same as above, but with heavy growth of spouts	0.050	0.060	0.080
4. Heavy stand of timber, a few down trees, little undergrowth, flood stage below branches	0.080	0.100	0.120
5. Same as above, but with flood stage reaching branches	0.100	0.120	0.160
3. Major Streams (top width at flood stage > 30 m). The n value is less than that for minor streams of similar description, because banks offer less effective resistance.			
a. Regular section with no boulders or brush	0.025	—	0.060
b. Irregular and rough section	0.035	—	0.100

Source: Chow, V.T.