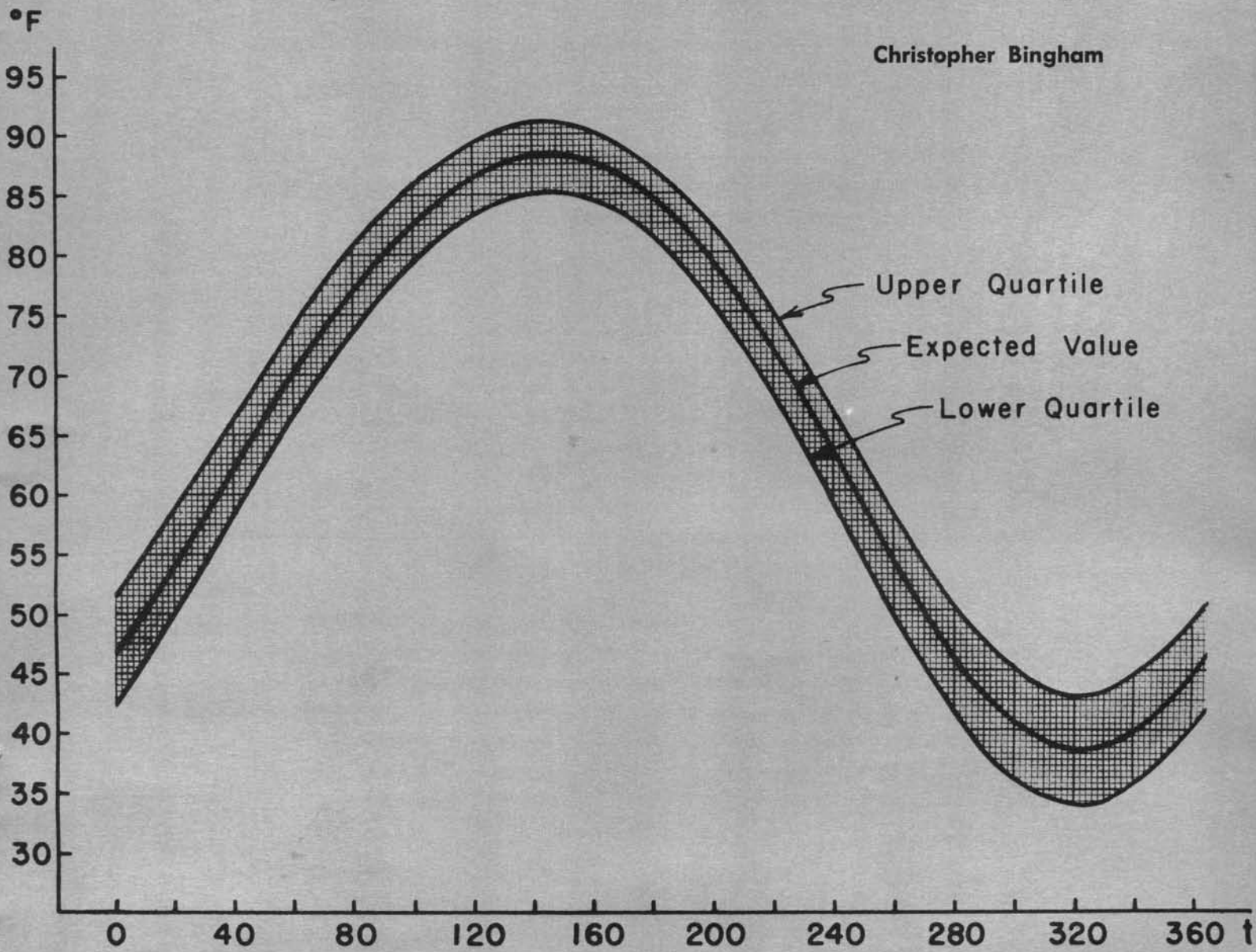




THE CLIMATE OF THE NORTHEAST

Probabilities of Weekly Averages of the Daily Temperature Maximum, Minimum, and Range

Christopher Bingham



Bulletin 659
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INTRODUCTION

A complete description of the climatology of a region requires not only the average values or normals of climatological variates, but also information about their probability distributions. This bulletin provides in a compact form the information necessary to compute all probability levels of certain temperature variates at all points in the Northeastern region of the United States: New England, New York, Pennsylvania, New Jersey, Delaware, Maryland, and West Virginia. These temperature variates are the following:

1. The average of any 7 consecutive values of the diurnal maximum temperature;
2. The average of any 7 consecutive values of the diurnal minimum temperature;
3. The average of any 7 consecutive values of the diurnal temperature range.

When we refer to the maximum temperature, the minimum temperature, or the diurnal temperature range, we will always mean these 7-day averages.

THEORY

It has been shown elsewhere (Bingham 1961) that the probability distributions of these three variates are not in general normal. That is, they do not have the bell-shaped frequency curve given by $\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{[x-\mu]^2}{2\sigma^2}\right)$. However, for most practical applications, it was shown that the normal distribution is an adequate approximation. This assumption underlies all that follows. Because of the non-normality, the method given herein is not recommended for estimating probabilities very far in the tails of a distribution, say beyond the 5% or 95% points. For probabilities in this region it should be preferable to use some form of extreme value analysis on the original data.

Given that a distribution is normal, we can easily see that it will be completely specified by its expected or average value μ , which determines the center of the bell-shaped curve, and by its standard deviation σ , which indicates the "spread" of the distribution. The parameters are illustrated

in Figure 1. It was also shown (Bingham 1961), using methods developed by Bliss (1958), that the values of these parameters for every week of the year for the maximum and minimum temperatures and the diurnal temperature range can be specified very concisely using Fourier or harmonic analysis as described below.

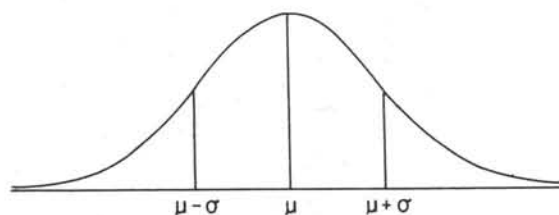


Figure 1. The frequency curve $\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ of the normal distribution with expected value μ and standard deviation σ . This approximates the distributions of the maximum and minimum temperatures and the diurnal temperature range.

In the following formulae and throughout this bulletin, t is the number of days after March 1, the start of the climatological year. The correspondence between the ordinary calendar and t is given in Table 1. For instance, $t=131$ refers to the week for which July 10 is the midpoint, i.e., the week of July 7 through 14. Thus $\mu(t)$ and $\sigma(t)$ are the expected value and standard deviation of a variate for the week whose midpoint comes t days after March 1. Clearly $t=0$ on March 1, that is, for the week of February 26 through March 3.

The expected value $\mu(t)$ and the common logarithm of the standard deviation $\sigma(t)$ for the maximum, minimum, or range can be expressed with notable accuracy by an equation of the form

$$y(t) = a_0 + A_1 \cos(t-\phi_1) + A_2 \cos 2(t-\phi_2) + A_3 \cos 3(t-\phi_3) \quad (1)$$

where $y(t)$ can represent either $\mu(t)$ or $\log \sigma(t)$. Here t , ϕ_1 , ϕ_2 , and ϕ_3 are measured in days after March 1,*

* For computational convenience, the notation here differs slightly from Bingham (1961). ϕ_2 and ϕ_3 in this bulletin are equal respectively to $1/2 \phi_2$ and $1/3 \phi_3$ in the previous paper.

TABLE 1

Date	t	Date	t
March 1	0	Sept. 1	184
10	9	10	193
20	19	20	203
30	29	30	213
April 1	31	Oct. 1	214
10	40	10	223
20	50	20	233
30	60	30	243
May 1	61	Nov. 1	245
10	70	10	254
20	80	20	264
30	90	30	274
June 1	92	Dec. 1	275
10	101	10	284
20	111	20	294
30	121	30	304
July 1	122	Jan. 1	306
10	131	10	315
20	141	20	325
30	151	30	335
August 1	153	Feb. 1	337
10	162	10	346
20	172	20	356
30	182	28	364

Table 1. t = number of days after March 1 of calendar dates.

while a_0 , A_1 , A_2 , and A_3 are either in °F or log °F. Strictly speaking, we should transform the units of days to units of angular measure such as radians or degrees. However, since a table of $\cos t$ in units of days is included in the bulletin (Table 2) no confusion should arise.

Formula (1) expresses $y(t)$ as the sum of a yearly average and three cosine terms, as is displayed in Figure 2. The average over the whole year is a_0 . The first cosine term, $A_1 \cos(t - \phi_1)$, has a single maximum at day ϕ_1 . Its semi-amplitude, A_1 , is the height of this maximum above a_0 . The second cosine term, $A_2 \cos 2(t - \phi_2)$, has two maxima during the year at days ϕ_2 and $(\phi_2 + 182\frac{1}{2})$. Its semi-amplitude, A_2 , measures its maximum effect. The third term, $A_3 \cos 3(t - \phi_3)$, has three maxima at days ϕ_3 , $(\phi_3 + 121\frac{2}{3})$, and $(\phi_3 + 243\frac{1}{3})$, with A_3 having a similar significance as A_1 and A_2 . The meaning of all seven parameters is made clear in Figure 2 where they are displayed separately in the decomposition of the curve $y(t)$ at the top of the figure. At any time t , $y(t)$ is found by adding to a_0 the values of the three cosine curves at that time t . Once the coefficients a_0 , A_1 , A_2 , A_3 , ϕ_1 , ϕ_2 , and ϕ_3 for a curve of the form (1) have been given, the value of $y(t)$ can be easily computed using Table 2.

Both the mean, $\mu(t)$, and the log standard deviation, $\log \sigma(t)$, for the maximum and minimum temperature and the diurnal temperature range can be approximated over the entire year by curves of the form (1). Hence no more than

14 parameters — 7 for $\mu(t)$ and 7 for $\log \sigma(t)$ — are needed to compute probabilities for any week of the year for any one of these variates. In fact, fewer are required, since the second and third terms do not always add to the accuracy of the curve. For example, the log standard deviation of both the maximum and minimum temperatures can be specified sufficiently accurately by $\log \sigma(t) = a_0 + A_1 \cos(t - \phi_1)$ which requires only three parameters. Similarly, $\log \sigma(t)$ for the range needs only five parameters, as does $\mu(t)$ for the maximum temperature.

A further simplification is possible. Since the diurnal temperature range is equal to (diurnal maximum temperature) - (diurnal minimum temperature), one can compute the curve for $\mu(t)$ for the range from the difference of the curves for the maximum and minimum. Hence separate parameters are not needed for $\mu(t)$ for the range. Thus a large amount of reduction is possible. With no more than 23 parameters for any single location (8 for $\mu(t)$ and $\log \sigma(t)$ of the maximum, 10 for $\mu(t)$ and $\log \sigma(t)$ of the minimum, and 5 for $\log \sigma(t)$ of the range), probabilities for any 7-day period in the year can be computed for all three variates.

One final and very important step can now be taken. Since the parameters a_0 , A_j , and ϕ_{j3} for both $\mu(t)$ and $\log \sigma(t)$, vary regularly with location, isolines can be drawn on maps so that parameter values for any location in the Northeast can be determined. This has been done in the present bulletin. Since ϕ_2 for $\log \sigma(t)$ is effectively constant over the region, 22 maps, one for each parameter, replace a voluminous set of tables of different probability levels for a dense net of locations. The price paid is some easy calculations. These are simplified by the use of Tables 1, 2, and 3, and Workforms A, B, and C. Detailed instructions for these are given below, although the reader may find the workforms self-explanatory.

DATA

The maps were constructed from calculations which were performed using data on summary cards from the 149 stations in the Northeast listed in Table 4. This was part of the program of the Regional Technical Committee on Agricultural Climatology (NE 35). This program is described more fully by Havens and McGuire (1961). All stations with a summarized record of 20 to 30 years were used. Most have 30 years of record. These stations are listed in Table 4 together with their station numbers. Their locations may be determined from Map 1 on which the station numbers are plotted. The summary cards, 52 for each station, from week 1 (March 1-7) to week 52 (February 21-27), contained the (usually) 30-year averages and the estimated variances of the weekly averages of diurnal maximum and minimum temperatures. In addition, the average and the estimated variance of the weekly "mean" temperature, $(1/2$

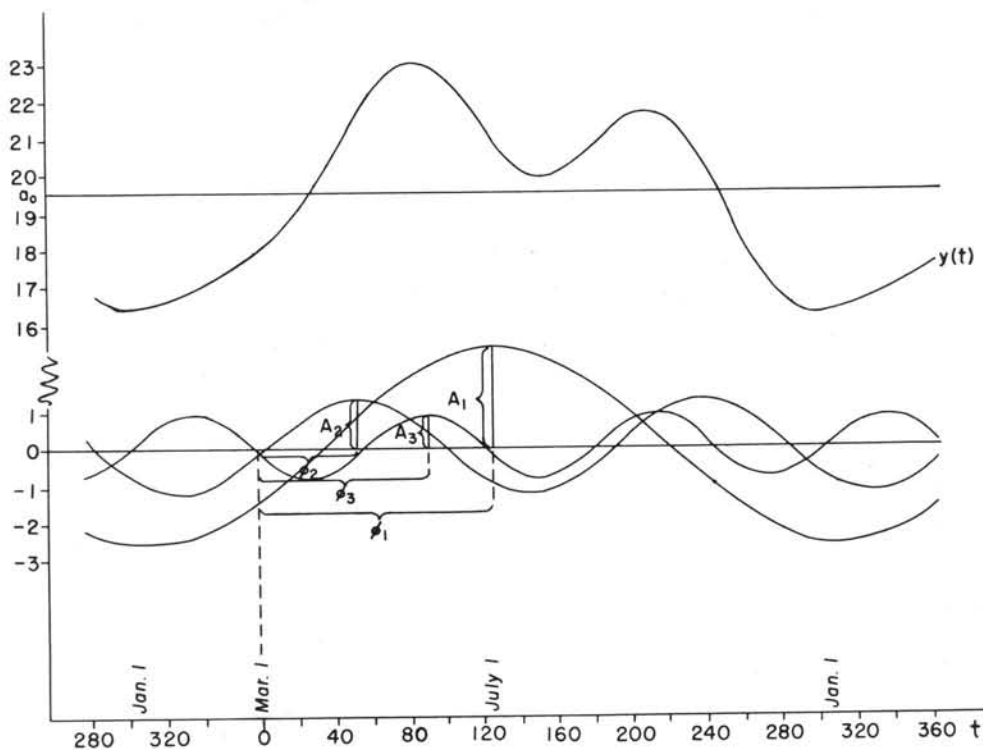


Figure 2. The decomposition of the curve $y(t)$ into the sum of a mean a_0 and three cosine terms.

(maximum + minimum)), were available on the cards. From these data, an IBM 650 program computed the average diurnal temperature range and its variance and then calculated the best fitting curve of type (1) to the variates and their log standard deviations. (The analysis was carried out on the "mean" temperature as well as on the other three variates. The "mean" was not included in this bulletin, since it has little intrinsic interest. Moreover, since it is derived by an even weighting of the maximum and minimum temperatures, it is not a fully accurate measure of the true mean temperature. This last is defined as an average over the entire course of temperature in a day, not just over the extreme values.) The maps in this bulletin were then compiled from the results of these calculations.

It will be noticed that the maps of a_0 for both the maximum and the minimum temperatures include exact calculated values for each station, while all other maps show only isolines. The relatively steep gradient of a_0 over the area, and the relatively greater influence of local topography upon a_0 make it impossible to get satisfactorily accurate values from an isoline map of the scale included. The exact values should enable fairly accurate interpolation on the map for any location not listed explicitly. However, a_0 is the long-term yearly average of each variate. Hence, whenever such an average is available for any location it can be used in place of a_0 from the maps.

FINDING TEMPERATURE PROBABILITIES FROM MAPS

Three elements go into computing probabilities for normally distributed temperatures — an expected value μ , a standard deviation σ , and probabilities of the normal distribution. Corresponding to these, three workforms are used. Workform A is used to compute the expected value μ for one or several weeks during the year. Workform B is used to compute the common logarithm of the standard deviation σ for these same weeks. Workform C presents a straightforward means of computing the 5, 10, 20, 25, 50, 75, 80, 90, and 95 percentiles for each of these same weeks.

Detailed instructions for these workforms are presented below. The workforms are compiled and their use described in greatest generality. That is, provision has been made for using all terms up to those in $\cos 3(t-\phi_3)$ for μ and up to $\cos 2(t-\phi_2)$ for $\log \sigma$. However, since the effect of including the second or third terms is never greater than their semi-amplitudes A_2 and A_3 , where these are small they contribute very little and hence should not be included. Thus, because A_3 for the maximum temperature is less than 0.5°F over practically the whole Northeast, no maps of A_3 (or ϕ_3) are included. Similarly, along the Atlantic coast and Lake Ontario, A_2 for the maximum temperature is less than 0.5°F and hence the term in $\cos 2(t-\phi_2)$ would generally not be included in this region. In cases in which a coefficient is

TABLE 2

t	cos t	t	cos t	t	cos t	
0	365	730	1.000	1095	730	365
5	370	735	0.996	1090	725	360
10	375	740	0.985	1085	720	355
15	380	745	0.967	1080	715	350
20	385	750	0.941	1075	710	345
25	390	755	0.909	1070	705	340
30	395	760	0.870	1065	700	335
35	400	765	0.824	1060	695	330
40	405	770	0.772	1055	690	325
45	410	775	0.715	1050	685	320
50	415	780	0.652	1045	680	315
55	420	785	0.584	1040	675	310
60	425	790	0.512	1035	670	305
65	430	795	0.438	1030	665	300
70	435	800	0.358	1025	660	295
75	440	805	0.276	1020	655	290
80	445	810	0.192	1015	650	285
85	450	815	0.107	1010	645	280
90	455	820	0.022	1005	640	275
95	460	825	-0.065	1000	635	270
100	465	830	-0.150	995	630	265
105	470	835	-0.234	990	625	260
110	475	840	-0.317	985	620	255
115	480	845	-0.398	980	615	250
120	485	850	-0.475	975	610	245
125	490	855	-0.549	970	605	240
130	495	860	-0.619	965	600	235
135	500	865	-0.684	960	595	230
140	505	870	-0.744	955	590	225
145	510	875	-0.799	950	585	220
150	515	880	-0.848	945	580	215
155	520	885	-0.890	940	575	210
160	525	890	-0.926	935	570	205
165	530	895	-0.955	930	565	200
170	535	900	-0.977	925	560	195
175	540	905	-0.992	920	555	190
180	545	910	-0.999	915	550	185
182.5	547.5	912.5	-1.000	912.5	547.5	182.5

Table 2. $\cos t$, t given in days, $\cos(-t) = \cos t$.

either omitted from the bulletin by the author or is considered by the user to be too small to be important, the lines in Workforms A and B corresponding to it can be skipped. The use of a short table for $\cos t$ (Table 2) is made possible by the equality $\cos(-t) = \cos t$. Hence in using Table 2 the sign of t is ignored, although the sign of $\cos t$ is always retained. Thus $\cos(-150) = \cos 150 = -0.848$.

In the following outline, Workforms A, B, and C will be referred to simply as A, B, and C respectively.

I. Parameters: μ and $\log \sigma$

1. Maximum temperature

a. Expected value μ .

i. Obtain ϕ_1 , ϕ_2 , A_1 , A_2 , and a_0 for maximum temperature from Maps 2, 3, 4, 5, and 6, respectively and enter them in the appropriate lines on the left side of A.

ii. Using Table 1, find the number of days, t , after March 1 of the midpoints of the weeks for which probabilities are sought and enter these values in row 1 of A, B, and C. (e.g., for the week of September 11 through 17 with midpoint September 14, $t = 197$).

iii. Fill in rows 1, 2, 3, 4, 5, 6, 10, 11, and 13 of A, using Table 2 to find $\cos t_1'$ and $\cos t_2'$.

iv. Enter row 13 of A into row 2 of C.

b. Log standard deviation, $\log \sigma$.

i. Obtain ϕ_1 , A_1 , and a_0 for log standard deviation of maximum temperature from Maps 7, 8, and 9 respectively and enter them on the left side of B.

ii. Fill in rows 2, 3, 7, and 9 of B (row 1 should already be completed from a. *ii.* above), using Table 2 to find $\cos t_1'$.

iii. Enter row 9 of B into row 3 of C.

c. See II below.

2. Minimum temperature

a. Expected value μ .

i. Obtain ϕ_1 , ϕ_2 , ϕ_3 , A_1 , A_2 , A_3 , and a_0 from Maps 10, 11, 12, 13, 14, 15, and 16 respectively, and enter them in the appropriate lines on the left side of A.

ii. Same as a. *ii.* for maximum temperature.

iii. Fill in all rows of A, using Table 2 to find $\cos t_1'$, $\cos t_2'$, and $\cos t_3'$.

iv. Enter row 13 of A into row 2 of C.

b. Log standard deviation, $\log \sigma$.

i. Obtain ϕ_1 , A_1 , and a_0 for the log standard deviation of the minimum temperature from Maps 17, 18, and 19 respectively and enter them on the left side of B.

ii. Same as b. *ii.* for maximum temperature.

iii. Enter row 9 of B into row 3 of C.

c. See II below.

3. Diurnal temperature range

a. Expected value μ .

i. Carry out a. *i.*, a. *ii.*, and a. *iii.* for both the maximum and minimum temperatures, using two separate workforms A, entering t in one copy each of B and C.

ii. Compute the difference, (row 13 of A_{\max}) - (row 13 of A_{\min}), and enter it into row 2 of C.

b. Log standard deviation, $\log \sigma$.

i. Obtain ϕ_1 , A_1 , A_2 , and a_0 for the log standard deviation of the range from Maps 20, 21, 22, and 23 respectively, and enter them on the left side of B. Insert $\phi_2 = 41$ on left of B.

ii. Fill in all rows of B (row 1 should already be completed from a. *i.*), using Table 2 to find $\cos t_1'$ and $\cos t_2'$.

iii. Enter row 9 of B into row 3 of C.

c. See II below.

II. Probability estimation

1. Check to see that rows 1, 2, and 3 of C are filled in as they should be if I has been completed.

2. From Table 3, interpolate the anti-logarithm of the value in row 3 of C and enter it in row 4 of C. The expected values $\mu(t)$ and the standard deviations $\sigma(t)$ of the

TABLE 3

Log σ	σ
0.45	2.82
0.50	3.16
0.55	3.55
0.60	3.98
0.65	4.47
0.70	5.01
0.75	5.62
0.80	6.31
0.85	7.08
0.90	7.94
0.95	8.91
1.00	10.00
1.05	11.22

Table 3. Values of σ corresponding to values of log σ obtained from row 3 of Workform C.

distributions for the weeks considered are now in rows 2 and 4 respectively.

3. a. Rows 5 through 16 can be filled in to provide estimates of the probability levels indicated on the left. Row 2 is, of course, the median or 50% level.

b. Alternatively, one may use μ and σ in rows 2 and 4 of C, together with a table of the normal distribution to compute other percentiles or to find the probability that a given level will be exceeded.

APPLICATIONS

Example 1: What are the 5 and 95 percentiles of the distribution of the diurnal maximum temperature for the week of August 1 through 7 at Washington, D. C.? The midpoint of this week is August 4 which corresponds to $t = 156$ (Table 1). In Table 5 are replicas of Workforms A, B, and C, the numbering of the rows being the same as in the workforms. The values of the parameters from Maps 2 through 9 have been entered in the appropriate places in Table 5. $\mu(t)$ and log $\sigma(t)$ for $t = 156$ are then computed as described above and the results entered on rows 2 and 3 of C. The percentiles are then computed in Workform C. Thus 1 year out of 20, the average maximum temperature in Washington should be above 93.8°F the first week in August, and 1 year out of 20 below 80.4°F.

Example 2: What is the probability that the average maximum temperature at Washington, D. C., exceeds 90°F for the week of August 1 through 7? We can use the expected value and standard deviation already computed in example 1 on rows 2 and 4 of Table 5C. Entering a table of the normal probability distribution (e.g., Fisher and Yates (1957), Table II,) with standardized argument $(90.0 - 87.1) / 4.1 = 0.71$, we find this probability to be 0.24. That is, 24 years out of 100 the average maximum temperature will exceed 90°F.

Example 3: What are the 20 and 80 percentiles of the distribution of the minimum temperature at Bangor, Maine,

for the week of January 1 through 7? For this week $t = 309$. Filling in the relevant lines of Workforms A, B, and C as shown in Table 6, we find that one year in five the minimum temperature will be less than 6.5°F and one year in five will be above 18.1°F.

Example 4: What is the probability that the average minimum temperature in Bangor falls below 5°F for the week of January 7? Using the values 12.3°F and 6.9°F for the mean and standard deviation which are already computed on rows 2 and 4 of Table 6C, we enter a table of the normal distribution with the standardized argument $(5.0 - 12.3) / 6.9 = -1.06$. The corresponding probability is 0.14.

Example 5: Since the diurnal temperature range appears to be of considerable importance as an element determining the suitability of the climate of a given location for growth of various species of plants (Brooks 1959 p. 174 ff.; Hocker 1956), the probability distribution of this variate is of interest. Although studies of the relationship of the range and ecological and phenological phenomena have used only the average range, its specification is incomplete without its probability distribution. To illustrate the determination of this distribution by maps we wish to find the 5, 10, 20, 25, 50, 75, 80, 90, and 95 percentiles of the distribution of the average of the diurnal range at Utica, New York, for the week of April 10 through 16. From Table 1 this week corresponds to $t = 43$. We computed the expected value μ for the maximum and minimum temperatures for this week on two separate Workforms A and calculate the expected value of the range from the difference. The log standard deviation is computed on Workform B using Maps 20 through 23 while the percentiles are computed in Workform C. In Table 7 the parameters and filled-in rows of these workforms are given. Row 2 and rows 9 through 16 of Table 7C contain the desired percentiles. The value of μ in row 2 of Table 7C is the difference (row 13 of Table 7A_{max}) - (row 13 of Table 7A_{min}). The cumulative probability distribution interpolated from these percentiles is graphed in Figure 3. It can be seen that, although the average range for this April week is 19.3°F, it exceeds 22°F about one year in four.

Example 6: One useful and intuitively appealing way of graphically depicting the statistical behavior of a climatological variate such as the maximum or minimum temperature, or diurnal temperature range is the following: The curve for the expected value $\mu(t)$ is plotted for the entire year. Then, above and below this curve and roughly parallel to it, curves are drawn corresponding to the upper and lower quartiles of the distribution for each week. One can then state that for any particular week the average value of the variate will fall between these upper and lower lines 5 years out of 10. Of course other percentiles could be used

Table 4. List of stations for which harmonic regression parameters were computed.

Station Name	Number	Elevation	Station Name	Number	Elevation	Station Name	Number	Elevation
CONNECTICUT			NEW JERSEY Continued			PENNSYLVANIA Continued		
Cream Hill	1715	1300	Charlotteburg	1582	760	Freeland	3056	1900
Hartford, Brainard Field	3451	15	Flemington 1NE	3029	134	George School	3200	135
Norwalk	5892	120	Indian Mills	4229	100	Gettysburg	3218	540
Storrs	8138	600	Layton	4735	480	Greenville	3526	1026
Waterbury	8911	288	Long Branch	4987	34	Huntingdon	4159	700
DELAWARE			Moorestown	5728	55	Johnstown	4385	1214
Bridgeville 1NW	1330	50	New Brunswick	6062	80	Lancaster 2NE	4758	255
Dover	2730	34	Pleasantville	7131	8	Lawrenceville 2S	4873	1000
Milford	5915	10	NEW YORK			Lock Haven	5104	570
MAINE			Albany	0047	19	Montrose	5915	1630
Eastport	2426	100	Alfred	0085	1760	Mount Pocono	6055	1915
Farmington	2765	390	Angelica	0183	1420	New Castle 1N	6233	825
Houlton 1NE	3897	410	Auburn, water works	0321	715	Newell	6246	805
Lewiston	4566	182	Binghamton	0691	858	Palmerton	6689	435
Old Town	6420	108	Bridgehampton	0889	60	Philadelphia-Shawmont	6904	38
Portland, airport	6905	61	Buffalo, airport	1012	693	Ridgway	7477	1371
Presque Isle	6937	606	Canton	1185	406	Selinsgrove	7931	437
Rockland	7250	40	Carmel 1SW	1207	500	Somerset	8249	2150
Rumford 1SSE	7330	505	Dannemora	1966	1338	State College	8449	1175
MARYLAND			Delhi	2036	1460	Stroudsburg	8596	440
Chewsville-Bridgeport	1790	560	Elmira	2610	863	Towanda	8905	760
Crisfield	2205	5	Fredonia	3033	750	Uniontown	9050	1040
Easton	2695	28	Geneva, Expt. Sta.	3177	615	Warren	9298	1280
Elkton	2860	28	Hemlock	3773	920	Wellsboro	9408	1920
Frederick, police barrack	3348	380	Ithaca, Cornell Univ.	4174	950	York 3SSW	9933	390
Frostburg	3410	2035	Jamestown	4206	1390	RHODE ISLAND		
Keedysville	4780	420	Little Falls, city res.	4791	890	Kingston	4266	100
Oakland 1SE	6620	2420	Lockport	4844	520	VERMONT		
Owings Ferry Landing	6770	120	Lowville	4912	860	Burlington	1081	331
Princess Anne 1E	7330	17	Morrisville	5512	1325	Cornwall	1580	500
Salisbury	8000	10	New York, Central Park	5801	132	Newport	5542	766
Westminster	9435	770	Norwich 1WNW	6085	1070	Northfield	5733	840
Woodstock	9750	415	Ogdensburg 3NE	6164	258	St. Johnsbury	7054	699
MASSACHUSETTS			Oswego	6314	292	WEST VIRGINIA		
Adams	0049	750	Port Jervis	6774	470	Bayard	0527	2375
Amherst	0120	210	Poughkeepsie	6817	103	Charleston	1575	600
East Wareham	2451	20	Rochester, airport	7167	543	Clarksburg	1677	977
Hyannis 2NNE	3821	35	Roxbury	7317	1494	Elkins	2718	1970
Lawrence	4105	57	Salisbury	7413	1300	Fairmont	2920	1298
Springfield	8046	190	Setauket	7633	40	Flat Top	3072	3225
Weston	9360	224	Syracuse	8383	419	Gary	3353	1426
Worcester	9928	628	Walden 2NE	8902	400	Glenville	3544	740
NEW HAMPSHIRE			Wanakena	8944	1510	Huntington	4378	675
Berlin	0690	1110	Watertown	9000	497	Mannington 1N	5621	974
Durham	2174	73	PENNSYLVANIA			Martinsburg	5707	537
Hanover	3850	603	Altoona	0134	1500	Parkersburg	6859	615
Keene	4399	490	Bethlehem	0634	436	Piedmont	7004	1053
NEW JERSEY			Brookville	1002	1417	Point Pleasant	7105	569
Belvidere	0729	289	Butler	1130	1100	Rainelle-McRoss	7306	2424
Boonton	0907	280	Carlisle	1234	460	Spencer	8384	789
			Claysville	1512	1150	Wardensville	9281	1200
			Corry	1790	1427	Wheeling, Warwood Dam	9492	659
			Donora	2190	814	White Sulphur Springs	9522	1914
			Ebensburg	2466	2090	Williamson	9605	673
			Emporium 1E	2633	1160			
			Franklin	3028	987			

TABLE 5

A			B			C		
	Row			Row			Row	
$\phi_1=142$	1	156	$\phi_1=329$	1	156	50%	1	156
	2	14		2	-173		2	87.1
	3	0.971		3	-0.986		3	0.61
$\phi_2=31$	4	125	$\phi_2=$	4			4	4.1
	5	250		5			5	6.7
	6	-0.398		6			6	
$\phi_3=$	7		$A_1=0.13$	7	-0.13		7	
	8		$A_2=$	8			8	
	9		$a_0=0.74$	9	0.61	5%	9	80.4
$A_1=21.5$	10	20.9				10%	10	
$A_2=1.4$	11	-0.6				20%	11	
$A_3=$	12					25%	12	
$a_0=66.8$	13	87.1				75%	13	
						80%	14	
						90%	15	
						95%	16	93.8

Table 5. Replicas of Workforms A, B, and C showing calculation of the 5 and 95 percentiles of the distribution of the average diurnal maximum temperature at Washington, D. C., for the week of August 1 through 7 (Example 1).

instead of the 25 and 75 percentiles. Such a graph can easily be constructed using the methods of this bulletin. It is sufficient to compute expected values and log standard deviations for only 12 equally spaced times using Workforms A, B, and C and employing graphical interpolation for the intervening times. The expected value is given by row 2 of C while the quartiles are obtained from rows 12 and 13 of C. Such a graph for the maximum temperature at Harrisburg, Pennsylvania, is shown in Figure 4.

Example 7: Another use for the maps in this bulletin is the determination of "homoclimes" for crops. Since the

TABLE 6

A			B			C		
	Row			Row			Row	
$\phi_1=147$	1	309	$\phi_1=309$	1	309	50%	1	309
	2	162		2	0		2	12.3
	3	-0.938		3	1.000		3	0.84
$\phi_2=45$	4	264	$\phi_2=$	4			4	6.9
	5	528		5			5	
	6	-0.943		6			6	
$\phi_3=23$	7	286	$A_1=0.17$	7	0.17		7	5.8
	8	858	$A_2=$	8			8	
	9	-0.591	$a_0=0.67$	9	0.84	5%	9	
$A_1=22.8$	10	-21.4				10%	10	
$A_2=1.0$	11	-0.9				20%	11	6.5
$A_3=1.5$	12	-0.9				25%	12	
$a_0=35.5$	13	12.3				75%	13	
						80%	14	18.1
						90%	15	
						95%	16	

Table 6. Replicas of Workforms A, B, and C showing calculation of the 20 and 80 percentiles of the distribution of the average diurnal minimum temperature at Bangor, Maine, for the week of January 1 through 7 (Example 3).

coefficients of the curves for expected values and log standard deviations characterize an important aspect of the temperature climate, they can be used to compare different locations in respect to suitability for crops. If a given plant is known to thrive at one location, the parameters for a proposed location can be compared with those at the established location. If they are similar, this indicates that the temperature climates are also similar. This was proposed by J. A. Prescott (1943) using a curve fit to mean temperatures. Clearly, however, information about the variance should increase the value of this method for it indicates how variable the climate is.

TABLE 7

A_{max}			A_{min}			B			C		
	Row			Row			Row			Row	
$\phi_1=144$	1	43	$\phi_1=146$	1	43	$\phi_1=0$	1	43	50%	1	43
	2	-101		2	-103		2	43		2	19.3
	3	-0.167		3	-0.200		3	0.738		3	0.62
$\phi_2=46$	4	-3	$\phi_2=63$	4	-20	$\phi_2=41$	4	2		4	4.2
	5	-6		5	-40		5	4		5	6.9
	6	0.994		6	0.772		6	0.997		6	5.4
$\phi_3=$	7		$\phi_3=30$	7	13	$A_1=0.03$	7	0.02		7	3.5
	8			8	39	$A_2=0.05$	8	0.05		8	2.8
	9			9	0.782	$a_0=0.55$	9	0.62	5%	9	12.4
$A_1=26.8$	10	-4.5	$A_1=23.1$	10	-4.6				10%	10	13.9
$A_2=1.1$	11	1.1	$A_2=1.0$	11	0.8				20%	11	15.8
$A_3=$	12		$A_3=1.0$	12	0.8				25%	12	16.5
$a_0=55.5$	13	52.1	$a_0=35.8$	13	32.8				75%	13	22.1
									80%	14	22.8
									90%	15	24.7
									95%	16	26.2

Table 7. Two replicas of Workform A and replicas of Workforms B and C showing calculation of 5, 10, 20, 25, 50, 75, 80, 90, and 95 percentiles of the average diurnal temperature range at Utica, New York, for the week of April 10 through 16.

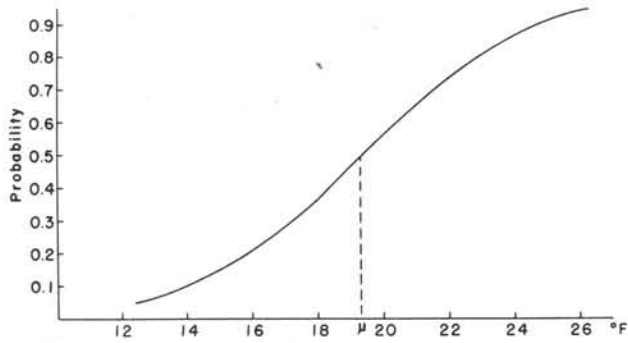


Figure 3. The cumulative probability distribution of the average diurnal range at Utica, New York, for the week of April 10 through 16.

ACKNOWLEDGEMENTS

This bulletin would not have been possible without the full cooperation of the members of the NE 35 Technical Committee on Agricultural Climatology. I am especially grateful to those of the Committee who skillfully and efficiently handled the machine computations on which this bulletin is based. Finally, my thanks are due to Jane Belcher for her long hours spent to make the maps and figures as nearly perfect as possible.

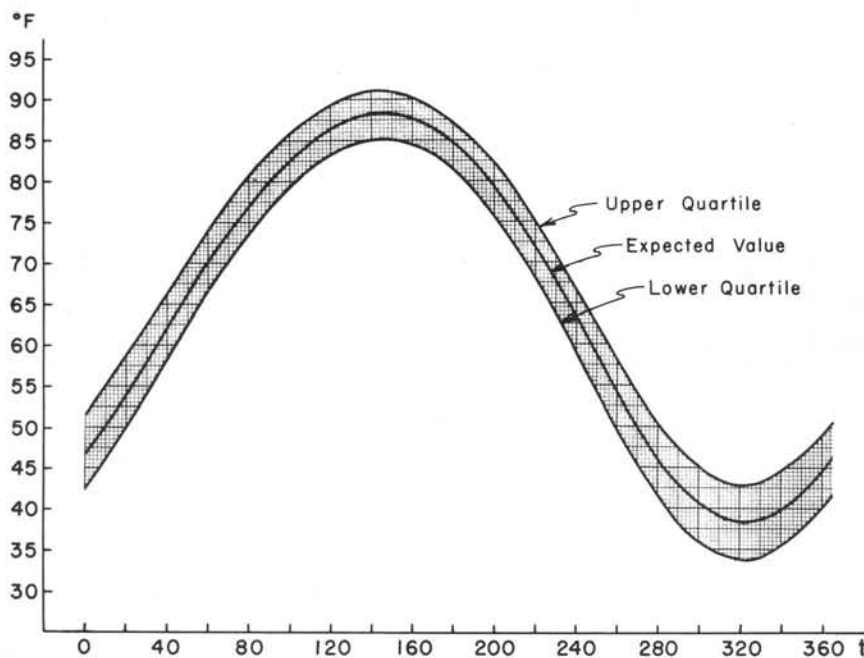


Figure 4. The expected value and upper and lower quartiles of the average maximum temperature at Harrisburg, Pennsylvania, through the year.

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WORKFORM A: Expected value

	1	t		
$\phi_1 = \dots\dots\dots$	2	$t'_1 = t - \phi_1$		
	3	$\cos t'_1$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t'_2 = 2(t - \phi_2)$		
	6	$\cos t'_2$		
$\phi_3 = \dots\dots\dots$	7	$t - \phi_3$		
	8	$t'_3 = 3(t - \phi_3)$		
	9	$\cos t'_3$		
$A_1 = \dots\dots\dots$	10	$A_1 \cos t'_1$		
$A_2 = \dots\dots\dots$	11	$A_2 \cos t'_2$		
$A_3 = \dots\dots\dots$	12	$A_3 \cos t'_3$		
$a_0 = \dots\dots\dots$	13	$a_0 + 10 + 11 + 12$		

WORKFORM A: Expected value

	1	t		
$\phi_1 = \dots\dots\dots$	2	$t'_1 = t - \phi_1$		
	3	$\cos t'_1$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t'_2 = 2(t - \phi_2)$		
	6	$\cos t'_2$		
$\phi_3 = \dots\dots\dots$	7	$t - \phi_3$		
	8	$t'_3 = 3(t - \phi_3)$		
	9	$\cos t'_3$		
$A_1 = \dots\dots\dots$	10	$A_1 \cos t'_1$		
$A_2 = \dots\dots\dots$	11	$A_2 \cos t'_2$		
$A_3 = \dots\dots\dots$	12	$A_3 \cos t'_3$		
$a_0 = \dots\dots\dots$	13	$a_0 + 10 + 11 + 12$		

WORKFORM B: Log standard deviation

	1	t		
$\phi_1 = \dots\dots\dots$	2	$t'_1 = t - \phi_1$		
	3	$\cos t'_1$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t'_2 = 2(t - \phi_2)$		
	6	$\cos t'_2$		
$A_1 = \dots\dots\dots$	7	$A_1 \cos t'_1$		
$A_2 = \dots\dots\dots$	8	$A_2 \cos t'_2$		
$a_0 = \dots\dots\dots$	9	$a_0 + 7 + 8$		

WORKFORM B: Log standard deviation

	1	t		
$\phi_1 = \dots\dots\dots$	2	$t'_1 = t - \phi_1$		
	3	$\cos t'_1$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t'_2 = 2(t - \phi_2)$		
	6	$\cos t'_2$		
$A_1 = \dots\dots\dots$	7	$A_1 \cos t'_1$		
$A_2 = \dots\dots\dots$	8	$A_2 \cos t'_2$		
$a_0 = \dots\dots\dots$	9	$a_0 + 7 + 8$		

WORKFORM C: Probability levels

	1	t		
50%	2	$\mu = \text{mean value}$		
	3	$\log \sigma$		
	4	$\sigma = \text{stand. dev.}$		
	5	1.64σ		
	6	1.28σ		
	7	0.84σ		
	8	0.67σ		
5%	9	(2) - (5)		
10%	10	(2) - (6)		
20%	11	(2) - (7)		
25%	12	(2) - (8)		
75%	13	(2) + (8)		
80%	14	(2) + (7)		
90%	15	(2) + (6)		
95%	16	(2) + (5)		

WORKFORM C: Probability levels

	1	t		
50%	2	$\mu = \text{mean value}$		
	3	$\log \sigma$		
	4	$\sigma = \text{stand. dev.}$		
	5	1.64σ		
	6	1.28σ		
	7	0.84σ		
	8	0.67σ		
5%	9	(2) - (5)		
10%	10	(2) - (6)		
20%	11	(2) - (7)		
25%	12	(2) - (8)		
75%	13	(2) + (8)		
80%	14	(2) + (7)		
90%	15	(2) + (6)		
95%	16	(2) + (5)		

WORKFORM A: Expected value

	1	t		
$\phi_1 = \dots\dots\dots$	2	$t_1' = t - \phi_1$		
	3	$\cos t_1'$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t_2' = 2(t - \phi_2)$		
	6	$\cos t_2'$		
$\phi_3 = \dots\dots\dots$	7	$t - \phi_3$		
	8	$t_3' = 3(t - \phi_3)$		
	9	$\cos t_3'$		
$A_1 = \dots\dots\dots$	10	$A_1 \cos t_1'$		
$A_2 = \dots\dots\dots$	11	$A_2 \cos t_2'$		
$A_3 = \dots\dots\dots$	12	$A_3 \cos t_3'$		
$a_0 = \dots\dots\dots$	13	$a_0 + 10 + 11 + 12$		

WORKFORM A: Expected value

	1	t		
$\phi_1 = \dots\dots\dots$	2	$t_1' = t - \phi_1$		
	3	$\cos t_1'$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t_2' = 2(t - \phi_2)$		
	6	$\cos t_2'$		
$\phi_3 = \dots\dots\dots$	7	$t - \phi_3$		
	8	$t_3' = 3(t - \phi_3)$		
	9	$\cos t_3'$		
$A_1 = \dots\dots\dots$	10	$A_1 \cos t_1'$		
$A_2 = \dots\dots\dots$	11	$A_2 \cos t_2'$		
$A_3 = \dots\dots\dots$	12	$A_3 \cos t_3'$		
$a_0 = \dots\dots\dots$	13	$a_0 + 10 + 11 + 12$		

WORKFORM B: Log standard deviation

	1	t		
$\phi_1 = \dots\dots\dots$	2	$t_1' = t - \phi_1$		
	3	$\cos t_1'$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t_2' = 2(t - \phi_2)$		
	6	$\cos t_2'$		
$A_1 = \dots\dots\dots$	7	$A_1 \cos t_1'$		
$A_2 = \dots\dots\dots$	8	$A_2 \cos t_2'$		
$a_0 = \dots\dots\dots$	9	$a_0 + 7 + 8$		

WORKFORM B: Log standard deviation

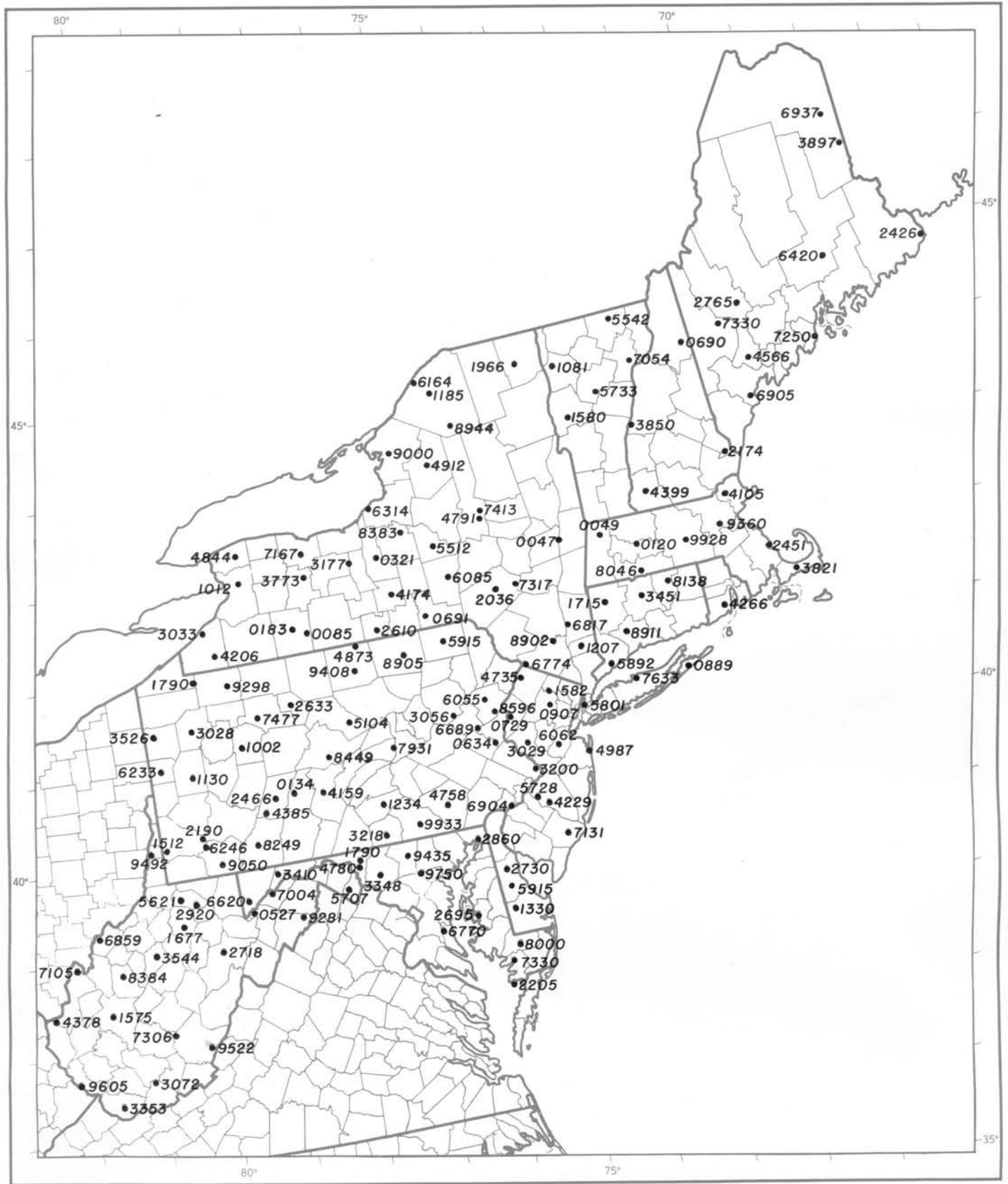
	1	t		
$\phi_1 = \dots\dots\dots$	2	$t_1' = t - \phi_1$		
	3	$\cos t_1'$		
$\phi_2 = \dots\dots\dots$	4	$t - \phi_2$		
	5	$t_2' = 2(t - \phi_2)$		
	6	$\cos t_2'$		
$A_1 = \dots\dots\dots$	7	$A_1 \cos t_1'$		
$A_2 = \dots\dots\dots$	8	$A_2 \cos t_2'$		
$a_0 = \dots\dots\dots$	9	$a_0 + 7 + 8$		

WORKFORM C: Probability levels

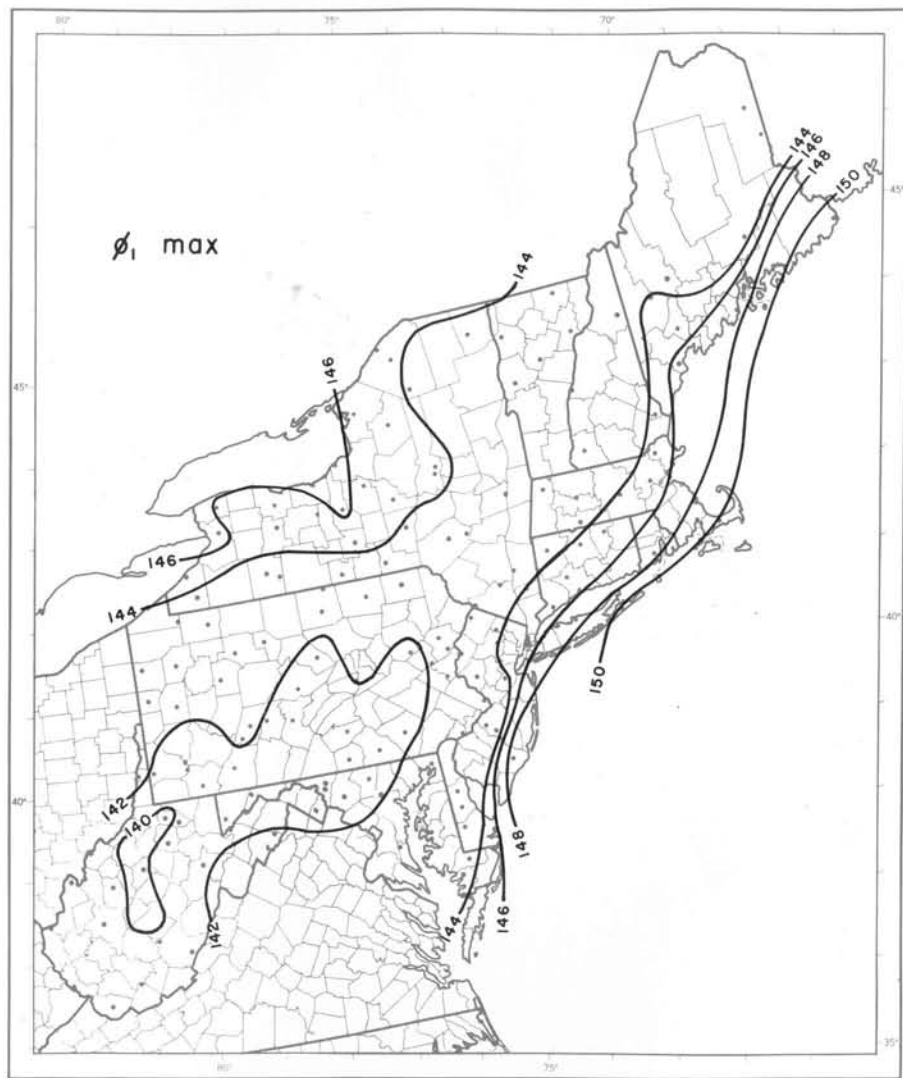
	1	t		
50%	2	$\mu = \text{mean value}$		
	3	$\log \sigma$		
	4	$\sigma = \text{stand. dev.}$		
	5	1.64σ		
	6	1.28σ		
	7	0.84σ		
	8	0.67σ		
	5%	9	(2) - (5)	
10%	10	(2) - (6)		
20%	11	(2) - (7)		
25%	12	(2) - (8)		
75%	13	(2) + (8)		
80%	14	(2) + (7)		
90%	15	(2) + (6)		
95%	16	(2) + (5)		

WORKFORM C: Probability levels

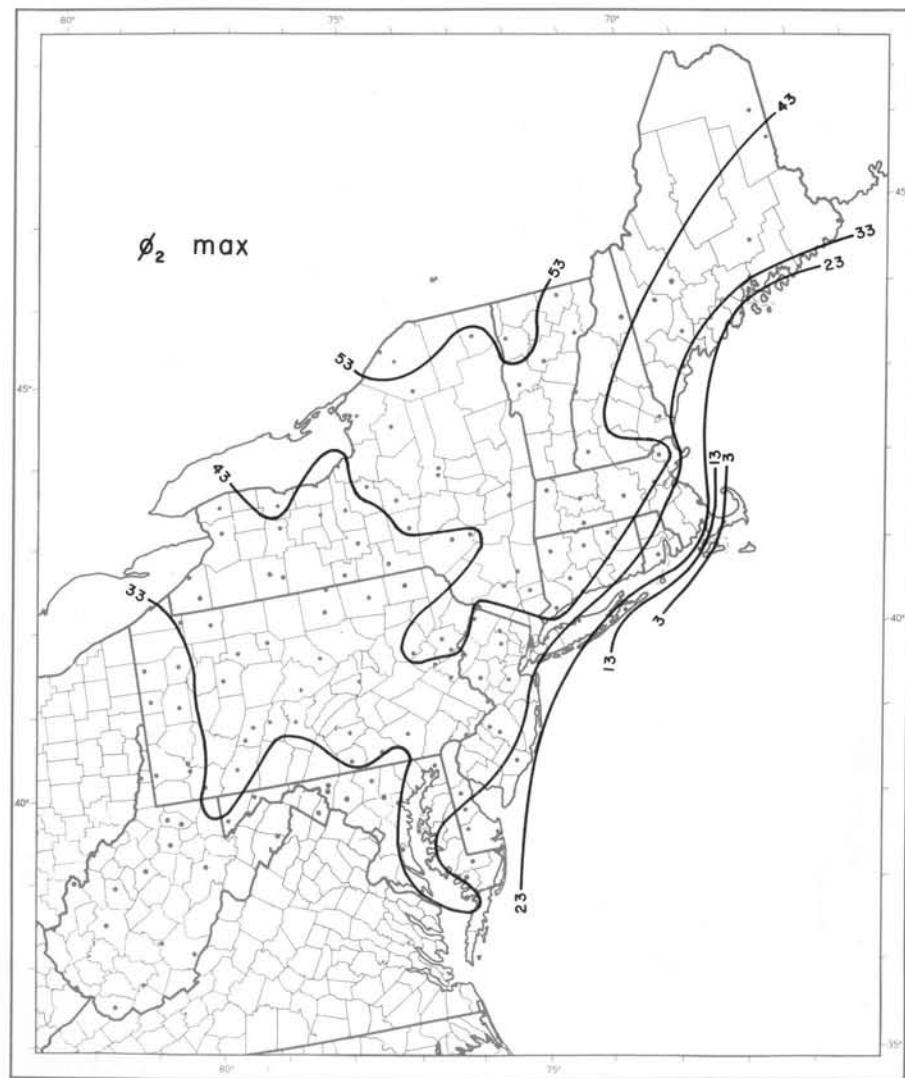
	1	t		
50%	2	$\mu = \text{mean value}$		
	3	$\log \sigma$		
	4	$\sigma = \text{stand. dev.}$		
	5	1.64σ		
	6	1.28σ		
	7	0.84σ		
	8	0.67σ		
	5%	9	(2) - (5)	
10%	10	(2) - (6)		
20%	11	(2) - (7)		
25%	12	(2) - (8)		
75%	13	(2) + (8)		
80%	14	(2) + (7)		
90%	15	(2) + (6)		
95%	16	(2) + (5)		



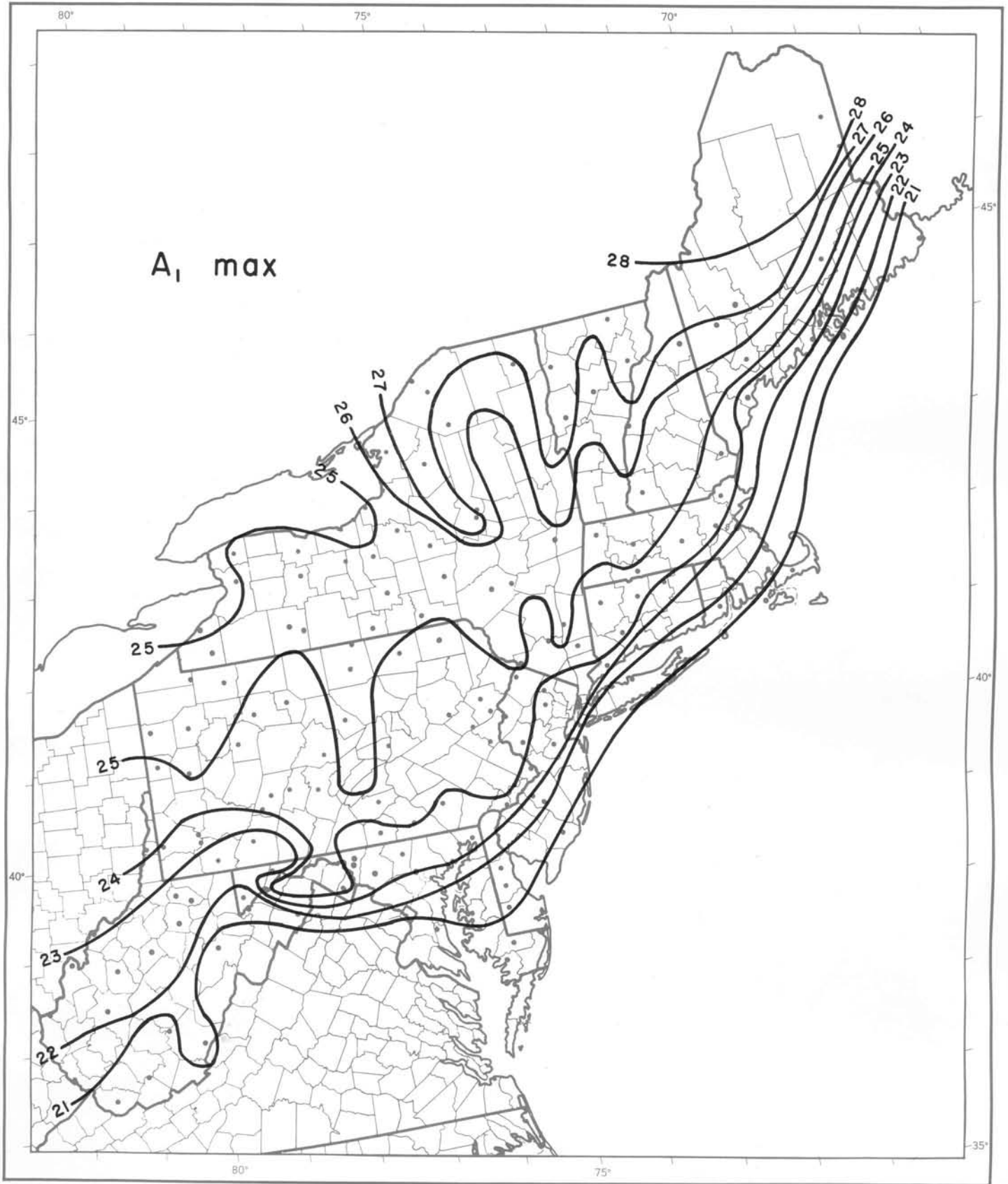
Map 1. Station locator map.



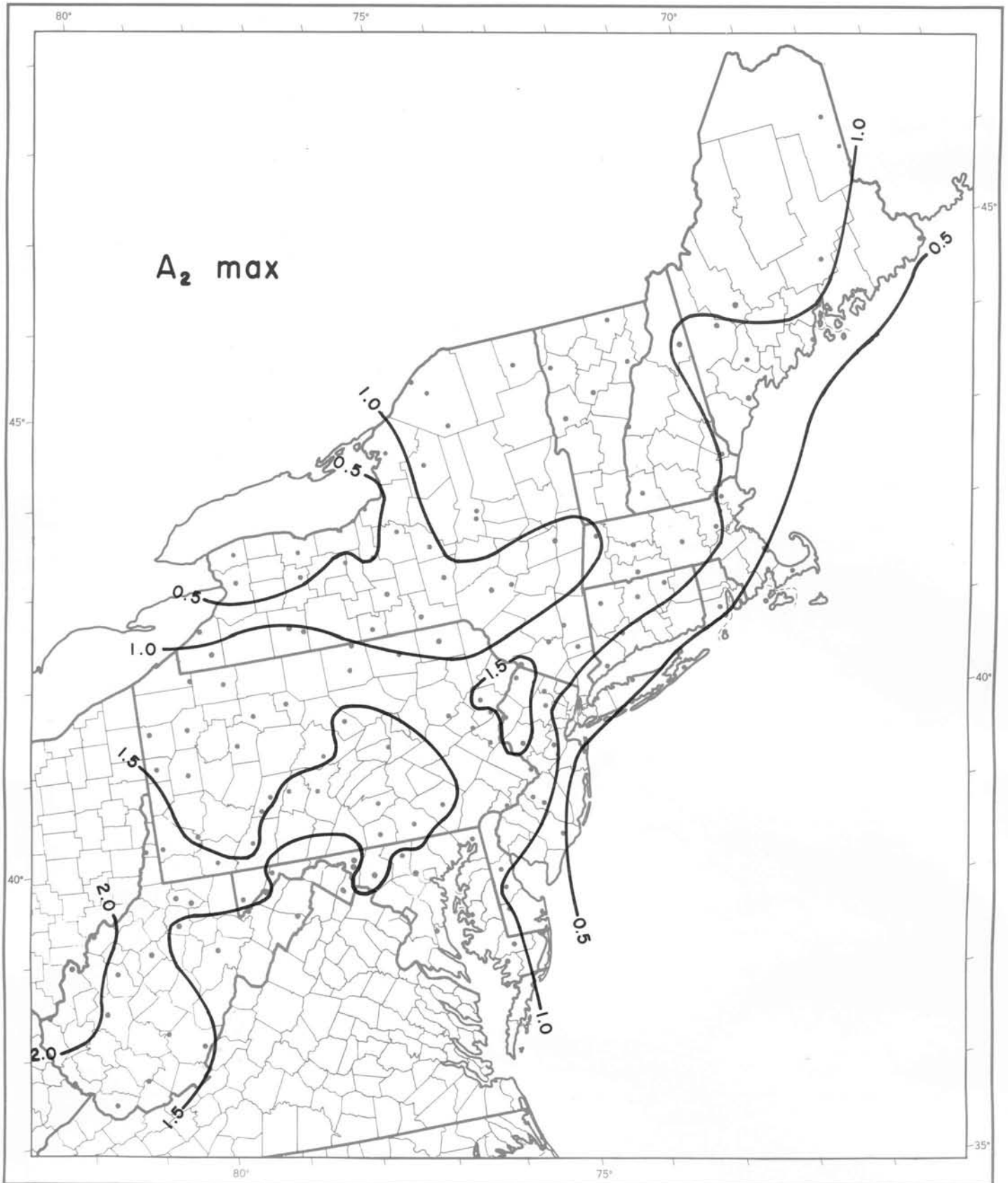
Map 2. ϕ_1 for the diurnal maximum temperature. In days after March 1.



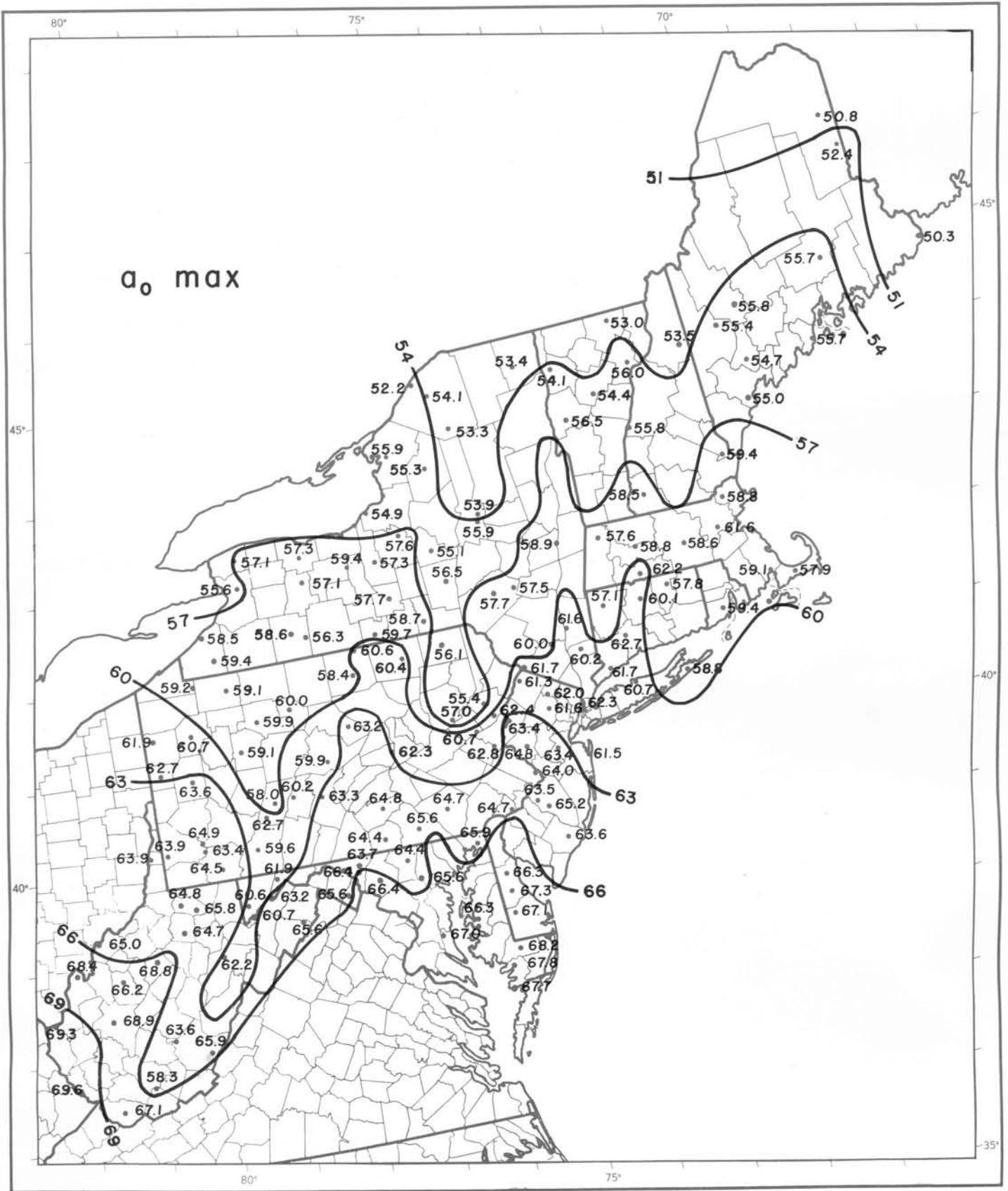
Map 3. ϕ_2 for the diurnal maximum temperature. In days after March 1.



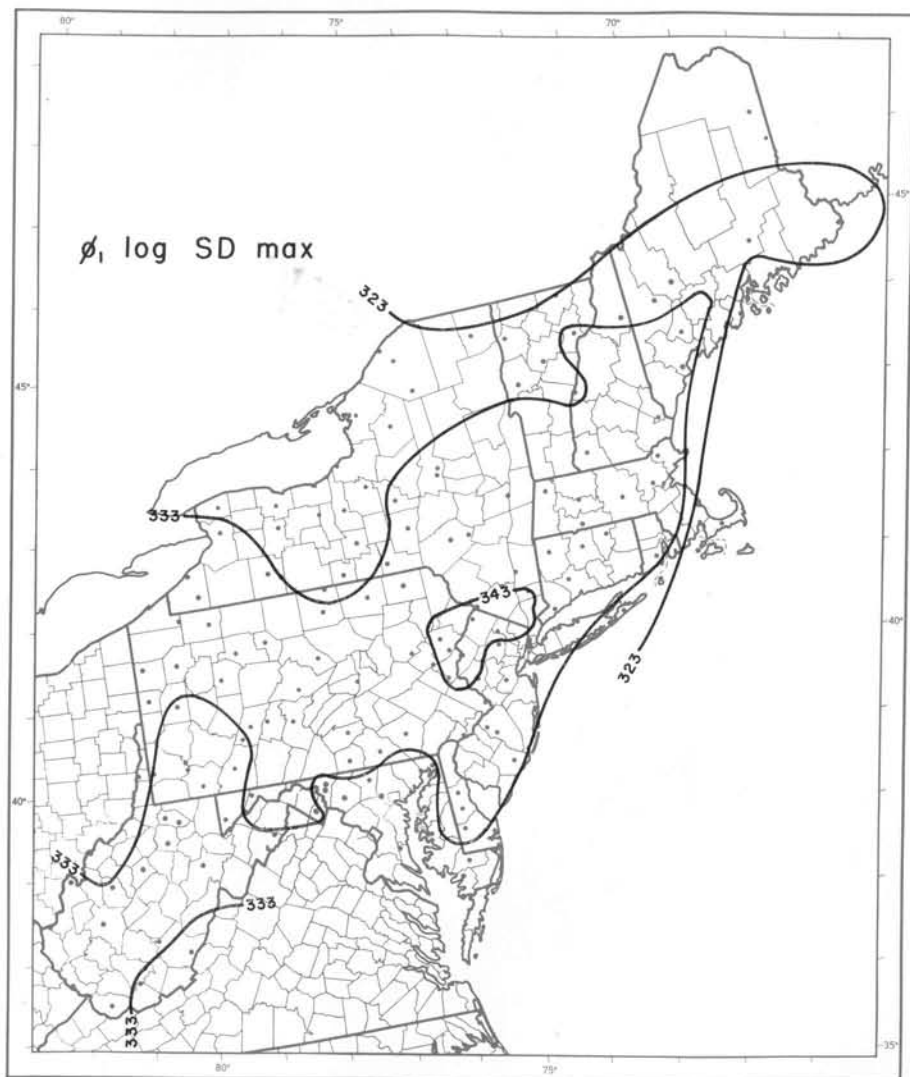
Map 4. A_1 for the diurnal maximum temperature, in °F.



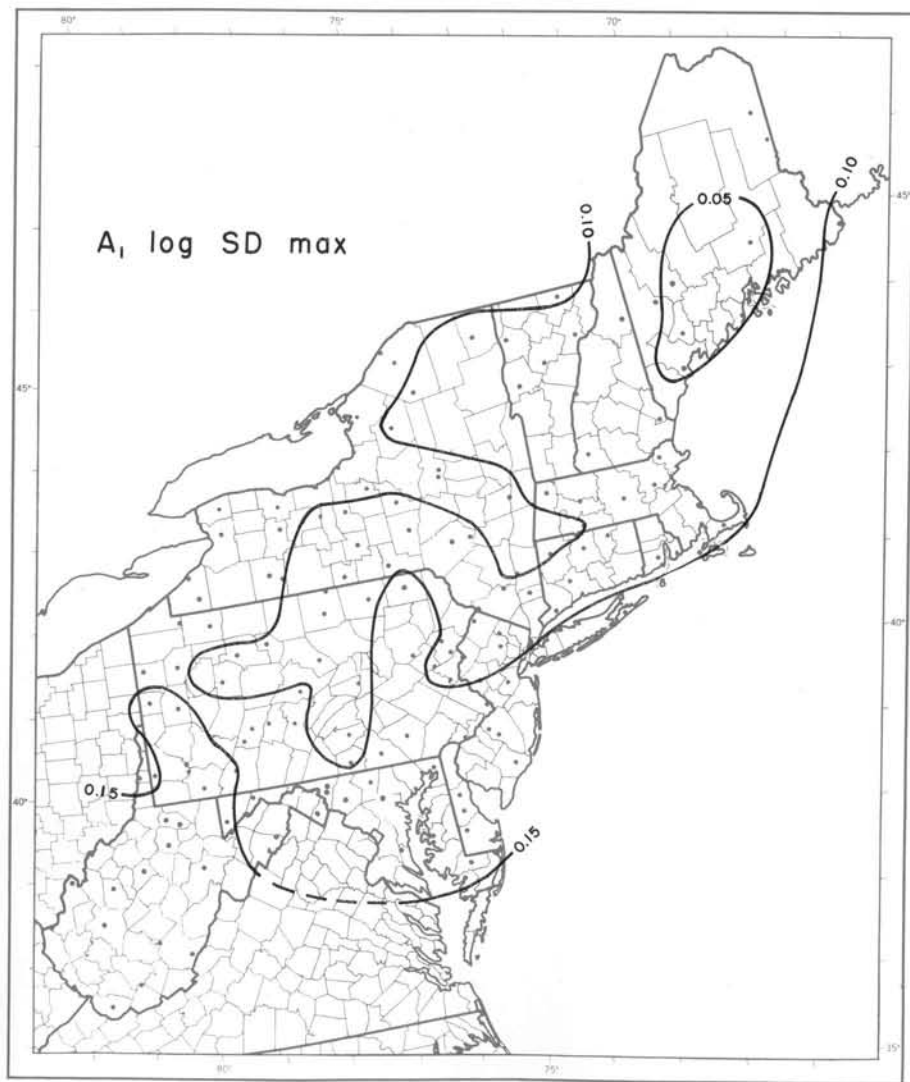
Map 5. A_2 for the diurnal maximum temperature, in °F.



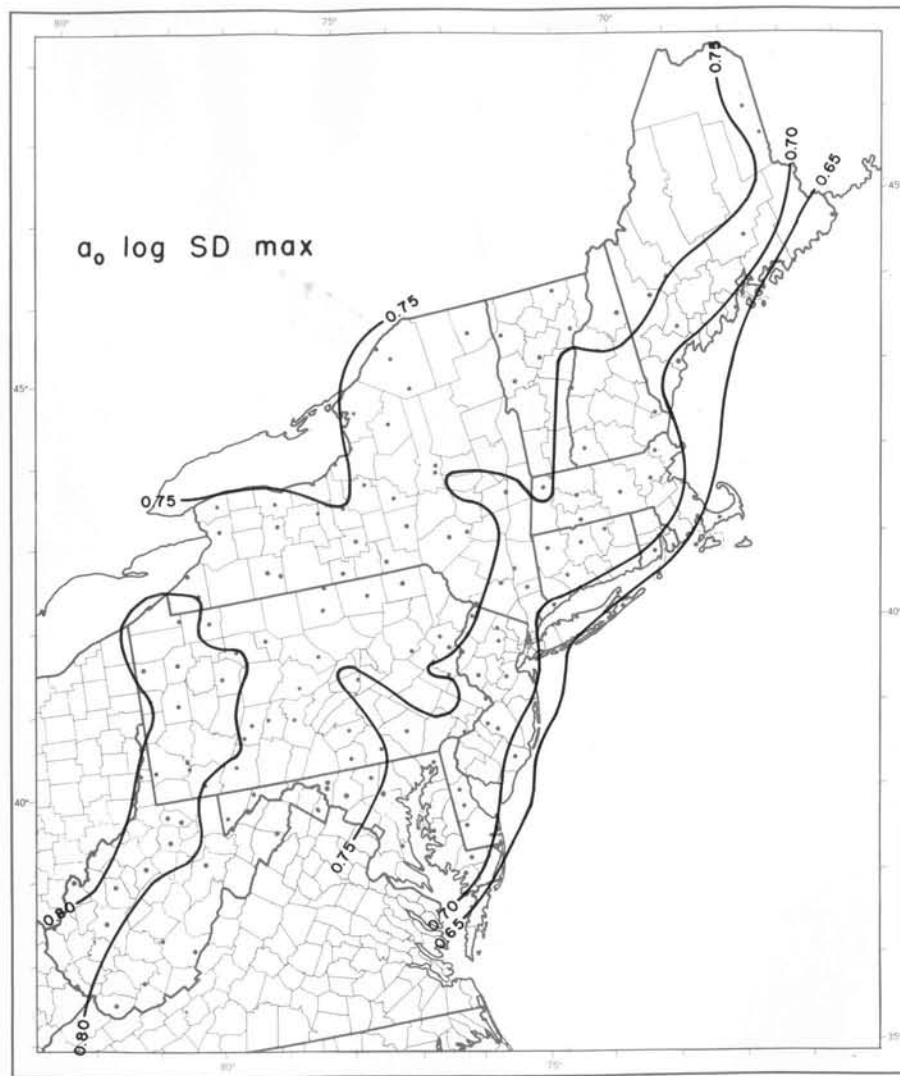
Map 6. a_0 for the diurnal maximum temperature, in °F.



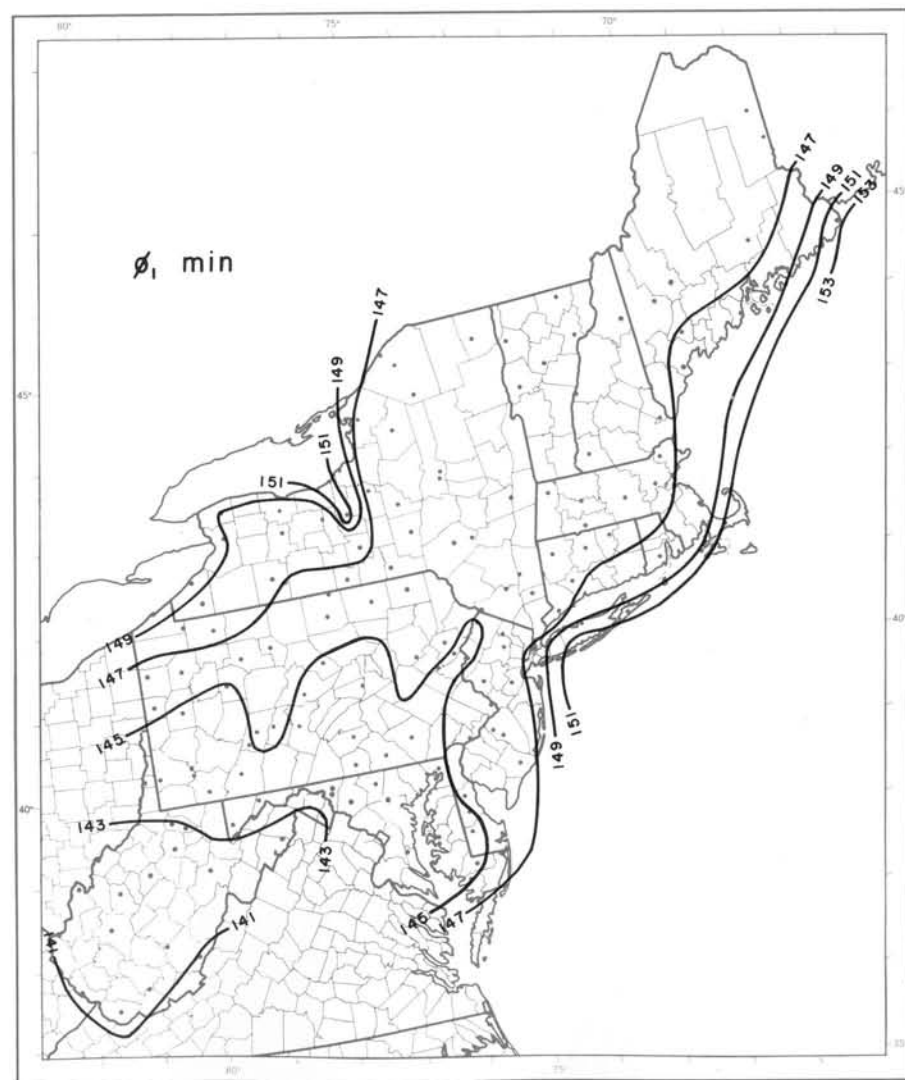
Map 7. ϕ_1 for the log standard deviation of diurnal maximum temperature. In days after March 1.



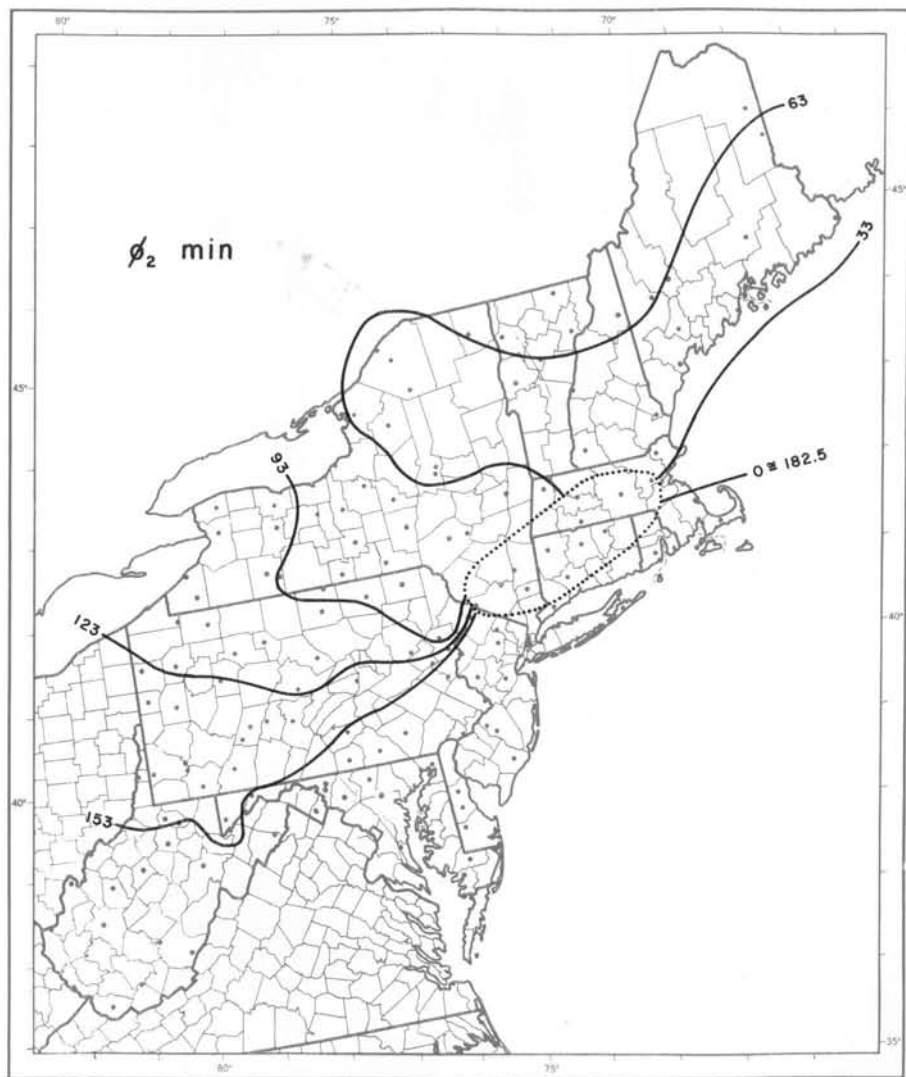
Map 8. A_1 for the log standard deviation of diurnal maximum temperature, in log ($^{\circ}\text{F}$).



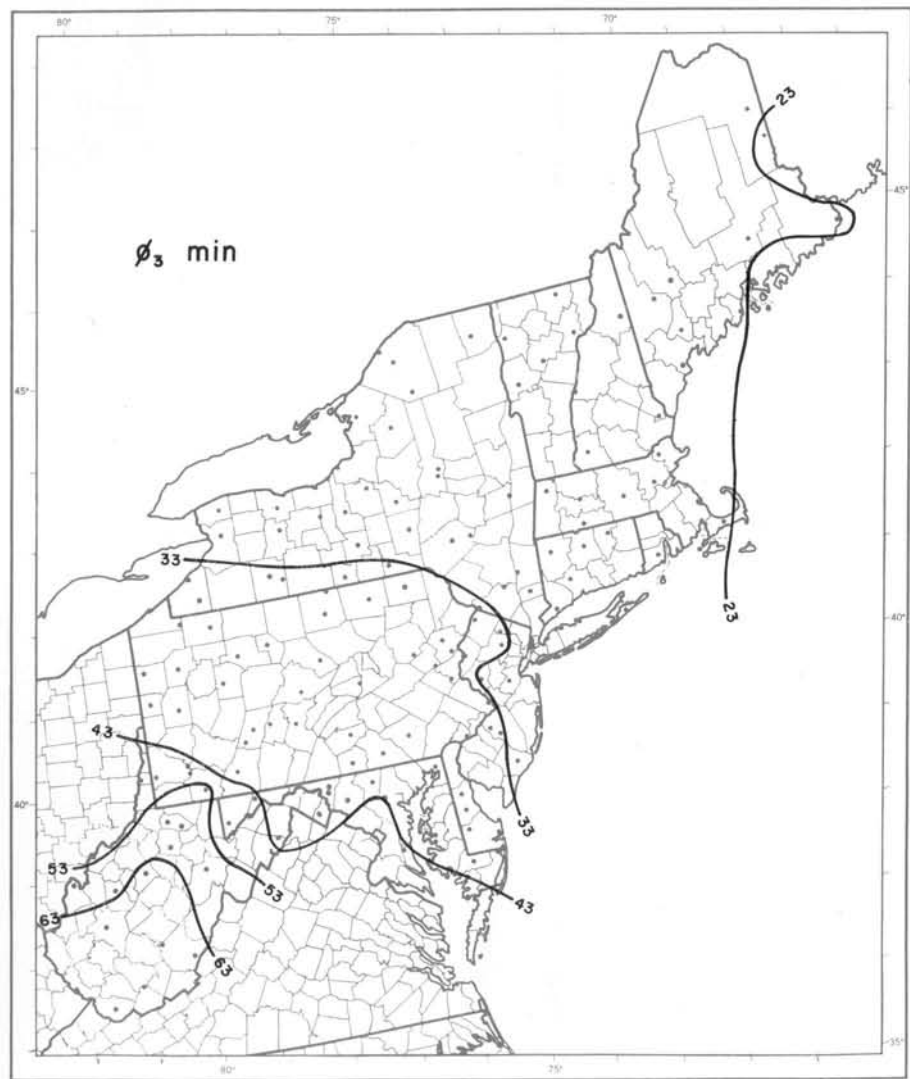
Map 9. a_0 for the log standard deviation of diurnal maximum temperature, in $\log(^{\circ}F)$.



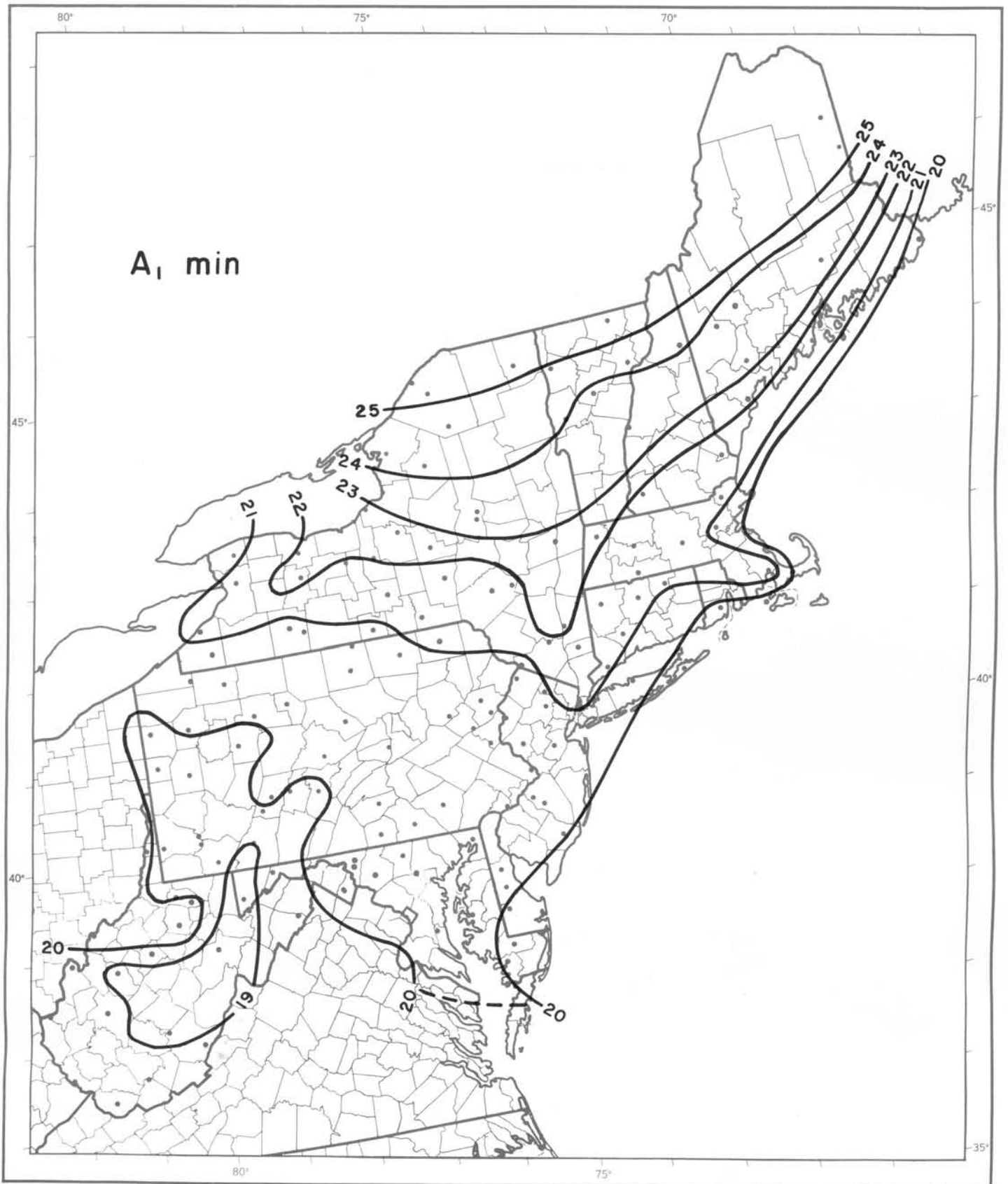
Map 10. ϕ_1 for the diurnal minimum temperature. In days after March 1.



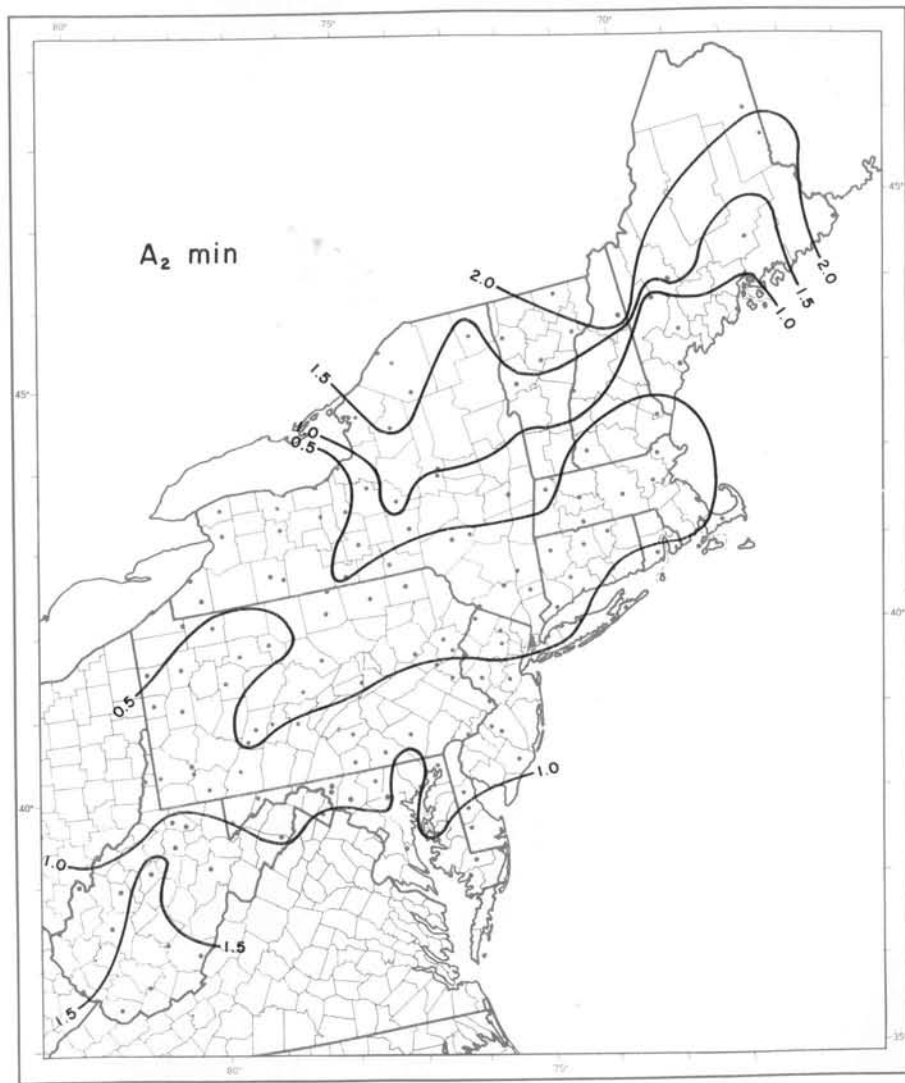
Map 11. ϕ_2 for the diurnal minimum temperature. In days after March 1.



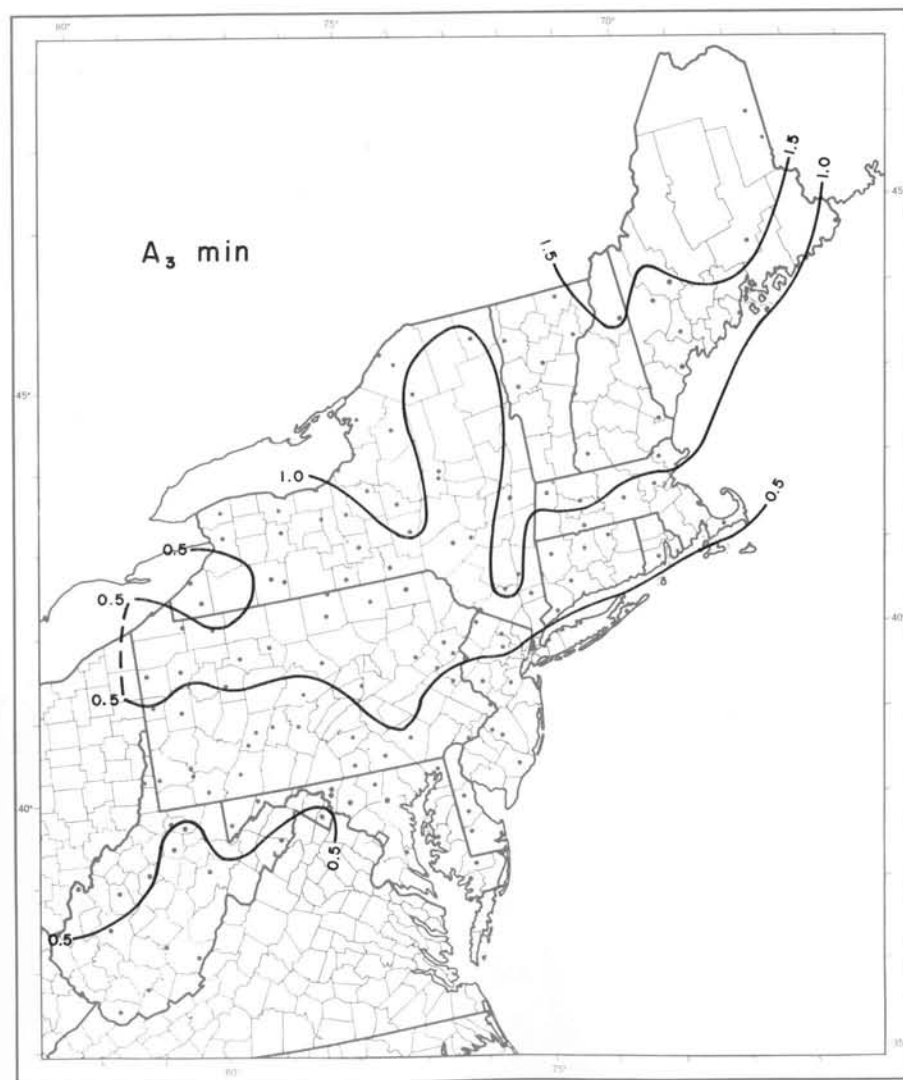
Map 12. ϕ_3 for the diurnal minimum temperature. In days after March 1.



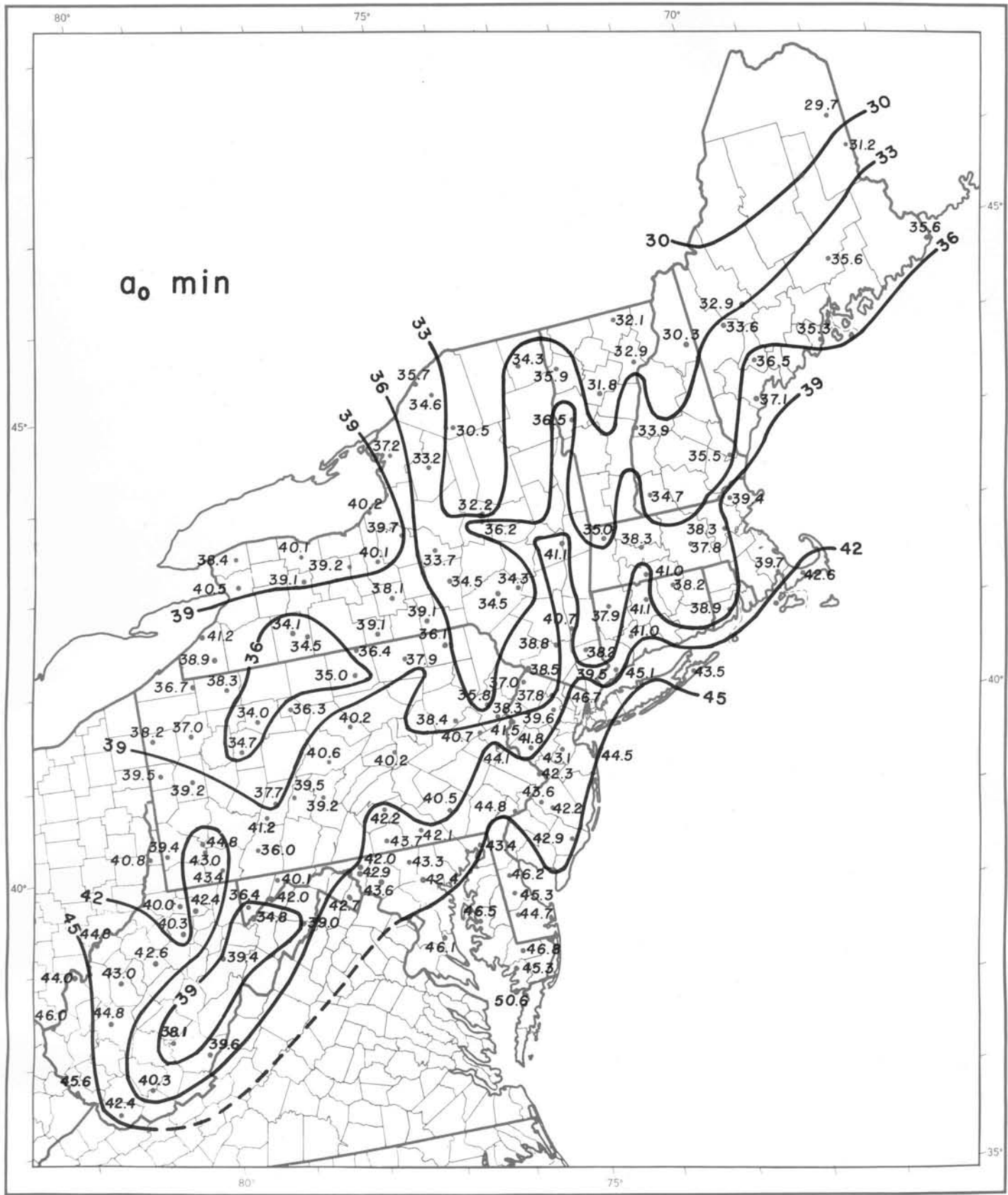
Map 13. A_1 for the diurnal minimum temperature, in °F.



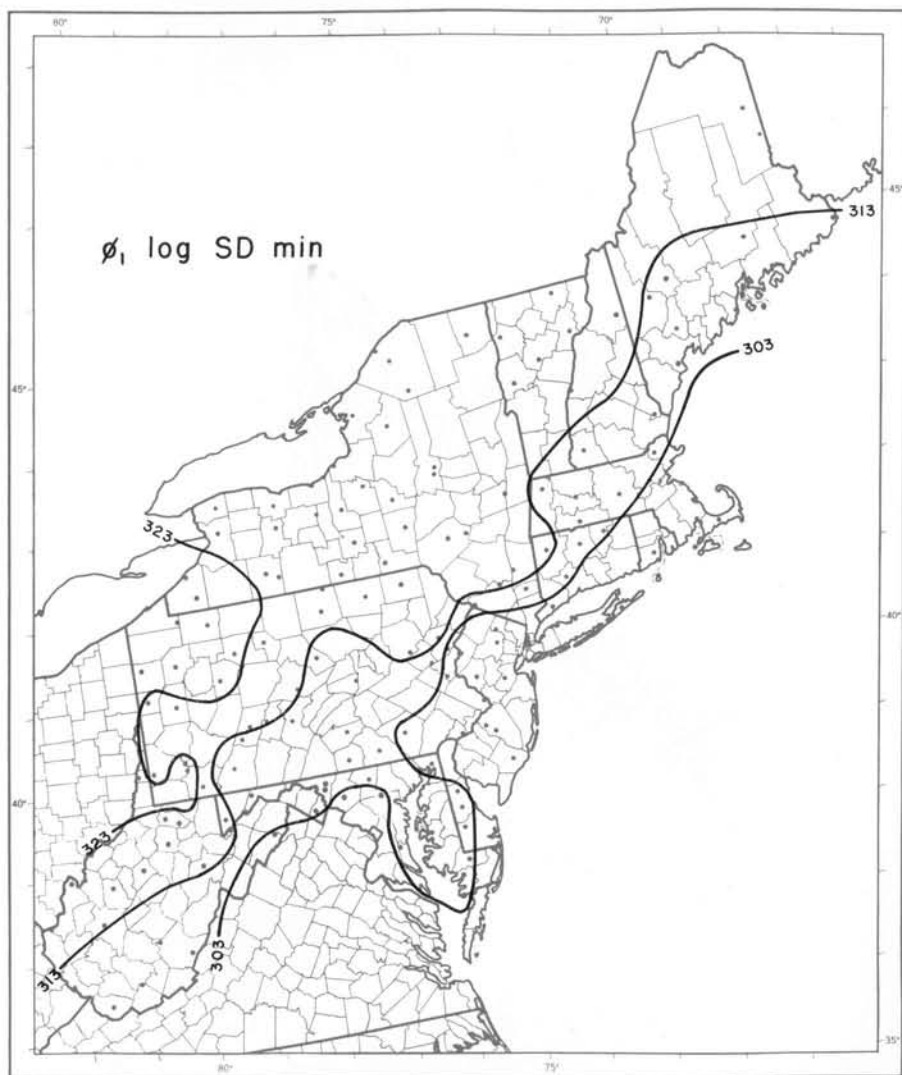
Map 14. A_2 for the diurnal minimum temperature, in °F.



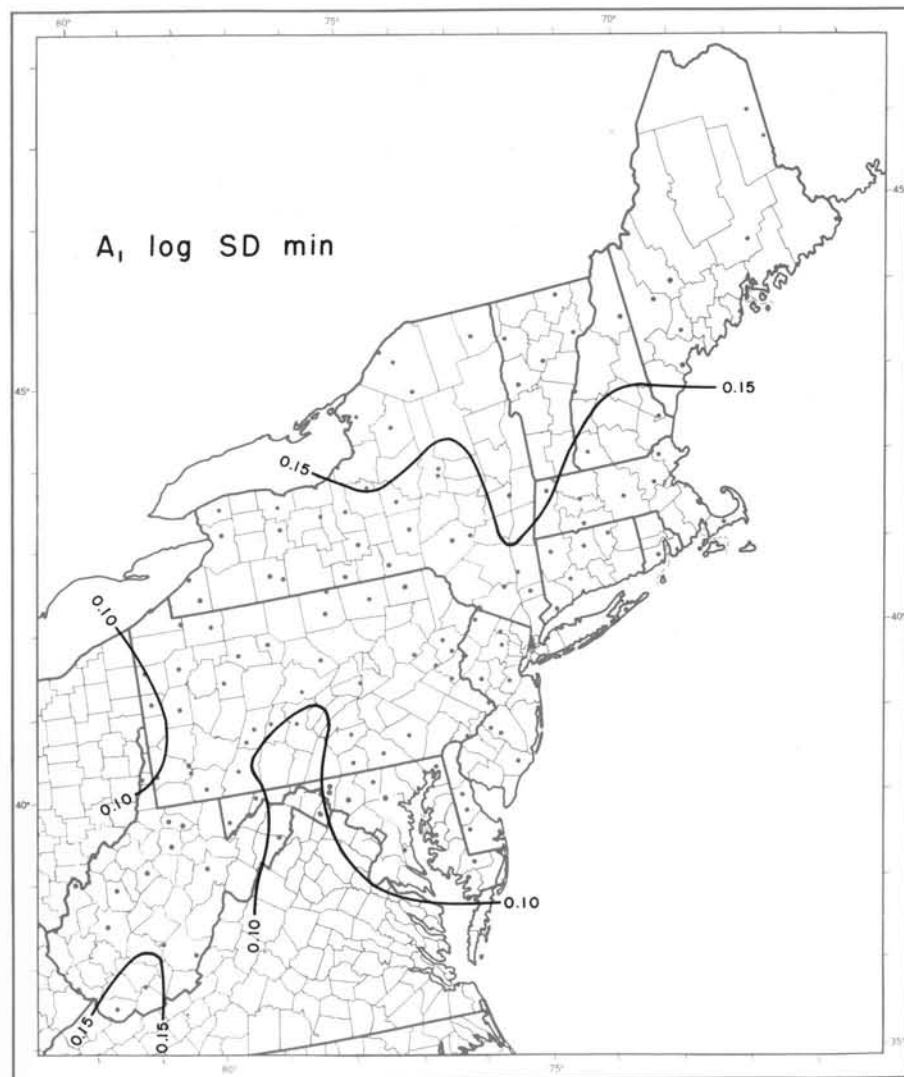
Map 15. A_3 for the diurnal minimum temperature, in °F.



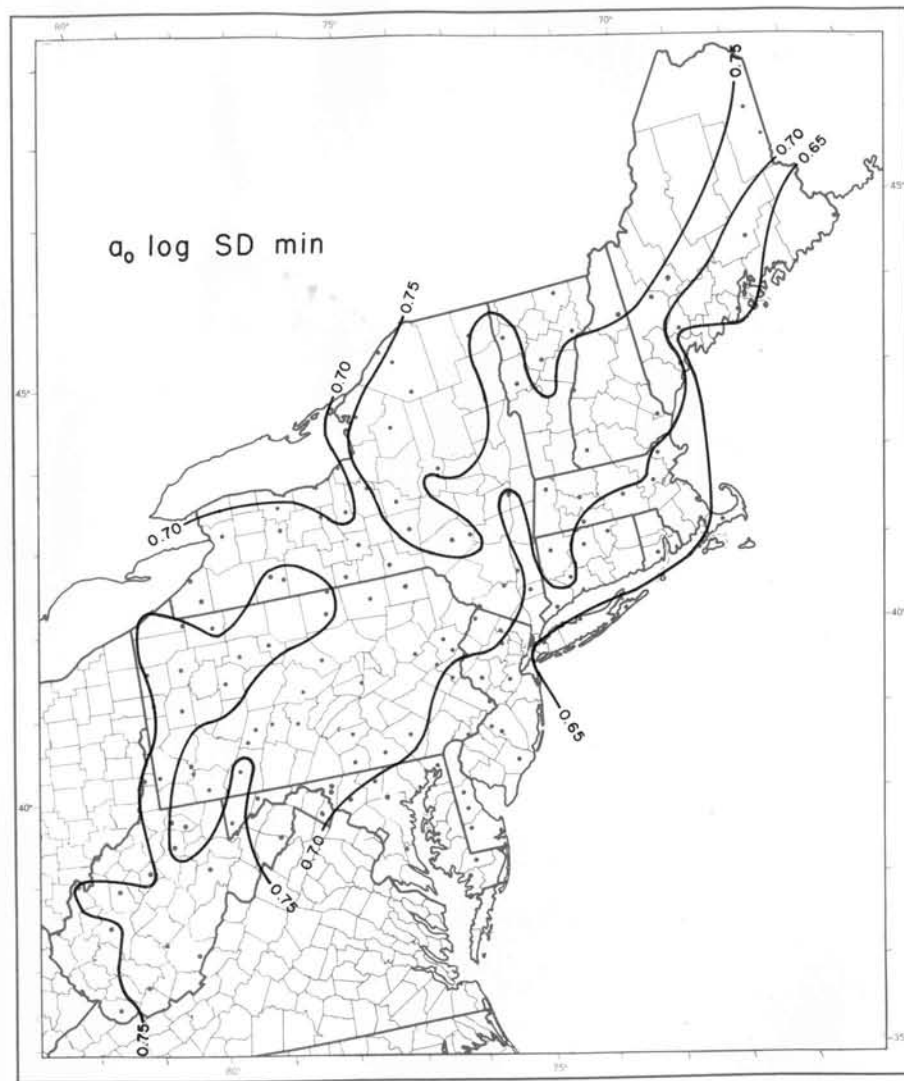
Map 16. a_0 for the diurnal minimum temperature, in °F.



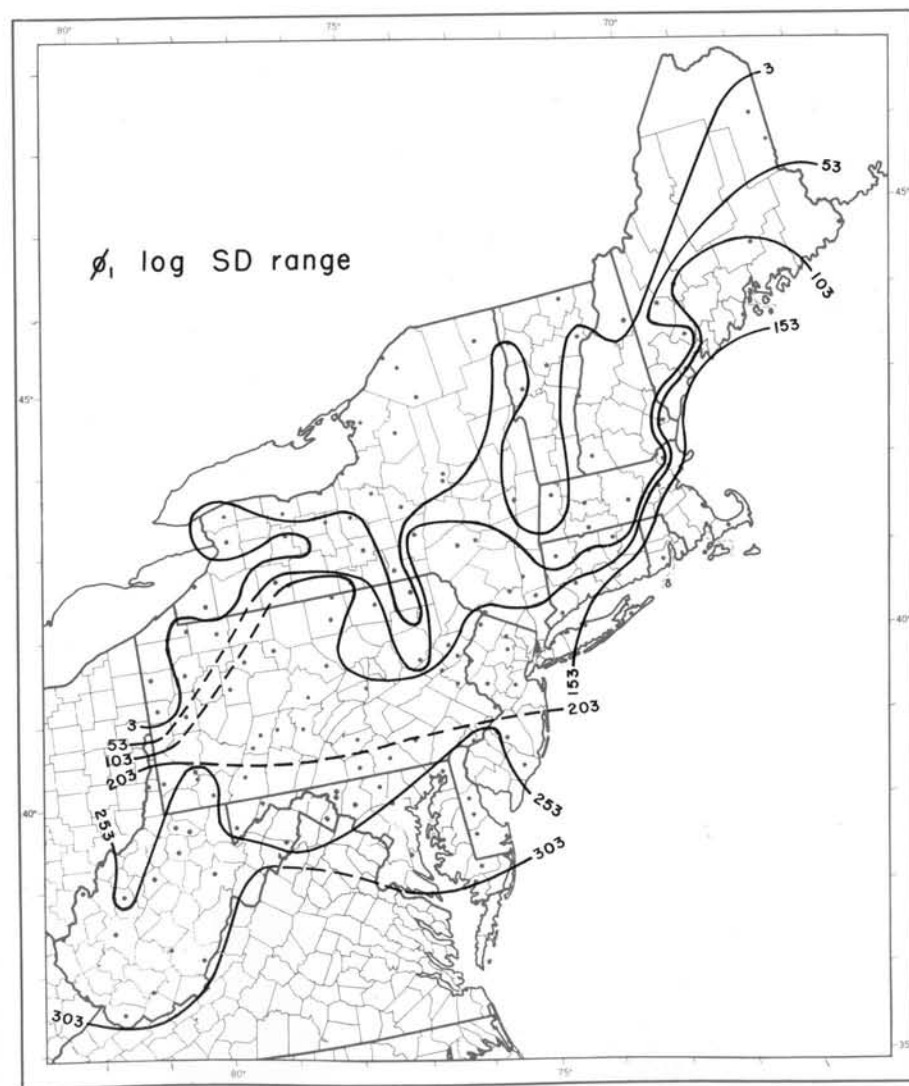
Map 17. ϕ_1 for the log standard deviation of diurnal minimum temperature. In days after March 1.



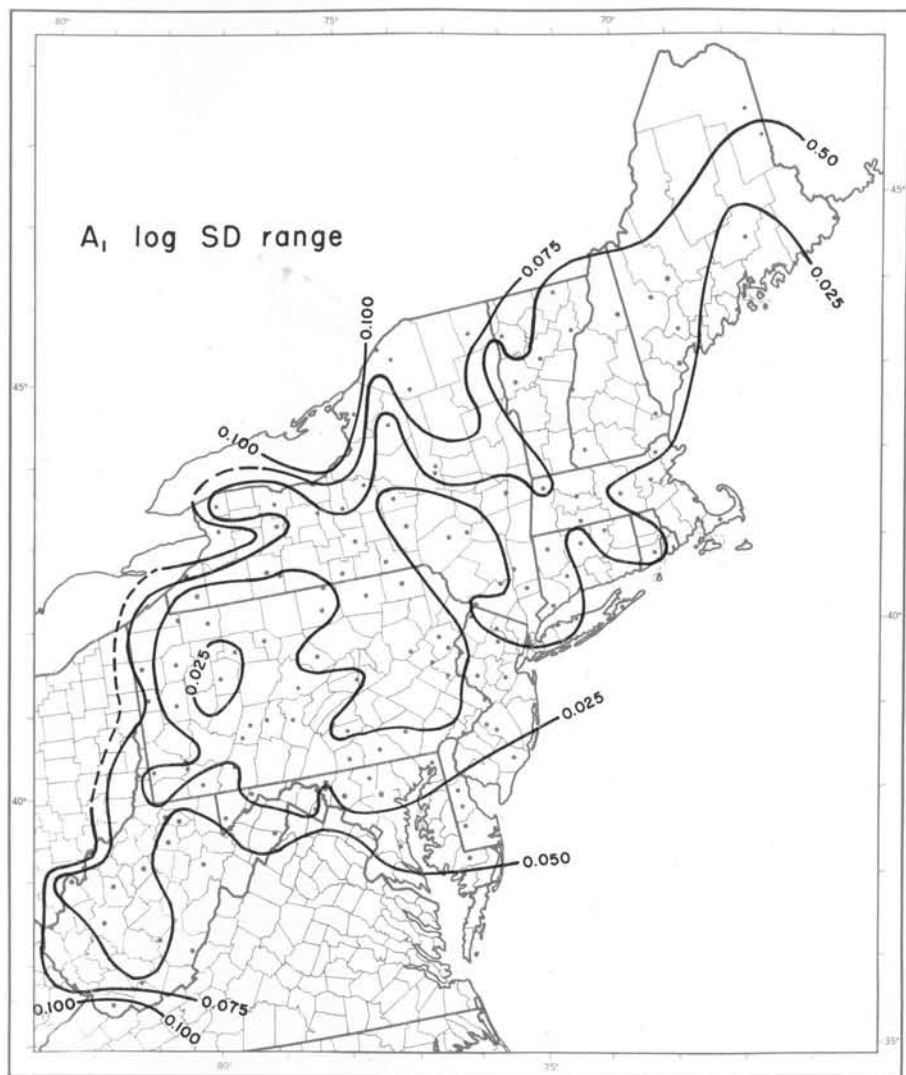
Map 18. A_1 for the log standard deviation of diurnal minimum temperature, in log ($^{\circ}$ F).



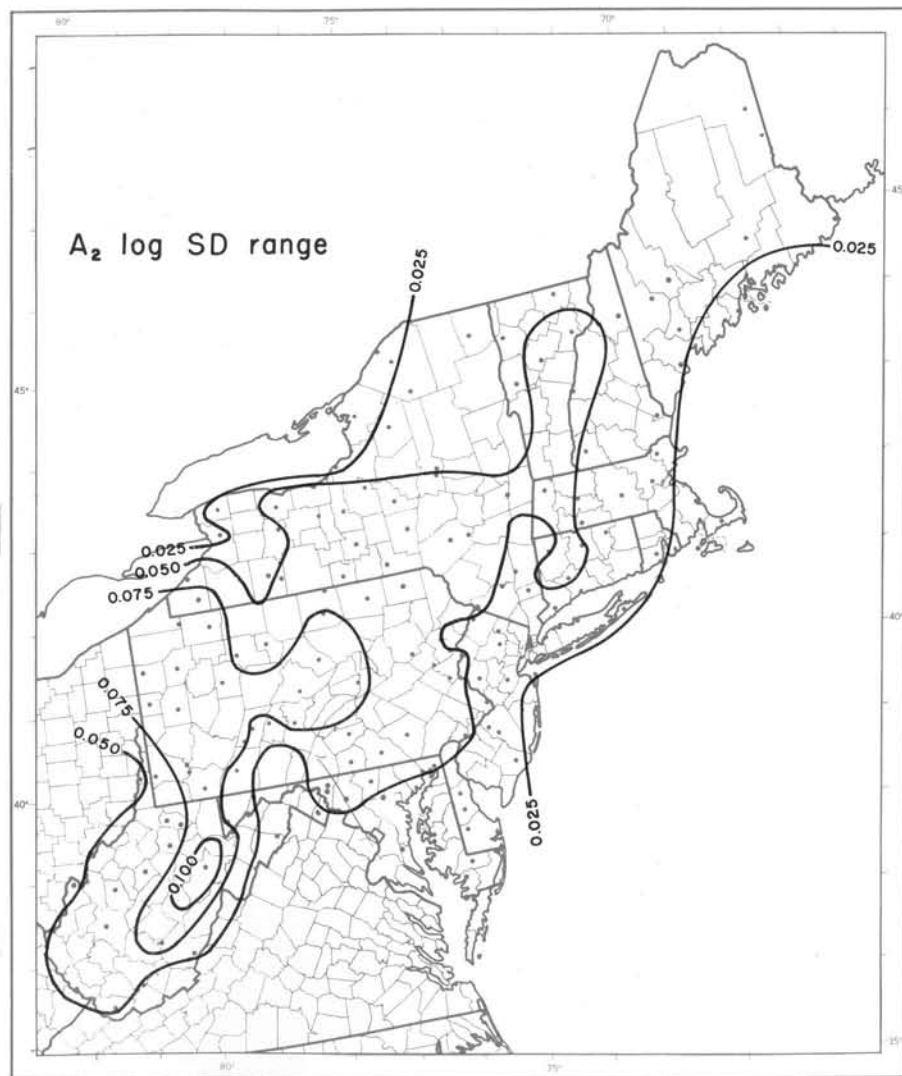
Map 19. a_0 for the log standard deviation of diurnal minimum temperature, in log ($^{\circ}$ F).



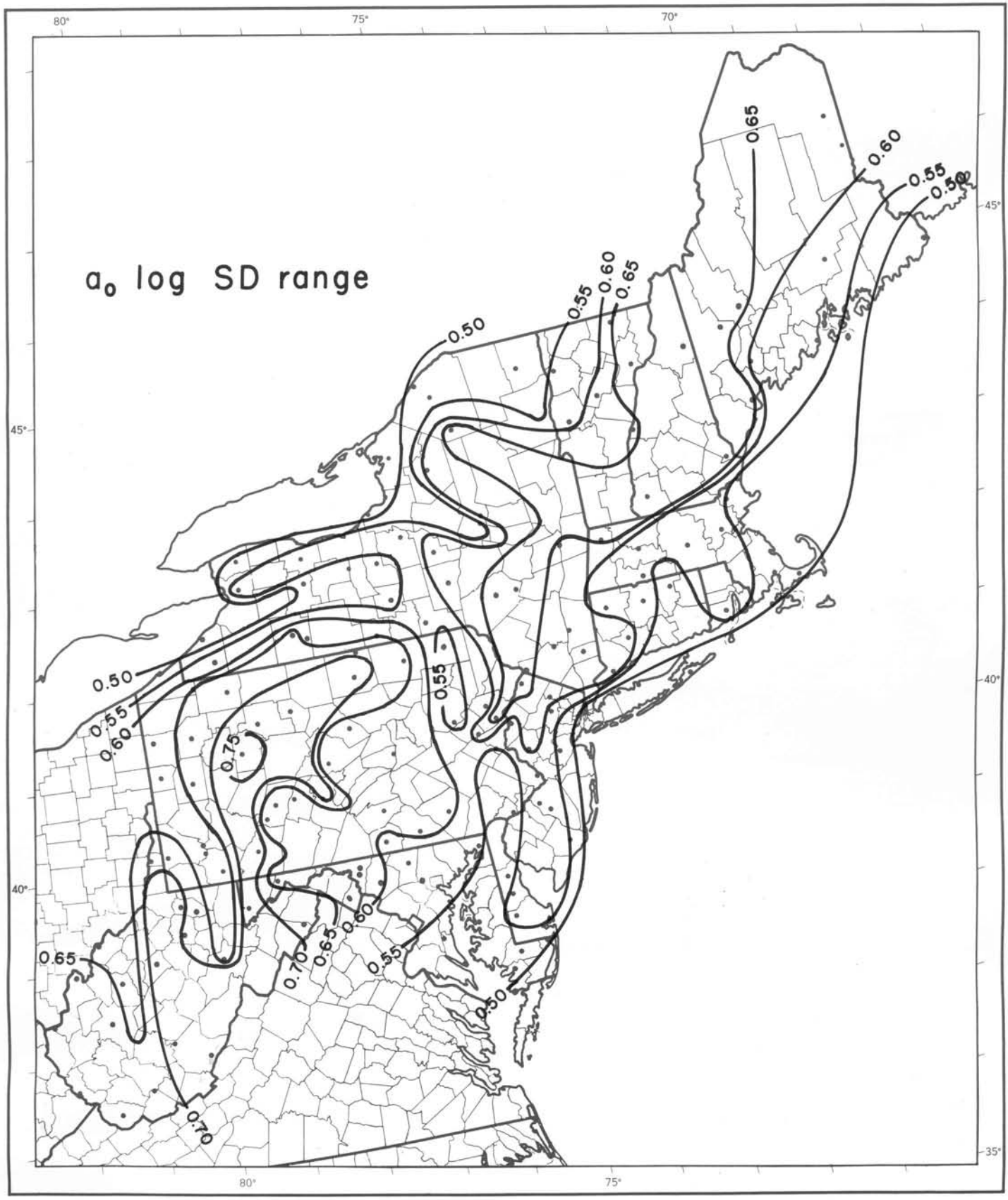
Map 20. ϕ_1 for the log standard deviation of diurnal temperature range. In days after March 1.



Map 21. A_1 for the log standard deviation of diurnal temperature range, in log ($^{\circ}$ F).



Map 22. A_2 for the log standard deviation of diurnal temperature range, in log ($^{\circ}$ F).



Map 23. a_0 for the log standard deviation of diurnal temperature range, in log ($^{\circ}$ F).

THE NE 35 TECHNICAL COMMITTEE

Members of the committee and the participating state and federal agencies that they represented follow:

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A. J. Loustalot	U. S. Department of Agriculture, Agricultural Research Service, State Experiment Stations Division
J. K. McGuire	U. S. Department of Commerce, Weather Bureau, Office of Climatology

¹ Succeeded J. J. Kolega

² Succeeded C. H. Blasberg

³ Succeeded A. A. Spielman