STATISTICAL RELATIONS IN FERTILIZER INSPECTION

C. I. BLISS

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The CONNECTICUT Agricultural Experiment Station
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Foreword

With pleasure I accept Dr. Bliss' invitation to write a foreword to his provocative paper on the statistical theory of sampling fertilizers for sale. Citizens in a modern society depend upon their government to protect them from fraud. The responsible government agencies must constantly examine their procedures to keep them current. In my view this paper is a contribution to the policy of quality control by government.

In 1857 S. W. Johnson, later professor of chemistry at Yale, pioneered the principle of quality control by analyzing fertilizers on the market for the Connecticut Agricultural Society. At his urging the Connecticut legislature passed the nation's first fertilizer law in 1869. The fertilizer law pioneered other laws on foods, drugs, and the like.

A little later, still at Johnson's urging, the Connecticut legislature pioneered again by chartering The Connecticut Agricultural Experiment Station as the first Experiment Station. Its first assignment was to analyze fertilizers. Quality control has been an important function of the Station ever since.

It is 108 years since Johnson began regular publication of fertilizer analyses, 96 years since the first law was passed, and 90 years since the Station was born. Over the intervening years, we have made many improvements in the methods of chemical analyses. In this paper we examine the publication policy in the light of modern statistical theory of sampling.

How do you know your sample is representative? How do you know when a low sample is due to fraud or to random variation? On the average how precise is a manufacturer's control of quality? These are among the questions that Dr. Bliss has looked at.

The central idea in Johnson's policy of fertilizer control was that the results must be published. You can see this in the accompanying facsimile of the first inspection report published in New Haven on August 18, 1877.

A policy for fertilizer testing was vigorously debated in the legislature of 1875 when the Station was being established. Orange Judd, publisher of the *American Agriculturist* and a trustee of Wesleyan University, argued that fertilizers should be analyzed before sale so that a farmer would know whether he was buying dried harbor mud or real fertilizer. Johnson argued that this policy would force the Station to be a party to the sale, that the Station in effect would be guaranteeing the quality. Since the Station could not possibly follow the product from manufacturer to user, it could not be a guarantor. Orange Judd's policy would give the Station responsibility without adequate authority. This would, and eventually did, put the Station in an untenable position.

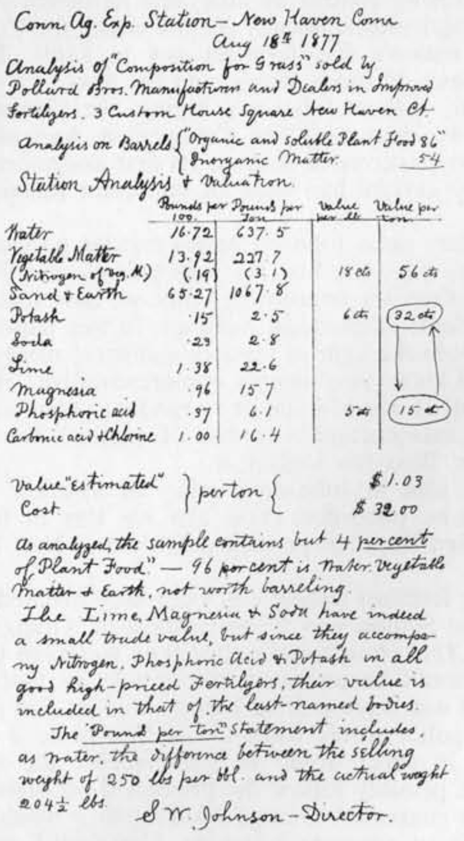
Johnson argued for a policy of collecting the fertilizer in the open market, analyzing it, and publishing the data for all to read. Any fraud found would, thus, be pilloried in the public stocks. The competitors would take care of the fraudulent salesman. It was simple, effective, and inexpensive. Moreover, it had a built-in policing system.

Orange Judd won the debate, however. The legislature set up the Station under his aegis and the Station operated on his policy for 2 years. But then the Legislature stepped in, the Station was moved to New Haven in 1877, and Johnson's policy was adopted.

On the basis of Dr. Bliss' study, we shall be able to publish the results much earlier, and they will be more easily read and understood.

I am happy to write a few words to introduce the study made by Dr. Bliss.

JAMES G. HORSFALL
Director


Conn. Ag. Exp. Station— New Haven Conn
Aug 18th 1877
Analysis of "Composition for Grass" sold by Pollard Bros. Manufacturers and Dealers in Improved Fertilizers. 3 Custom House Square New Haven Ct.

Analysis on Barrels { "Organic and soluble Plant Food 86" / Inorganic Matter 54 }

Station Analysis & Valuation.

	Pounds per 100.	Pounds per Ton	Value per lb	Value per ton
Water	16.72	637.5		
Vegetable Matter	13.92	227.7		
(Nitrogen of Veg. M.)	(.19)	(3.1)	18 cts	56 cts
Sand & Earth	65.27	1067.8		
Potash	.15	2.5	6 cts	32 cts
Soda	.23	2.8		
Lime	1.38	22.6		
Magnesia	.96	15.7		
Phosphoric acid	.37	6.1	5 cts	15 cts
Carbonic acid & Chlorine	1.00	16.4		

Value "estimated" } per ton { $1.03
Cost $32.00

As analyzed, the sample contains but 4 percent of "Plant Food" — 96 per cent is Water, Vegetable Matter & Earth, not worth barreling.

The Lime, Magnesia & Soda have indeed a small trade value, but since they accompany Nitrogen, Phosphoric Acid & Potash in all good high-priced Fertilizers, their value is included in that of the last-named bodies.

The "Pounds per ton" statement includes, as water, the difference between the selling weight of 250 lbs per bbl. and the actual weight 204½ lbs.

S. W. Johnson — Director.

First bulletin of the Station sent to newspapers 88 years ago reported on a fertilizer "Composition for Grass" that was sold for $32 a ton and had a plant food value of about $1 a ton.

STATISTICAL RELATIONS IN FERTILIZER INSPECTION

C. I. BLISS
with the assistance of Mrs. Paula Salerno

This study of fertilizer control in Connecticut is concerned with ways of making the program more useful to fertilizer buyers. These include possible improvements in the selection of samples, the interpretation of analytical results, and the publication of more timely and more informative reports to the industry and its customers.

On July 1, 1965 a new Connecticut Fertilizer Law replaced the one operative when this study was made. Several changes made effective by the new law are in accord with the suggestions reported here. The principles of fertilizer control in Connecticut were not changed.

1. *The Collection of Fertilizer Samples*

The term "commercial fertilizer" in the old law included "any and every substance imported, manufactured, prepared or sold for fertilizing or manuring or soil amending purposes, except barnyard manure and stable manure which have not been artificially treated or manipulated, marl and lime." Guarantees of constituents in both the old and new laws must be in percentages of the elements, except phosphorus, which must be declared as available phosphoric acid (P_2O_5) or, in natural organic materials, as total phosphoric acid, and potassium as water-soluble potash (K_2O). Fertilizers sold in Connecticut are expected to comply with these and other standards of the Association of American Fertilizer Control Officials.

The old law stipulated that one or more analyses should "be made annually of all commercial fertilizers registered in the state." Although not required, many fertilizers sold in packages of 10 pounds or less were also registered. In enforcing the statute, samples of all registered brands that could be found on the market have been collected by the Station Agent. Each brand is considered a unit and its composition is assumed independent of the place at which it is sampled, either as to location or type of firm. In 1963, 624 brands were registered and the number of official samples totalled 577. This represented only 431 or 69% of the registered brands, multiple sampling accounting for the difference.

Incompleteness in coverage has been practically unavoidable. Some companies register all of their fertilizers at the start of the season but apparently send only part or none of them into the state for sale. Other brands may be stocked in so few outlets and during so short a season that they are missed. From the standpoint of protecting the public against fraudulent claims, however, the omission of many registered brands was less important than the fact that 6 or 7 out of every 10 brands were sampled and any deficiencies reported publicly.

In general, the brands with multiple samples were those sold in larger quantities, as shown in Table 1 for 1,954 annual brand registrations with reported sales of 0.1 ton or more in the period from July 1, 1959 to June 30, 1963. For the 802 brands or 41% that were not sampled, the median amount sold per year was 8 tons of each brand. Of those which were sampled, this quantity increased from 21 tons for brands sampled once to 89 tons for those sampled twice, to 240 tons for those with three or more samples. Brands selling less than 10 tons per year were still sampled at a rate of 46%.

Table 1. Correlation between the reported tonnage of each registered brand in each year and the number of samples analyzed, from July 1, 1959 to June 30, 1963

Reported sales	No. of brands with 0 to 7 samples per year								Total	
in tons	0	1	2	3	4	5	6	7	Brands	Samples
None reported	(202)	399	40	4	2	2	5	0	452	539
0.1-2.0	201	146	5						352	156
2.1-5.0	127	115	3	1	1				247	128
5.1-10.0	121	99	15	2					237	135
10.1-20.0	79	115	8	1					203	134
20.1-30.0	52	56	8	1					117	75
30.1-50.0	52	89	12	2					155	119
50.1-70.0	37	40	12	3					92	73
70.1-100.0	27	52	6	4	1				90	80
100.1-140.0	27	39	8	6	2				82	81
140.1-200.0	22	40	11	2	1				76	72
200.1-300.0	20	52	14	5	1	2			94	109
300.1-500.0	15	42	6	6	2	1	1		73	91
500.1-700.0	8	18	13	2	2		1		44	64
700.1-1000.0	5	16	8	1	1	1		1	33	51
1000.1-1500.0	2	13	5	3			1		24	38
over 1500.0	7	22	1	2	3				35	42
Brands sampled	—	1353	175	45	16	6	8	1	1604	1987
Brands reporting sales	802	954	135	41	14	4	3	1	1954	1448
Tons at										
50th percentile	8	21	89			240				
80th percentile	52	141	479			724				

Analyses in the first line of Table 1 of brands which reported no sales included fertilizers sold in less than 10-pound lots, samples submitted by other State agencies, and companies that had failed to register or had reported "no sales." Sales would have to exceed 16 tons in a 6-month period before the statutory fee of 6 cents a ton would amount to $1. In the new law sales of less than 10 tons in a 6-month period are exempted from this fee.

A continuing check on brand performance through the years has been complicated by changes in brand names and the short life that many of them enjoyed. A formulation would often be changed before the start of the next season. In other cases, one company would buy out another but continue the brand name. The relation between tonnage sold and brands recording sales in 1, 2, 3, and 4 years is shown in Table 2 for this same 4-year period. For those listed in only 1 year, the median quantity sold per brand was 4½ tons, for brands reporting sales in 2 or 3 years it was 12 tons, and for those reporting sales in all 4 years it was 50 tons.

Table 2. Brands reporting tonnage sales in one or more of the four years, July 1, 1959 to June 30, 1963

Average tons per year	No. of brands reporting sales in					
	1 year	2 years	3 years	4 years	Total	%
0.1-2.0	119	27	20	13	179	21.3
2.1-5.0	51	21	19	20	111	13.2
5.1-10.0	41	23	18	21	103	12.2
10.1-20.0	37	29	16	25	107	12.7
20.1-30.0	24	16	8	15	63	7.5
30.1-50.0	15	12	9	23	59	7.0
50.1-70.0	11	8	6	15	40	4.8
70.1-100.0	4	5	4	14	27	3.2
100.1-140.0	6	6	4	14	30	3.6
140.1-200.0	7	5	5	11	28	3.3
200.1-300.0	2	2	0	21	25	3.0
300.1-500.0	7	4	4	18	33	3.9
500.1-700.0	1	1	0	7	9	1.1
700.1-1000.0	0	2	0	6	8	1.0
1000.1-1500.0	0	1	0	8	9	1.1
over 1500.0	1	2	2	6	11	1.3
Total	326	164	115	237	842	
%	38.7	19.5	13.7	28.1		100
Tons at						
50th percentile	4.5	12		50		
80th percentile	27	60		269		

It is clear from this study that multiple sampling has given a wider coverage of the quantities actually sold, and that the regulation calling for one or more analyses annually of all registered commercial fertilizers was not realistic or essential. Accordingly, the new law states more simply that "The director, who may act through his authorized agent, shall sample, inspect, make analyses of, and test commercial fertilizers distributed within this state at such times and places and to such extent as he may deem necessary to determine whether such commercial fertilizers are in compliance with the provision of this act." The fact that every registered brand is subject to sampling and analysis each year and that a majority of them are so sampled, should have the same effect as the earlier stipulation in "keeping people honest."

In line with this objective, an initial sampling program would cover all available registered brands, especially those with reported sales in the preceding year of more than 10 tons. Apart from samples submitted by other State agencies, priority in multiple sampling would then be given to (1) brands with one or more components below guarantee in the first sample or in the preceding year, (2) brands from manufacturers whose products in the preceding year varied markedly in the percentage of claim (an indication of poor quality control), and (3) brands with sales of more than 300 tons in the preceding year. Brands sampled more than once should be collected at as widely separated places and dates as feasible, so as to increase the chance of sampling different runs.

In collecting a representative sample of given brand, the Station Agent selects five or more bags or containers at random at a store or warehouse, inserts a collecting tube in one corner of each bag and pushes it through to the furthest corner. The cores removed with the tube sample each level in each of the five or more bags in all three dimensions. These cores are thoroughly mixed when combining them into a single representative sample. The risk of inaccurate sampling is thus held to a minimum.

After a sample reaches the laboratory, it is pulverized and again mixed by successive quartering before weighing out a portion for analysis. One analysis is made normally of each guaranteed constituent. If the sample falls below claim in one or more ingredients, that analysis is repeated with a newly weighed portion. To guard against reporting a sample deficient due to analytical errors, the determination most favorable to the manufacturer has been reported in the past. Because of variation between analyses of the same sample, this has introduced an upward bias in the reporting of deficient samples.

The percentage tolerance allowed on a guaranteed element before declaring a sample deficient is based upon the laboratory analytical error. The tolerance for the element nitrogen is 0.1%, and for the compounds of phosphorus (P_2O_5) and of potassium (K_2O) it is 0.2%. Any sample falling below these tolerances has been marked "deficient" in the Fertilizer Reports of this Station.

The analytical techniques are those recommended by the Association of Official Agricultural Chemists or are demonstrably equal or better. Before a new technique is approved, it is tested in a collaborative study in which uniform samples from a common source are sent to several laboratories for analysis. These analytical errors have been based upon the agreement between analyses of separately-weighed portions of the same sample within each laboratory and are assumed to be independent of the percentage content. This may account, however, for only part of the real error. Statistical analyses of AOAC collaborative studies have revealed repeatedly other sources of variation, such as between analysts and between laboratories, and one may question whether tolerances of 0.1% for nitrogen and of 0.2% for phosphoric acid and soluble potash cover more than the analytical phase of the error.

2. *Analysis of the Results of Sampling*

As published in the Station bulletins on commercial fertilizers, each guaranteed constituent in the Station sample of a given brand is paired with its content by chemical analysis. After allowing for the tolerance, all determinations that fall below claim are marked as deficient. How are these results to be interpreted?

Comparing replicate samples of a single brand in one year, the variation between them in a given constituent usually exceeds by a considerable margin that assumed in the tolerances of 0.1% or 0.2% from the laboratory analytical error. The agreement between different samples of the same brand will depend upon factors such as the efficiency of the manufacturer's mixing apparatus, the nature of the materials involved, and the way the fertilizer is handled after bagging. To measure their collective effect would require many more independent samples of each brand than is possible or would be justified. It is not unlikely, however, that many of the brands and mixtures produced by a given company are compounded with the same machinery and by similar methods. On this assumption, the different brands from a single producer could be compared in terms of their percentage agreement with claim, giving a sufficient number of cases for the larger manufacturers to permit a check on their consistency in meeting labelled guarantees and on their improvement over the years.

Two approaches are possible within this framework. One is to compare producers and years in terms of the proportions of analyses for a given component that fall below 100% of claim. The other is based upon the actual amount of each component reduced to a percentage of the amount claimed for each sample. The latter provides more information on the performance of a manufacturer than the percentage of samples below a given level. The comparisons by both procedures cover several years.

Analysis of the percentage deficient. The first approach, in terms of the percentage of deficient claims for each firm in each year, has been based upon the 9 years of sampling from 1955 to 1963. To insure a mean of 20 or more analyses per year in each firm, the statistical study has been restricted to 11 firms that were represented in each year, the total number of analyses per firm ranging from 224 to 1,553. The data in Table 3 then formed a cross-classification of the percentages of analyses which fell short of guarantee, each row representing a single firm, and each column a given year.

As a binomial-type variate, the variance of a percentage in Table 3 depends upon both the number of analyses and the expected percentage deficient in each cell. Both differed sufficiently from cell to cell to require adjustment before years and firms could be compared. To remove the dependence upon the expected response, each percentage was transformed for statistical analysis to its equivalent angle by the inverse sine transformation (Fisher and Yates, 1963). Zero percentages or cells with only a single deficient analysis were corrected additionally to working angles. To adjust systematic differences between cells in the number of analysis, each individual angle has been weighted proportionately to its row and column totals in computing an analysis of variance from the transformed data.

The resulting analysis in Table 4 shows a clearly significant difference between firm means at $P < 0.02$, but less variation between the years of this 9-year interval than might be expected from the error of the comparison. Since this error exceeded its binomial expectation significantly ($P = 0.01$), by a factor of 1.412, the differences between firms from year to year about their respective means were greater than would be expected by

Table 3. Percentages of individual guarantees falling below 100% in samples from all firms with more than 200 analyses in 1955 to 1963

Firm	1955	1956	1957	1958	1959	1960	1961	1962	1963	Weighted mean per cent	Total No. of claims
	Per cent of analyses below claim in										
A	14.0	12.5	13.9	12.5	11.8	15.7	6.5	8.9	3.3	10.30	848
B	14.5	21.9	21.1	10.1	12.6	14.0	11.7	11.3	13.8	14.07	482
C	12.2	19.2	12.4	10.9	17.6	14.2	12.7	12.1	15.5	13.95	1073
D	10.7	0	17.3	16.2	25.0	21.1	16.0	7.7	24.3	15.16	256
E	0	6.8	7.4	12.9	18.1	3.0	4.3	2.0	5.2	6.08	334
F	16.2	12.5	11.7	7.6	13.8	7.9	9.3	7.7	21.9	11.72	338
G	11.4	8.6	8.7	5.8	10.9	14.2	9.4	13.1	15.2	10.81	1553
H	19.3	20.8	17.1	5.2	9.0	15.4	29.4	8.9	5.6	13.37	280
I	0	14.2	10.7	21.4	16.6	21.5	22.2	7.1	12.3	13.62	444
J	0	7.1	14.2	12.1	4.7	8.3	7.9	12.2	4.3	7.60	290
K	22.2	16.6	13.3	26.7	8.3	12.5	0	14.3	2.0	11.07	224
Mean per cent	10.78	12.35	12.25	10.56	13.13	13.66	10.72	10.24	11.33	11.60	
No. of claims	590	599	589	679	652	692	703	815	803		6122

Firms: American Agricultural Chemical Co., Armour Agricultural Chemical Co., Consolidated Rendering Co., Davison Chemical Division of W. R. Grace and Co., Eastern States Farmers' Exchange, Inc., Goulard and Olena, Inc., Hubbard-Hall Chemical Co., Lebanon Chemical Corp., Old-Fox Agricultural Sales, Inc., Sears, Roebuck and Co., Swift and Co., Agrichemical Division.

chance alone. The weighted mean angles for both rows or firms and columns or years have been transformed back to percentages in the margins of Table 3, and supplemented with the total number of analyses for each row and column. Although the analyses increased in number over this period, the percentage deficient showed no systematic change about its mean of 11.6%. The row means for firms, however, varied much more, as would be expected from the significant F test in Table 4. Two firms (E and J) with weighted means of less than 9% had significantly fewer analyses below claim than any of the remaining nine firms. The latter fell into a group of four firms (A, G, K, F) with an average of 10.8% of analyses below claim, and one of five firms (H, I, C, B, D) averaging 14.0% below claim.

Table 4. Analysis of variance of the percentage of analyses below claim from Table 3, in angular measure weighted by proportionate sub-class frequencies

Term	DF	Sum of Squares	Mean Square	F
Between firms	10	27943.91	2794.39	2.41
Between years	8	6500.43	812.55	.70
Error	80	92701.40	1158.77	
Total	98	127145.74		

Analysis of percentages of claim. Apart from the occasional markedly discrepant value, which occurred as often above claim as below and is presumably due to an error in manufacture or labelling, differences in the number of analyses falling below claim can be traced to two factors. One is the size of the increase above claim (the "overage") in a fertilizer component that is added by a manufacturer as a margin of safety for meeting his guarantees. The other is the uniformity achieved in the manufacturing process. The more uniform the mixing the smaller is the margin over claim which is needed to reduce the per cent of analyses falling below guarantee to an acceptable level. By a statistical analysis of the observed percentages of claim for each guaranteed constituent in the Station samples, both factors could be estimated.

This evaluation depends upon the distribution of the percentages of claim for each ingredient in the samples from a given firm. If there is no change in the size of the margin or the method of mixing in a given year, the variation from bag to bag should be due to a number of factors, some increasing and others decreasing the concentration of a given ingredient. From their combined effect, the frequency distribution of the amount of an element may be expected to agree with the normal curve of error. The mean of the individual percentages of claim would then measure the relative concentration of the ingredient which is regularly put in the mixture. The standard deviation of these percentages would be an independent estimate of the uniformity of the total manufacturing and distribution process for the given firm, which may not be the same for all ingredients in a mixture.

The most sensitive test for normality with a small number of analyses is graphic. The N determinations for an individual firm, year, fertilizer constituent, and concentration, in units of the observed per cent of claim in each Station sample, are listed in increasing order from the smallest to the largest percentage in the series. The corresponding rankits or mean expected normal deviates for a sample of N measurements (Fisher and Yates, 1963, Table XX) are then plotted on the ordinate against the ranked percentage fullfillment of claim on the abscissa. If the plotted points form a homogeneous sample from a normal population, they should define a straight line.

The percentage of claim at which a line fitted to these points passes through zero rankit is an estimate of the mean relative concentration of the constituent put into the fertilizer by the manufacturer. The reciprocal of the slope of this line is an estimate, in the same units of per cent of claim, of the standard deviation between samples as analyzed in the Station analytical laboratory. The test is illustrated in Table 5 and Fig. 1 by the analyses for nitrogen in each of two years, 1959 and 1960, from 26 samples of mixed fertilizers from the same firm. Each series had a guaranteed content of 4 to 6 per cent of nitrogen and the observed content of each sample has been transformed to its percentage of this claim.

Table 5. Distribution of analyses for nitrogen in samples of mixed fertilizers from Firm G (Table 3) with a guaranteed content of 4 to 6 per cent (1959 and 1960) and of 8 to 12 per cent (1963)

Rank	Per cent of claim			Rankit	Rank	Per cent of claim			Rankit
order	1959	1960	1963	for N=26	order	1959	1960	1963	for N=26
1	92.8	93.3	*60.0	−1.98	14	103.7	104.8	104.8	.05
2	95.6	93.6	89.6	−1.54	15	104.7	106.4	105.0	.14
3	96.4	96.3	91.0	−1.29	16	105.2	106.4	105.0	.24
4	97.3	97.2	91.6	−1.09	17	105.3	108.3	106.2	.34
5	100.0	98.3	92.0	− .93	18	107.6	108.4	106.5	.44
6	100.0	100.8	93.6	− .79	19	108.0	108.4	106.7	.55
7	100.8	102.0	95.5	− .67	20	108.4	108.7	107.5	.67
8	101.2	102.0	100.0	− .55	21	111.5	108.8	110.8	.79
9	101.7	102.0	100.4	− .44	22	113.3	108.8	111.6	.93
10	102.4	102.3	100.8	− .34	23	115.2	111.5	112.0	1.09
11	102.4	102.5	101.6	− .24	24	115.5	113.6	115.2	1.29
12	102.8	103.6	102.0	− .14	25	115.7	113.7	116.0	1.54
13	102.8	104.4	102.8	− .05	26	118.4	117.5	*134.4	1.98
					27		*146.0		

* Outliers

The mean percentages for 1959 and 1960, marked by a cross in Fig. 1, averaged 104.95 and 104.75 per cent or an excess of 5 per cent over the claimed concentration, as could be interpolated from the abscissa at zero rankit on the ordinate. The two years also agreed in the slope of the lines, the reciprocals of their standard deviations of 6.74 and 6.12 per cent. The mean and standard deviation, together with the number of samples, define each curve completely. With this method of plotting, some "weaving" around the fitted straight lines is due to correlation between successive points and has no significance. The underlying trend is still

linear rather than a systematic curve. Even though, on the average, the the manufacturer had increased the nitrogen component by 5% over what he claimed when mixing these fertilizers, four and five samples respectively of the 26 in each series fell below claim.

The two series plotted in Fig. 1 are homogeneous, with 26 samples belonging to the same basic population or family of brands. This is not always the case. At the lower end of a series one or more samples may have an observed content that is much less than would be predicted from the straight line, or at the upper end much more than is claimed. An

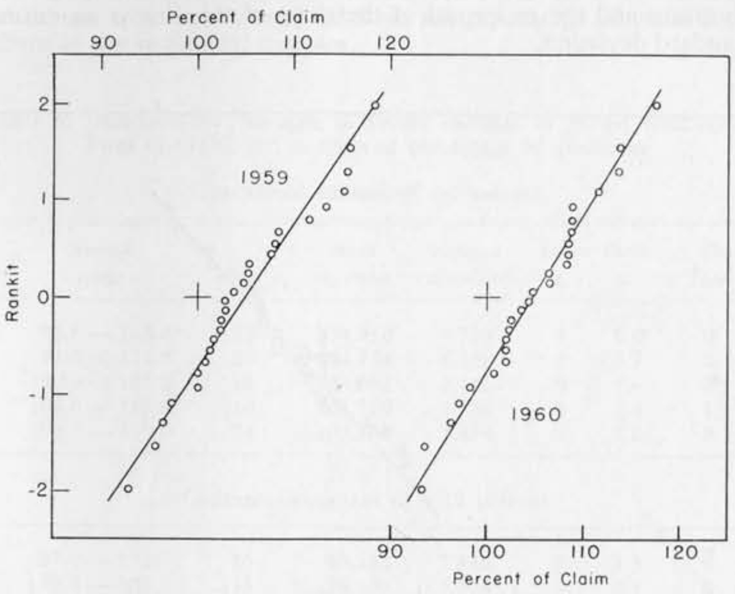


Fig. 1. Rankit diagram of nitrogen content as a percentage of a 4 to 6 per cent claim in brands of mixed fertilizers, from Table 5.

example is the percentage of claim for nitrogen in 8-12% mixtures in 1963, as listed in Table 5 and plotted in Fig. 2. Both the low value at 60% of claim and the high values at 135% of claim fall far enough from the curve that they could be rejected by inspection as not belonging to the population or family that has been fitted by the straight line.

Outlying samples such as these can be tested numerically as "outliers" from the ratio of two differences. The difference between the extreme value and the second or third next observation is divided by the difference between the extreme value and the second or third observation from the other end. Each ratio is referred to a table (Dixon, 1951) of the probability of finding such disparate values in a random sample from a normal distribution. For the example in Fig. 2, the probability of a departure this extreme was $P < 0.01$ for the lower sample and $P < 0.06$ for the upper one. When the main body of the curve is as satisfactorily linear as that in Figs. 1 or 2, most outliers can be spotted by looking at the diagram.

Outliers in a series are usually omitted completely and the rankits for the remaining N values plotted against the ranked percentages of claim.

This has been done for the curve for 1960 in Fig. 1, where a sample with 146% of its claimed nitrogen was omitted before plotting the remaining 26 consistent observations. The mean and standard deviation are then computed by elementary statistical methods from all samples retained in the series. Alternatively, the analyses in Fig. 2 have been fitted with a straight line as a series that has been "censored" at both ends, as if the two end samples were smaller and larger than expected but were otherwise of undetermined value. If analysis as a censored distribution is preferred, its much lengthier calculation can be avoided by fitting a line by inspection to the retained observations. Its intercept at zero rankit is an estimate of the mean, and the reciprocal of the slope of this line is an estimate of the standard deviation.

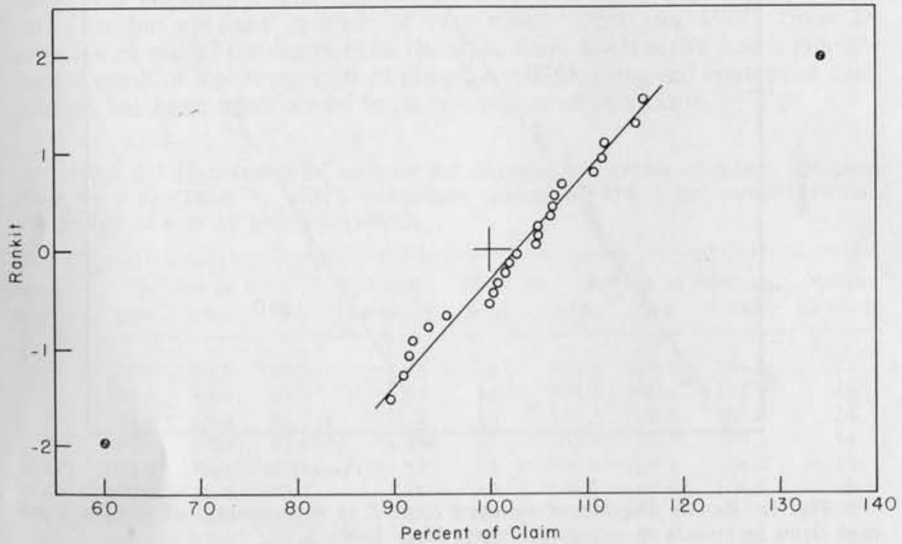


Fig. 2. Rankit diagram of nitrogen content as a percentage of an 8 to 12 per cent claim in brands of mixed fertilizers, from Table 5.

If the plotted points in a rankit diagram bend more or less abruptly in the middle of a series, the two sections of the curve may reflect different policies on the part of the manufacturer. They might then be plotted separately. Alternatively, what is apparently two sections may represent random variation in sampling a single population or non-normality in its distribution.

This analysis of percentages of claim has been applied to the determinations of mixed fertilizers manufactured by Company G in Table 3, for which the number of samples was the largest. The data were analyzed separately for guaranteed levels of 4 to 6% and of 8 to 12% for each constituent for each year from 1959 to 1963, as shown for nitrogen in Table 6. In 113 guarantees of 4-6% of nitrogen over the 5 years, five determinations were omitted as outliers, one below claim and the other four above claim. For the 85 samples at the higher level, two were omitted as

outliers, one below and the other above the assumed normal range of per cent claim.

Each of the 10 distributions of nitrogen content was then analyzed as a complete series of 13-26 observations in computing its mean and standard deviation of the percentage of guaranteed content. Within each guaranteed content of ingredient, the means for 1959-61 seemed essentially in agreement with one another but differed appreciably from the means for 1962-63. Accordingly, the data for each of the two periods have been analyzed separately in Table 7, and supplemented with the analyses for phosphoric acid and for potash from the same set of samples. Means and standard deviations were then computed from the percentage of claim for each ingredient in the individual samples.

Table 6. Analyses for nitrogen in yearly samples of mixed fertilizers from Firm G (Table 3), in units of percentage of guarantee

Year	Normal range	No.	Mean % claim	Standard deviation	Below 100% f	Below 100% ø	Outliers Low	Outliers High
			Guaranteed content of 4-6 percent					
1959	92.8 — 118.4	26	104.950	6.739	4	6.0	0	0
1960	93.3 — 117.5	26	104.754	6.118	5	5.7	0	1
1961	100.0 — 107.2	16	103.062	2.411	0	1.6	0	2
1962	100.0 — 112.0	16	105.719	4.254	0	1.4	1	0
1963	100.0 — 119.0	24	107.162	5.414	0	2.2	0	1
			Guaranteed content of 8-12 percent					
1959	87.0 — 112.2	16	99.381	7.816	7	8.5	0	0
1960	89.8 — 108.5	15	98.693	5.554	9	8.9	0	0
1961	92.0 — 109.8	13	99.138	4.639	6	7.5	0	0
1962	94.4 — 112.4	15	102.153	5.142	3	5.1	0	0
1963	89.6 — 116.0	24	102.842	7.643	6	8.5	1	1

Pooling analyses in this way increased their number sufficiently that it was no longer practicable to plot each analysis individually as in Figs. 1 and 2. Instead, the ranked observations have been averaged in successive groups of three to five determinations in each ranked sequence of percentages of claim. When the analyses in a series did not form an exact multiple of the number per group, they were averaged in equal intervals, starting from both ends of the ranked analyses, and the central one or two intervals adjusted for differences in size.

For plotting against these mean percentages, the rankits for an equal number of sub-divisions could be averaged similarly from a long sequence of rankits, as in the last column of Table 8 for N.= 20. Equivalent values could be determined more exactly, however, from tables of the deviates and ordinates of the normal curve, leading to units which P. C. Mahalanobis (1960) has termed "fractiles." These have been computed for series of 6 to 20 intervals in Table 8.

Table 7. Average analyses for nitrogen, available phosphoric acid, and potash in the samples in Table 6

Content claimed	Years	Normal range	No.	Mean % of claim	Standard deviation	Below 100% f	Below 100% φ	"Outliers" Low	"Outliers" High	"Outliers" Mean %
Analyses for nitrogen										
4-6%	1951-61	92.8 — 118.4	68	104.431	5.728	9	14.9	0	3	141.93
	1962-63	100.0 — 119.0	40	106.585	4.976	0	3.7	1	1	96.80
8-12%	1959-61	87.0 — 112.2	44	99.075	6.119	22	24.6	0	0	—
	1962-63	89.6 — 116.0	39	102.577	6.724	9	13.7	1	1	97.20
Analyses for "available" phosphoric acid (as P_2O_5)										
4-6%	1959-61	100.3 — 126.2	21	112.257	6.588	0	0.7	0	5	148.16
	1962-63	93.2 — 120.5	24	107.108	5.653	2	2.5	5	5	107.45
8-12%	1959-61	95.5 — 118.7	78	105.973	4.940	7	8.8	1	4	117.10
	1962-63	91.5 — 113.8	48	103.944	4.778	6	9.8	3	2	109.98
Analyses for potash (as K_2O)										
4-6%	1959-61	93.6 — 123.4	55	109.247	6.197	2	3.7	0	8	132.49
	1962-63	91.8 — 122.0	43	107.242	5.695	1	4.4	2	9	130.10
8-12%	1959-61	84.5 — 117.9	43	102.998	5.506	6	12.6	1	0	81.00
	1962-63	86.2 — 109.3	27	98.619	6.445	9	15.8	1	3	140.58

The individual analyses of the three ingredients, nitrogen, phosphoric acid, and potash, in the samples summarized in Table 7 have been grouped and plotted in Fig. 3 for guarantees of 4 to 6% and in Fig. 4 for guarantees of 8 to 12%. Each has been fitted with a straight line passing through zero

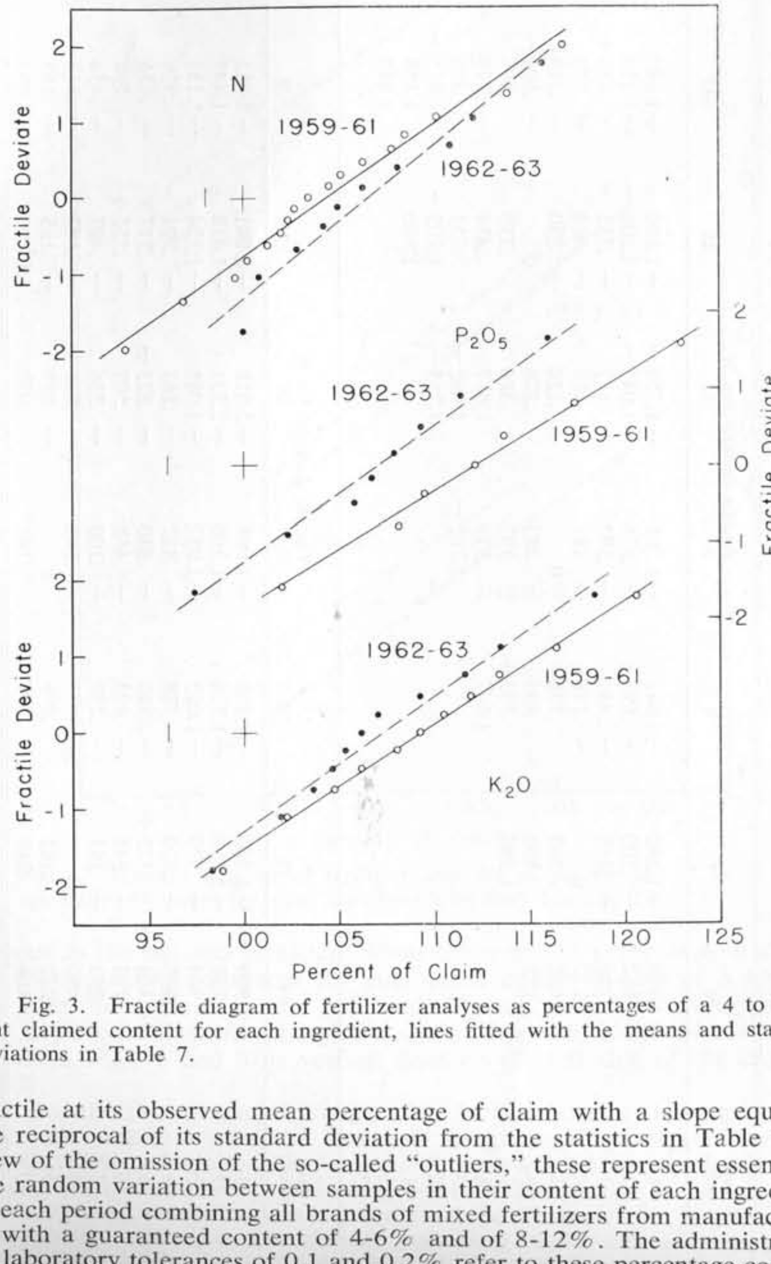


Fig. 3. Fractile diagram of fertilizer analyses as percentages of a 4 to 6 per cent claimed content for each ingredient, lines fitted with the means and standard deviations in Table 7.

fractile at its observed mean percentage of claim with a slope equal to the reciprocal of its standard deviation from the statistics in Table 7. In view of the omission of the so-called "outliers," these represent essentially the random variation between samples in their content of each ingredient, in each period combining all brands of mixed fertilizers from manufacturer G with a guaranteed content of 4-6% and of 8-12%. The administrative or laboratory tolerances of 0.1 and 0.2% refer to these percentage concen-

Table 8. Fractile mean deviates of the normal curve corresponding to successive equal fractions of the area under the curve, for plotting against the mean of N_e equal groups of ranked variates

Rank order	Fractile deviates for N_e 6	7	8	9	10	11	12	13
1	−1.499	−1.580	−1.647	−1.705−	−1.755−	−1.800	−1.840	−1.876
2	− .682	− .800	− .895+	− .976	−1.045−	−1.105−	−1.158	−1.206
3	− .212	− .368	− .491	− .592	− .677	− .751	− .815+	− .872
4	.212	0	− .158	− .283	− .386	− .474	− .550	− .617
5	.682	.368	.158	0	− .126	− .230	− .319	− .396
6	1.499	.800	.491	.283	.126	0	− .105−	− .194
7		1.580	.895+	.592	.386	.230	.105−	0
8			1.647	.976	.677	.474	.319	.194
9				1.705−	1.045−	.751	.550	.396
10					1.755−	1.105−	.815+	.617
11						1.800	1.158	.872
12							1.840	1.206
13								1.876

Rank order	Fractile deviates[1] for N_e 14	15	16	17	18	19	20	20[2]
1	−1.909	−1.940	−1.968	−1.994	−2.018	−2.041	−2.063	−2.051
2	−1.250	−1.290	−1.326	−1.360	−1.391	−1.418	−1.447	−1.446
3	− .924	− .970	−1.013	−1.052	−1.088	−1.124	−1.153	−1.152
4	− .676	− .730	− .778	− .822	− .863	− .901	− .936	− .935+
5	− .464	− .525+	− .580	− .630	− .675+	− .717	− .756	− .756
6	− .272	− .348	− .403	− .458	− .509	− .555+	− .598	− .598
7	− .090	− .161	− .237	− .300	− .356	− .407	− .454	− .454
8	.090	0	− .078	− .148	− .211	− .267	− .319	− .319
9	.272	.161	.078	0	− .070	− .133	− .189	− .189
10	.464	.348	.237	.148	.070	0	− .063	− .063

[1] Positive values are the negative values in reversed order with a positive sign. [2] Approximations (for comparison) from the average rankits for a series of 400 in successive groups of 20.

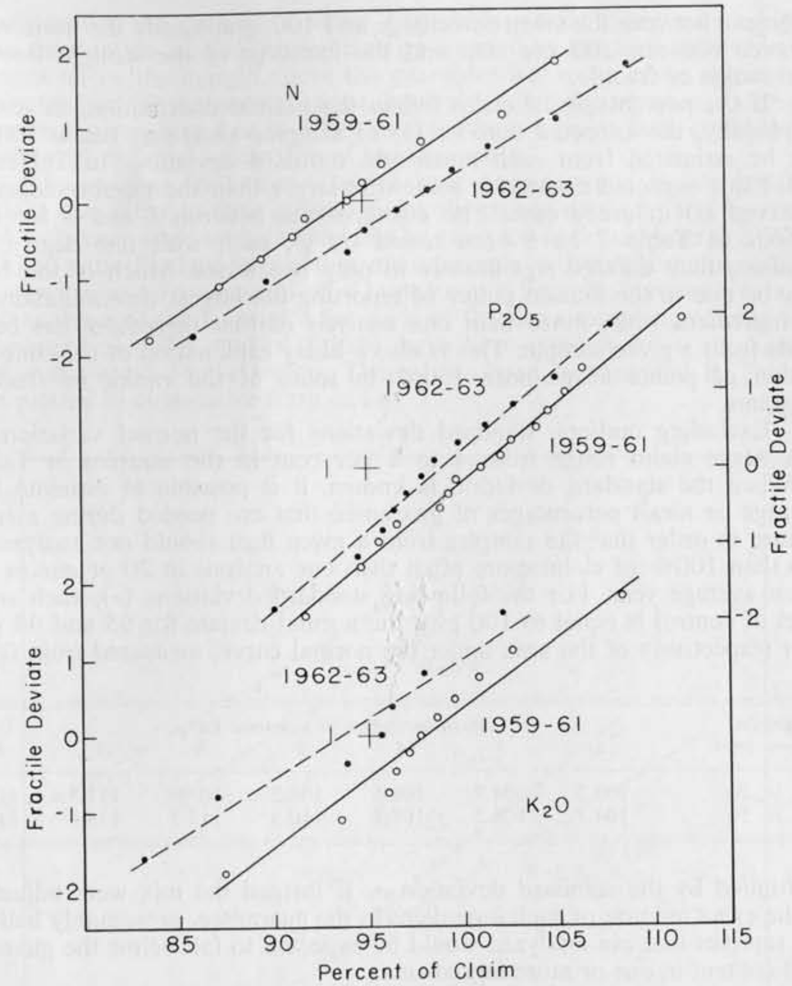


Fig. 4. Fractile diagram of fertilizer analyses as percentages of an 8 to 12 per cent claimed content for each ingredient with lines fitted in Table 7.

trations in the finished fertilizer. When converted to percentage of claim, the corresponding tolerances for guaranteed concentrations of 5 and 10 per cent, for example, are 2% and 1% of claim for nitrogen and 4% and 2% of claim for phosphoric acid and potash. These tolerances are shown in Figs. 3 and 4 as vertical lines on the left side of the cross at 100%.

Because the standard deviation exceeds the laboratory tolerance, some analyzed samples within the "normal" range could be expected to fall below 100% of claim. The numbers observed (f) for each element, guaranteed content and period are given in Tables 6 and 7. To keep this number to a small fraction of the total production, the manufacturer customarily adds an excess of each ingredient to the mix as a margin of safety above each claim. An estimate of this margin or "overage" is the difference between the mean percentage and 100, graphically the horizontal distance between 100 per cent and the intercept of the straight line at zero rankit or fractile.

If the percentages of claim follow the normal distribution, as tested graphically, the expected number (ϕ) of samples analyzing below 100% can be estimated from each mean and standard deviation. In Tables 6 and 7 this expected number is somewhat larger than the number actually observed (f) in every case. The discrepancies between f and ϕ for the periods in Table 7 have been tested by χ^2, each with one degree of freedom; they differed significantly in only two series. Much of the bias may be due to the Station policy of reporting the largest determination of an ingredient when more than one analysis of that ingredient has been made from a given sample. This is also a likely explanation of the concentration of points at or near 100% in some of the rankit or fractile diagrams.

Excluding outliers, standard deviations for the normal variation in percentage claim range from 4 to 8 per cent in the samples in Table 7. When the standard deviation is known, it is possible to estimate the overage or mean percentages of guarantee that are needed during manufacture in order that the samples from a given firm should not analyze at less than 100% of claim more often than one analysis in 20 or one in 50 in an average year. For the following standard deviations (σ), each such level of control is equal to 100 plus the normal deviate for 95 and 98 per cent respectively of the area under the normal curve, measured from $-\infty$,

Samples below 100%	Mean percentage of guarantee for σ = 2	3	4	5	6	7	8
1 in 20	103.3	104.9	106.6	108.2	109.9	111.5	113.2
1 in 50	104.1	106.2	108.2	110.3	112.3	114.4	116.4

multiplied by the standard deviation σ. If instead the mix were adjusted to the exact amount of each ingredient in the guarantee, presumably half of the samples that are analyzed would be expected to fall below the guaranteed content in one or more ingredients.

In preparing Tables 6 and 7, the data for each year were plotted separately to establish the so-called "normal range" of actual analyses within which the above inferences would apply. These ranges are given in the tables together with the number of analyses covered by each range. Samples falling outside these zones have been termed "outliers." Their numbers below and above the normal zone and their overall means are listed in the last three columns of Table 7. They represent a much larger fraction of all analyses for phosphoric acid and for potash (12.8 and 12.5 per cent) than for the more costly nitrogen (3.5 per cent). Equally striking is the larger number of samples with a content above (41) than below (15) guarantee, so that the outliers average 116.26 per cent of claim for nitrogen, 118.03 per cent for phosphoric acid, and 130.60 per cent for potash.

Some of these may have been mistakes in labeling, such as filling bags with the wrong mixture. In other cases, it may have been more practicable to increase the margin of safety than to reset the machinery. If the number of outlier "mistakes" above claim were equal to the number below claim,

nearly half of the outliers in Table 7 would represent filling with a higher ratio than on the label. Within the range covered by the high outliers in the present series, the margin above the guarantee was not a cause of concern. A content below the normal range, however, when recurrent in successive years in a given brand or in many samples from a given firm, could be indicative of dishonest intent or of incompetence.

The analyses in Tables 6 and 7 have been grouped in two separate periods that differ in their means, giving the paired diagrams in Figs.3 and 4. A difference such as this would not be expected to occur generally. When checked with the data for another manufacturer with many samples (Firm C), there was no suggestion of a systematic difference between years, so that analyses of their samples for each of the three main ingredients with guarantees of 4-6% and of 8-12% have been combined for all samples within the same 5-year period. These have been summarized in Table 9 and plotted in cumulative form in Fig. 5.

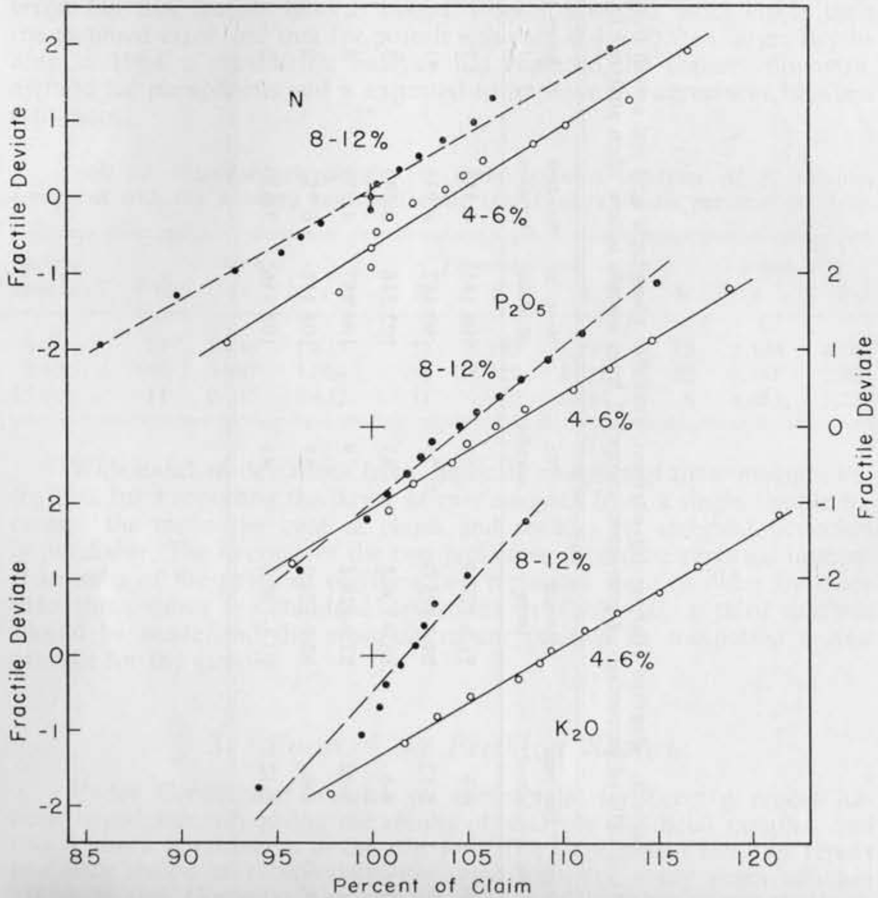


Fig. 5. Fractile diagram of fertilizer analyses as precentages of claimed contents of 4 to 6 and of 8 to 12 per cent, from Table 9.

Table 9. Average analyses for nitrogen, available phosphoric acid and potash from the 1959-1963 samples for Firm C (Table 3) in units of percentage of guarantee, computed from sample means when analyzed in duplicate

Ingredient	Per cent claimed	Normal range	No. samples	Mean % of claim	Standard deviation	Below 100% f	Below 100% φ	No. duplicate analyses	"Outliers" Low	"Outliers" High	"Outliers" Mean %
N	4-6	91.6-120.0	69	104.143	6.353	7	17.7	6	0	3	124.0
	8-12	84.4-128.0	75	99.347	6.931	31	40.3	27	1	0	65.6
P_2O_5	4-6	88.0-120.8	43	107.216	6.689	3	6.0	4	0	7	151.3
	8-12	93.0-118.1	79	104.957	4.924	8	12.4	10	0	1	123.7
K_2O	4-6	92.0-124.2	58	109.312	6.757	2	4.9	1	0	18	146.6
	8-12	90.8-108.6	41	101.756	3.620	5	12.9	5	2	0	81.9

When there were two analyses from a given sample, the fertilizer reports have reported the larger of the two. Since replicate analyses were nearly always of samples which fell below claim, the dip in the plotted points at the lower end in many initial curves could be traced to this source. As a test, the mean percentages of claim and their standard deviations in Table 9 and the curves in Fig. 5 have been computed with the means of all duplicate analyses.

The empirical analytical error could be computed directly from the standard deviation (s) between the duplicate analyses of N samples taken in 1959 to 1963. These have been summarized in Table 10 in terms of the percentage of claim for guarantees of 4-6%, 8-12% and 15-20% of each ingredient. For comparison, the assumed analytical errors of 0.1% of the guaranteed content of nitrogen and of 0.2% of the content of phosphoric acid and of potash have been converted to percentages of claim and averaged for the samples in each interval. During this period, the standard deviation for nitrogen agreed substantially with its established analytical error, but that for phosphorus ranged from 1.6 to 2.0 times larger than the assumed error and that for potash was only 0.4 to 0.7 as large. Beginning in 1964 a gravimetric analysis has replaced the earlier volumetric method for phosphorus and is expected to improve the agreement between replicates.

Table 10. Standard deviations (s) from replicate analyses of N samples compared with the assumed analytical error (e) in units of the per cent of claim

Per cent claimed	Nitrogen N	Nitrogen s	Nitrogen e	Phosphoric Acid N	Phosphoric Acid s	Phosphoric Acid e	Potash N	Potash s	Potash e
4-6	11	1.456	1.818	22	6.795	3.739	19	2.734	4.544
8-12	48	0.997	1.084	36	4.122	2.026	22	1.368	2.023
15-20	11	0.745	0.612	31	1.929	1.184	6	0.485	1.236

With standard deviations from duplicate analyses of these magnitudes, the bias from reporting the larger of two analyses from a single sample increases the mean per cent of claim and reduces its standard deviation unjustifiably. The average of the two replicates should be reported instead. If in units of the per cent of claim two replicates were to differ by more than three times the standard deviations in Table 10, a third analysis should be made and the most discrepant omitted in computing a new average for the sample.

3. *Form of the Fertilizer Report*

Under Connecticut statutes on commercial fertilizers a report has been issued annually giving the results of analyses of official samples, and this is also a requirement in the new law. The Connecticut fertilizer report has been issued in substantially the same form for many years and has grown in size. Corresponding reports are issued in other states, the form varying from state to state. In a comparison of a recent Connecticut report with its equivalent in six other states (Table 11), the tonnage of fertilizer sold in Connecticut was the smallest, but more samples were analyzed per 1000 tons and the report contained more detail. A study of its format has suggested changes that would make its essential information more accessible to the reader in fewer pages and the bulletin as a whole more effective in its objective of "keeping people honest." A number of these proposals have been adopted in the report for 1964 (Fisher, 1965).

One of the first questions of interest to a farmer or home gardener concerns the fertilizer he will need in growing the crops he plans. Information on the soil analysis service provided by the Station and a short list of publications on their agronomic applications might be noted. The Fertilizer Report has listed the ratios of mixed fertilizers recommended by New England agronomists, a practice which might well be continued. Both the old and new statutes call for a report of the tonnage sold of different types of fertilizers.

Table 11. Comparison of the Connecticut Fertilizer Report (Bulletin 666) with corresponding reports from six other states

State	Year	Tons sold	No. of samples	Samples per 1000 tons
Connecticut	1963	73,100	577	7.9
Massachusetts	1963	79,900	491	6.1
North Dakota	1963	138,400	133	1.0
Maine	1963	148,900	323	2.2
Kentucky[1]	1962	632,000	642	1.6
Pennsylvania	1961	637,700	1200	1.9
Indiana	1962	1,236,500	4520	3.7

[1] July-December only.

When a prospective purchaser has selected the type of fertilizer that he needs, he is interested in comparing the brands that have been sampled and their analyses. The majority of these are mixed fertilizers, many of them in small packages. A convenient arrangement would list them all under increasing ratios of the three ingredients, nitrogen, available phosphoric acid, and potash, and under each ratio the grade in order of increasing concentration, with reference numbers for each sample and its manufacturer.

This recommendation has been followed in Table 5 of the Fertilizer Report for 1964 (Bulletin 671), from which the first page is shown in Table 12. With the available brands arranged alphabetically, the reader has in successive rows all sampled brands for a given ratio and grade of NPK, their guaranteed content of each ingredient as part of the brand name, and the compliance of each sample with claim as measured analytically. These could be given as percentages of each constituent in the fertilizer, as in Table12, or converted to percentages of claim as in the preceding section of this bulletin. As supplementary information of increasing interest, the percentages of the observed nitrogen that are water soluble organic and water insoluble organic have been retained. A similar table could be prepared separately for the relatively few mixed fertilizers

Table 12. Format for analyses of mixed fertilizers with guaranteed levels of nitrogen, available phosphoric acid and potash, from Table 5 in Bulletin 671

Station No.	Ratio, grade and brand		Mfr. No.	Per cent organic nitrogen Sol.	Insol.	Per cent total nitrogen	Per cent available phosphoric acid	Per cent water-soluble potash
	0- 1- 1							
2630/[1]	0-20-20	AA Quality Fertilizer (20 lbs. Borax/Ton)	6	0.00	0.10	1.36	18.18	20.04
3055	0-20-20	Corenco Fertilizer	36	----	----	----	19.78	18.18
2671	0-20-20	Davco 3-G Granulated Fertilizer	65	----	----	----	17.55	23.93
2489	0-20-20	Liberty Fertilizer	80	----	----	----	20.13	20.16
	0- 1- 2							
2421[1]	0-15-30	AA Quality Fertilizer (20 lbs. Borax/Ton)	6	----	----	----	14.34	31.94
2426[1]	0-15-30	AA Quality Fertilizer (20 lbs. Borax/Ton)	6	----	----	----	15.29	31.18
2609	0-15-30	Corenco Fertilizer	36	----	----	----	15.41	26.81
2768	0-15-30	Davco 3-G Granulated Fertilizer	65	----	----	----	17.59	32.20
3032	0-15-30	Liberty Fertilizer	80	----	----	----	14.13	33.49
2360	0-15-30	Old Fox Fertilizer	99	----	----	----	12.70	32.84
	0- 1- 3							
2766	0-10-30	Davco 3-G Granulated Fertilizer	65	----	----	----	9.91	33.79
2245	0-10-30	Eastern States with 0.3 Boron	3	----	----	----	10.95	30.33
	1- 1- 1							
2960	4- 4- 4	Brookside's Nature's Thrift	24	0.34	4.06	4.40	5.01	3.50
3069*	5- 5- 5	Chemton Liquid Plant Food	73	4.00	0.00	5.20	5.12	5.01
2231	5- 5- 5	Cross Country Organic Lawn Food	116	0.38	5.10	5.64	7.72	----
2604	7- 7- 7	AA Quality Fertilizer	6	0.16	0.14	5.48	9.47	10.10
2788	7- 7- 7	Corenco Granulated Form Fertilizer	36	0.40	0.00	6.76	8.05	7.48

* Small package sample. / Unregistered. ~~~~ Indicates deficiencies. [1] State purchase.

Table 13. Compliance of 1964 fertilizer samples with guarantees, from Circular 226

Manufacturer	No. of samples	No. of guaranties N	$P_2O_5+K_2O$	total	Samples deficient N	$P_2O_5+K_2O$	Average % of claims N	$P_2O_5+K_2O$
Agricultural Products Co., Inc.	6	6	10	16	1	6	115.1	101.7
Agricultural Supply Co.	1	1	2	3	0	0	122.0	151.1
Agway, Inc.	17	14	31	45	0	4	102.3	100.4
Alaska Fertilizer Co.	1	1	2	3	0	0	120.8	143.5
Allied Chemical Corp.	3	3	0	3	0	0	100.6	-----
American Agr. Chemical Co.	36	29	67	96	9	15	101.4	104.5
American Cyanamid Co.	3	3	0	3	1	0	99.9	-----
American Fertilizer Co.	5	5	10	15	1	0	112.6	120.2
A-Penn Oil Co.	2	2	4	6	0	0	115.2	127.7
Armour Agr. Chemical Co.	15	15	28	43	5	3	108.3	115.9
Asgrow Seed Co.	4	4	8	12	1	2	104.7	118.5
Atlantic Seed Co.	1	1	2	3	0	1	100.0	95.4
Atlas Fish Fertilizer Co.	1	1	2	3	0	0	107.2	124.0
Baker Castor Oil Co.	3	0	0	0	---	---	-----	-----
F.A. Bartlett Tree Expert Co.	1	1	2	3	0	1	102.7	105.7
Baugh Chemical Co.	3	2	3	5	0	1	180.0	88.5
Bishop Processing Co.	1	1	1	2	1	0	92.3	116.3
Black Magic, Inc.	2	2	4	6	0	1	125.0	118.6
Blackstone Guano Co., Inc.	8	8	14	22	2	1	101.7	109.4
Bonide Chemical Co., Inc.	2	2	4	6	1	0	108.3	118.3
Borden Chemical Co. Div.	1	1	0	1	0	0	103.2	-----
Boyle-Midway, Inc.	1	1	0	1	0	0	102.5	-----
Breck's Div., Asgrow Seed Co.	6	6	11	17	0	4	101.8	108.4
Brookside Nurseries, Inc.	6	5	7	12	0	2	106.2	109.5

containing magnesium as an additional guaranteed element instead of listing magnesium in a separate column (omitted here from Table 12). The present tables for materials supplying chiefly nitrogen, superphosphate, potash, and products of animal origin could be shortened considerably by a corresponding arrangement.

As his next question a grower would be interested in comparing the firms which make the type of fertilizer he needs. Although he could compare analyses and claims in the brand samples from the previous year, it is evident from the present study that a given sample may analyze somewhat below its guarantee purely by chance, even though the average production of the manufacturer in the same year may fall well above his guarantee. A more reliable indicator of the performance of different manufacturers would be their average percentage compliance with their guarantees.

This proposal for summarizing fertilizer analyses was adopted in Part I of the Fertilizer Report for 1964, issued separately as Circular 226. The average percentages of claim were computed for each firm that was sampled and had guaranteed contents for one or more of the major constituents, nitrogen, phosphoric acid, and potash, combining the phosphoric acid and potash. These averages are given in the last two columns of the table in Circular 226, a portion of which is reproduced in Table 13 of the present bulletin. Except for outliers, the averages above 100 per cent measure the "overage" or safety margin put in by the manufacturer as insurance against uncontrolled or uncontrollable variation in production, mixing and handling, and lack of representativeness in the particular sample that has been analyzed. This type of summary permits a comparison between the manufacturers that handle the mixtures of interest to a prospective purchaser.

Of equal importance is the proportion of samples falling below 100% of the guarantee. In general, the larger the average per cent of claim or the "overage," the smaller will be the proportion of deficient samples, but, equally, the less uniform the product the larger will be the proportion of deficient samples for a given average per cent of claim. An average which included two or three "outliers" with 150% of claim or better could balance a much larger number of deficient samples that were below claim.

Caution is needed in comparisons based upon relatively few samples. In 1964, for example, 99 of the 127 manufacturers listed in Circular 226 were represented by four or fewer samples, too small a number to give a reliable index to an individual manufacturer's performance. Yet their percentages of claim averaged 110.2% for nitrogen and 119.2% for phosphoric acid and potash, even though 21.3% of their guarantees for nitrogen and 15.2% of their guarantees for the other two ingredients fell below 100%. For comparison, the 12 firms represented by eight or more samples averaged 104.8% of claim for nitrogen and 112.3% for phosporic acid and potash, with 23.6% and 19.9% of these analyses falling below 100% of claim.

Both to check on the policy of a manufacturer over a number of years and to increase the number of analyzed samples by which it can be judged, an additional table would be helpful, restricted to firms with an average of eight or more samples or 20 or more analyses of NPK per year in the last 4 years. It would show the cumulative results as percentages of guarantee of nitrogen and of phosphoric acid and potash in the samples from each firm over this period. Patterned after Tables 7 and 9 of this Bulletin, it would give the number of analyses within the normal range of approximately 85 to 125 per cent of claim, their mean percentages of claim, and the number of analyses below 100% of claim within this range. The total number of outliers or analyses below and above this range would be listed in two additional columns. It is evident from part 2 of this Bulletin that an occasional percentage of claim falling below 100%, but still well in the 90's, is no source of alarm if the mean percentage of claim exceeds 100%.

Summary

The operation of the Connecticut Fertilizer Law is examined statistically in relation to the selection of samples for analysis, the interpretation of the analytical results, and the form of their publication.

Although it has not been possible to sample more than 6 or 7 brands in every 10 that were registered in the past 4 to 9 years, the tonnage sold in Connecticut was least for the brands that were not sampled, but increased progressively by two to threefold or more for brands from which one to three or more samples were collected in each year. The median tonnage reported sold increased more than tenfold from that for brands listed in only one of the last 4 years to brands listed in all of those years.

The percentage of analyses falling below 100% of claim has been examined for the 11 firms for which 20 or more analyses were reported in each of the last 9 years. A statistical study showed no discernible trend in the percentage of deficient analyses over this period, but very significant differences between firms, ranging from means of 6.1 to 15.2% of individual guarantees falling below 100%.

For a more detailed study, the analytical findings for nitrogen, for phosphoric acid, and for potash in mixed fertilizers have been converted to percentages of claimed content for all samples collected in the last 5 years from two of the firms with the largest number of samples. The distribution of these percentages of claim for each year was then examined separately for each constituent with guaranteed contents of 4-6% and 8-12%. By graphic tests the analyses with a range of approximately 85 to 125% of claim could be identified as normally distributed. Within this range the mean percentage of claim estimated any overage adopted by the manufacturer to meet his guarantee, while its standard deviation measured the uniformity of his products or the effectiveness of his quality control. Of the "outliers" falling outside the normal range, those above claim were three times as numerous as those below claim. Although contributing to the variation between samples, the analytical error in measurements of content, as determined from duplicate analyses of the same sample, was of a much smaller magnitude. The variation in nitrogen analyses agreed with the assumed analytical error; for phosphorus it exceeded the assumed error by as much as twofold, and for potash it dropped to less than half as large as the assumed error.

A review of the form of the Fertilizer Report proposed several changes. For easy comparison of manufacturers in the current year, their total performance could be averaged separately for content of nitrogen and of phosphoric acid and potash in terms of the mean percentage of claim over all their brands, supplemented by the number falling below claim in the two constituents. This recommendation has been followed in Part I of the Fertilizer Report for 1964 (Circular 226). As a second recommendation, listing the analyses of mixed fertilizers by the ratio of their claimed content of nitrogen, available phosphoric acid, and potash, and under each ratio in order of increasing concentration, would facilitate the comparison of competing products. This arrangement has been adopted in Part II of the Fertilizer Report for 1964 (Bulletin 671). A third recommendation proposes that the cumulative results over the last 4 years for firms with an average per year of 8 or more samples, or 20 or more analyses of NPK be reported routinely. After separating the outliers from those in the normal range of 85-125% of claim, the mean percentages of claim for nitrogen and for phosphoric acid and potash would be listed separately as well as the number within this range below 100% of claim, and the number of outliers below and above this range in additional columns. Other suggestions on format complete the Bulletin.

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