Shifting Math Instruction: Rigor in the Classroom

Presented by: Jennifer Michalek
Learning Targets

• Participants will gain a deeper understanding of the CCS-Math instructional shifts of focus, coherence, and rigor.

• Participants will deepen their knowledge and understand the importance of Connecticut Core Standards (CCS) aligned instruction.

• Participants will gain a clear vision of how the CCS should look in practice within the mathematics classroom.
The Shifts for CCS - Math
The Need for Change

- High college remediation rates
- Stagnant academic progress
- Larger gap with international peers
Three Instructional Shifts for CCS - Mathematics

**Focus** on the Standards; teach less but for understanding.

**Coherence** – Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.

**Rigor** – Means a balance of solid conceptual understanding, procedural skill and fluency, and application of skills in problem solving situations.
The Shape of Math in A+ Countries

Mathematics topics intended at each grade by at least two-thirds of A+ countries

Mathematics topics intended at each grade by at least two-thirds of 21 U.S. states

Focus Through Domains

Table 1. The Pre-CCSSM Strand Picture of Mathematical Content

Table 2. The CCSSM Domain Picture of Mathematical Content
Shift 1: Focus

- Narrow the scope of content
- Focus deeply on what is emphasized in the Standards
- Move away from "mile wide, inch deep"
- Less topic coverage can be associated with higher scores on those topics covered
Levels of Focus

First level of focus: Knowing what is to be taught at each grade level and what is not.

• Serve as the foundation for the grade
• Essential mathematical ideas for each grade level
• Narrow the scope of content and deepen how time and energy is spent in the math classroom
Critical Areas at Each Grade

• 2 to 4 critical areas are identified at each grade from K-8

• Outline the essential mathematical ideas for each grade level

• Form a firm foundation on which to build concepts and procedures in later years
<table>
<thead>
<tr>
<th>Grade</th>
<th>Topic</th>
<th>How Did You Do?</th>
<th>Next Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Compare numbers</td>
<td>Use tally marks</td>
<td>Understand meaning of addition and subtraction</td>
</tr>
<tr>
<td>1</td>
<td>Add and subtract within 20</td>
<td>Measure lengths indirectly and by iterating length units</td>
<td>Create and extend patterns and sequences</td>
</tr>
<tr>
<td>2</td>
<td>Work with equal groups of objects to gain foundations for multiplication</td>
<td>Understand place value</td>
<td>Identify line of symmetry in two dimensional figures</td>
</tr>
<tr>
<td>3</td>
<td>Multiply and divide within 100</td>
<td>Identify the measures of central tendency and distribution</td>
<td>Develop understanding of fractions as numbers</td>
</tr>
<tr>
<td>4</td>
<td>Examine transformations on the coordinate plane</td>
<td>Generalize place value understanding for multi-digit whole numbers</td>
<td>Extend understanding of fraction equivalence and ordering</td>
</tr>
<tr>
<td>5</td>
<td>Understand and calculate probability of single events</td>
<td>Understand the place value system</td>
<td>Apply and extend previous understandings of multiplication and division to multiply and divide fractions</td>
</tr>
<tr>
<td>6</td>
<td>Understand ratio concepts and use ratio reasoning to solve problems</td>
<td>Identify and utilize rules of divisibility</td>
<td>Apply and extend previous understandings of arithmetic to algebraic expressions</td>
</tr>
<tr>
<td>7</td>
<td>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers</td>
<td>Use properties of operations to generate equivalent expressions</td>
<td>Generate the prime factorization of numbers to solve problems</td>
</tr>
<tr>
<td>8</td>
<td>Standard form of a linear equation</td>
<td>Define, evaluate, and compare functions</td>
<td>Understand and apply the Pythagorean Theorem</td>
</tr>
<tr>
<td>Alg.1</td>
<td>Quadratic inequalities</td>
<td>Linear and quadratic functions</td>
<td>Creating equations to model situations</td>
</tr>
<tr>
<td>Alg.2</td>
<td>Exponential and logarithmic functions</td>
<td>Polar coordinates</td>
<td>Using functions to model situations</td>
</tr>
</tbody>
</table>
Levels of Focus

*Second level of focus:* Knowing the major work of each grade.

- Not all content is emphasized equally
- Directly related to the critical areas
- Majority of the time should be dedicated to the major work of the grade
Cluster Emphases

Student Achievement Partners (SAP)
• Non-profit founded by three of the contributing authors of CCSSM
• Develops and makes available tools and resources free of charge
• Focus by grade level

Resource
## Widely Applicable Prerequisites

<table>
<thead>
<tr>
<th>Number and Quantity</th>
<th>Algebra</th>
<th>Functions</th>
<th>Geometry</th>
<th>Statistics and Probability</th>
<th>Applying Key Takeaways from Grades 6–8**</th>
</tr>
</thead>
</table>
| N-RN, Real Numbers: Both clusters in this domain contain widely applicable prerequisites. | Every domain in this category contains widely applicable prerequisites. | F-IF, Interpreting Functions: Every cluster in this domain contains widely applicable prerequisites. | The following standards and clusters are relatively important within this category as widely applicable prerequisites: | Solving problems at a level of sophistication appropriate to high school by:  
* Applying ratios and proportional relationships.  
* Applying percentages and unit conversions, e.g., in the context of complicated measurement problems involving quantities with derived or compound units (such as mg/mL, kg/m³, acre-feet, etc.).  
* Applying basic function concepts, e.g., by interpreting the features of a graph in the context of an applied problem.  
* Applying concepts and skills of geometric measurement e.g., when analyzing a diagram or schematic.  
* Applying concepts and skills of basic statistics and probability (see 6-8.SP).  
* Performing rational number arithmetic fluently. |
| N-Q*, Quantities: Every standard in this domain is a widely applicable prerequisite. Note, this domain is especially important in the high school content standards overall as a widely applicable prerequisite. | Note, the A-SSE domain is especially important in the high school content standards overall as a widely applicable prerequisite. | Additionally, standards F-BF.1 and F-LE.1 are relatively important within this category as widely applicable prerequisites. | The following standards are relatively important within this category as widely applicable prerequisites: |  
S-ID.2  
S-ID.7  
S-IC.1 |
| | Note, the above standards in turn have learning prerequisites within the Geometry category, including: | G-CO.A  
G-CO.B  
G-SRT.A | Note, the above standards in turn have learning prerequisites within 6-8.SP. | |

*Table excerpted from the High School Publishers Criteria for the Common Core State Standards for Mathematics*
What is a “large majority of time”?

At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the Major Work of the grade.

- K-8 Publishers’ Criteria, Spring 2013, p. 8
What does student learning look like if the teacher is focusing on the critical areas?
<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K–2</td>
<td>Addition and subtraction - concepts, skills, and problem solving and place value</td>
</tr>
<tr>
<td>3–5</td>
<td>Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving</td>
</tr>
<tr>
<td>6</td>
<td>Ratios and proportional reasoning; early expressions and equations</td>
</tr>
<tr>
<td>7</td>
<td>Ratios and proportional reasoning; arithmetic of rational numbers</td>
</tr>
<tr>
<td>8</td>
<td>Linear algebra and linear functions</td>
</tr>
</tbody>
</table>
## Look Fors

<table>
<thead>
<tr>
<th>Conceptual Category</th>
<th>Focus Clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers and Quantity</td>
<td>Real numbers and Quantities</td>
</tr>
<tr>
<td>Algebra</td>
<td>Seeing Structure in Expressions, Arithmetic with Polynomial and Rational Expressions, Creating Equations, Reasoning with Equations and Inequalities</td>
</tr>
<tr>
<td>Functions</td>
<td>Interpreting Functions, Building Functions, and Linear, Quadratic and Exponential Models</td>
</tr>
<tr>
<td>Geometry</td>
<td>Similarity, Right Triangles and Trigonometry</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Interpreting Categorical and Quantitative Data and Making Inferences and Drawing Conclusions</td>
</tr>
</tbody>
</table>
A Summary of Focus

The Importance of Focus in Mathematics

Jason Zimba, Ph.D.
Math Team Coordinator
Three Instructional Shifts for CCS - Mathematics

**Focus** on the Standards; teach less but for understanding.

**Coherence** – Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.

**Rigor** – Means a balance of solid conceptual understanding, procedural skill and fluency, and application of skills in problem solving situations.
Shift 2: Coherence

Think across grades and link to major topics within grades

- Connect learning within and across grades
- Each standard is not a new event, but an extension of previous learning
- Mathematics makes sense
- Based on the mathematical progressions
Why Coherence?

- The standards are designed around coherent progressions from grade to grade
- Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years.

2014 Common Core State Standards Initiative
# Domains and Conceptual Categories

## K-12

### Common Core State Standards – Mathematics

<table>
<thead>
<tr>
<th>Kindergarten</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>HS</th>
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<tbody>
<tr>
<td><strong>Counting and Cardinality</strong></td>
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<td><strong>Number and Operations in Base Ten</strong></td>
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<td><strong>Number and Operations - Fractions</strong></td>
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<td><strong>Operations and Algebraic Thinking</strong></td>
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<td><strong>Geometry</strong></td>
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<td><strong>Ratios and Proportional Relationships</strong></td>
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<td><strong>The Number System</strong></td>
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<td><strong>Expressions and Equations</strong></td>
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<td><strong>Statistics and Probability</strong></td>
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</tbody>
</table>

### Learning Progressions

- **Number and Quantity**
- **Algebra**
- **Geometry**
- **Statistics and Probability**
Coherence is Built Into the Standards

Number and Operations—Fractions  5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator \(\frac{a}{b} = a \div b\). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret \(\frac{3}{4}\) as the result of dividing 3 by 4, noting that \(\frac{3}{4}\) multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size \(\frac{3}{4}\). If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
Two Levels of Coherence

- Coherence within a grade
  - Reinforce a major topic in a grade by utilizing a supporting topic
  - Meaningful introduction to topics in the same grade that complement each other

- Coherence across grades
  - Apply learning from previous grades to learn new topics
  - Progressions of mathematics that are meaningful and make sense
Coherence Within a Grade

Example: Data Representation

1.MD.C.4  Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Coherence Within a Grade

Example: Statistics

8.SP.A.3  **Use the equation of a linear model** to solve problems in the context of bivariate measurement data, **interpreting the slope and intercept.**  For example, in a linear model for a biology experiment, interpret a slope of 1.5cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
A Look Within the Conceptual Categories

<table>
<thead>
<tr>
<th>Reasoning with Equations and Inequalities</th>
<th>Reasoning with Equations and Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
<td>A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</td>
</tr>
</tbody>
</table>
### Coherence Across the Grades

| K.OA.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. | 1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 – 4 = 13 – 3 – 1 = 10 – 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 – 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). | 2.OA.2: Fluently add and subtract within 20 using mental strategies. (Note: See standard 1.OA.6 for a list of mental strategies). By end of Grade 2, know from memory all sums of two one-digit numbers. |
Putting It Into Practice

Fractions Progression Module

Grade 3
- The meaning of fractions
- The number line and number line diagrams
- Equivalent fractions
- Comparing fractions
- Decimals

Grade 4
- Equivalent fractions
- Adding and subtracting fractions
- Comparing fractions
- Multiplication of a fraction by a whole #

Grade 5
- Adding and subtracting fractions
- Multiplying and dividing fractions
- Multiplication as scaling
### Coherence in the Middle Grades

| 6.EE.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p$, $q$ and $x$ are all nonnegative rational numbers. | 7.EE.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers.** | 8.EE.8: Analyze and solve pairs of simultaneous linear equations. **b. Solve systems of two linear equations in two variables algebraically,** and estimate solutions by graphing the equations. Solve simple cases by inspection. **c. Solve real-world and mathematical problems leading to two linear equations in two variables.** |
# A Look Across the Conceptual Categories

<table>
<thead>
<tr>
<th>Algebra 1</th>
<th>Geometry</th>
<th>Algebra 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.CED.1 Create equations in one variable and use them to solve problems.</td>
<td>G.GPE.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</td>
<td>F.BF.1 Write a function that describes a relationship between two quantities.</td>
</tr>
</tbody>
</table>
Coherence Map

• Build student understanding by linking together concepts within and across grades.

• Identify gaps in a student's knowledge by tracing a standard back through its logical pre-requisites.

• Visualize and understand how supporting standards relate to the major work of the grade.
What does student learning look like if the teacher is building coherence?
Look Fors

• Students experience a launch or initial review that anchors them before diving into new material.

• Students make comments like, “Oh! This is like when we...” or “It’s the same as when we used decimals” or “Last year we...”

• Students notice and ask about connections.

• Students can explain how some other concept they’ve studied is related to the current one.
Coherence Take-Aways

Coherence allows for:

- Linking work to major focus areas of the grade level.
- A natural progression of content from grade to grade, course to course.
- Support of mathematical connections between and among domains
Three Instructional Shifts for CCS - Mathematics

**Focus** on the Standards; teach less but for understanding.

**Coherence** – Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.

**Rigor** – Means a balance of solid conceptual understanding, procedural skill and fluency, and application of skills in problem solving situations.
Rigor: The three legged stool

Conceptual understanding

Procedural skill and fluency

Application
Shift 3: Rigor

✓ The CCSSM require a balance of:
  - Conceptual understanding
  - Procedural skill and fluency
  - Application in problem-solving

✓ Equal intensity in time, activities, and resources

In major topics, pursue conceptual understanding, procedural skill and fluency, and application
Conceptual Understanding
Conceptual Understanding

• Teach more than “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives

• Conceptual understanding is about big ideas in math

• Students are able to see math as more than a set of mnemonics or discrete procedures

• Conceptual understanding is produced through rich problems, hands-on activities, skillful questioning, student discussion, etc.

• Conceptual understanding supports the other aspects of rigor (procedural skill and fluency, and application)
Building Conceptual Understanding

8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

• The language in the standard speaks to conceptual understanding.
• Students need to understand more than just a process of creating a table or graphing a line.
• Students need to compare proportional relationships presented in different ways.
Questioning

• High-quality problems provide opportunities for discussion
• Good quality questions are easy to discuss
• Questions should elicit a range of responses from students which will enable class discussions to lead to conceptual understanding of a topic
• Questions do not have to be complex, but need to elicit student thinking about the conceptual understanding required in the Standards
Conceptual Understanding

Examples

**4.NF.C** Understand decimal notation for fractions, and compare decimal fractions.

1.7 or 17 twelfths

a. Which number is larger?

b. Explain how you can tell without drawing a picture.

**8.EE.C.8.a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

a. Draw a line that intersects this line.

b. Write a system of equations that is represented by these two lines.

c. What is the solution to the system?
Procedural Skill and Fluency
Procedural Skill and Fluency

• The Standards require speed and accuracy in calculation.

• Materials structure class time and/or homework time for students to practice core functions so that they are more able to understand and manipulate more complex concepts.

• The high school standards do not set explicit expectations for fluency, but fluency helps students get past the need to manage computational details so that they can observe structure and patterns in problems.
<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2, 2.NBT.5</td>
<td>Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7, 3.NBT.2</td>
<td>Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
</tr>
<tr>
<td>6</td>
<td>6.NS.2,3</td>
<td>Multi-digit division Multi-digit decimal operations</td>
</tr>
</tbody>
</table>
Procedural Skill and Fluency

5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm

1. \(49 \times 975 = \) 

2. 

\[
\begin{array}{c}
6,751 \\
\times 609 \\
\end{array}
\]

3. What is the product of 9 and 740?

Fluency in the Middle Grades

• Standards no longer use the key words of fluent or fluently
• Middle school fluency less about calculation
• Fluency at the middle level is about ease of manipulation
  • Expressions
  • Equations
  • Notations
Procedural Skill and Fluency

8.EE.C.7: Solve linear equations in one variable.

Decide whether the solution to each equation is positive, negative, zero, or there are no solutions. Check the correct box for each row.

<table>
<thead>
<tr>
<th>Equation</th>
<th>solution is positive</th>
<th>solution is negative</th>
<th>solution is zero</th>
<th>there are no solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x = 5$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$5z + 7 = 3$</td>
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<tr>
<td>$7 - 5w = 3$</td>
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<tr>
<td>$4a = 9a$</td>
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<tr>
<td>$y = y + 1$</td>
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</tbody>
</table>
Procedural Skill and Fluency in Practice

<table>
<thead>
<tr>
<th>Course</th>
<th>Fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra 1</td>
<td>• Solving characteristic problems involving the analytic geometry of lines</td>
</tr>
<tr>
<td></td>
<td>• Fluency in adding, subtracting and multiplying polynomials</td>
</tr>
<tr>
<td></td>
<td>• Fluency in transforming expressions and seeing parts of an expression as a single object</td>
</tr>
<tr>
<td>Geometry</td>
<td>• Fluency with triangle congruence and similarity criteria</td>
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<tr>
<td></td>
<td>• Fluency with the use of coordinates</td>
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<tr>
<td></td>
<td>• Fluency with the use of construction tools</td>
</tr>
<tr>
<td>Algebra 2</td>
<td>• Divide Polynomials with remainder by inspection in simple cases</td>
</tr>
<tr>
<td></td>
<td>• See structure in expressions and use this structure to rewrite expressions</td>
</tr>
<tr>
<td></td>
<td>• Fluency in translating between recursive definitions and closed forms</td>
</tr>
</tbody>
</table>
Developing Fluency

What
- More than just timed tests
- Deliberate selection of problems
- Different forms of repeated practice

Why
- Build automaticity

How
- Classroom instruction must emphasize the development and application of strategies
- Distributed over time, rather than in a single sitting
Application
Application

• Students can use appropriate concepts and procedures for application even when not prompted to do so

• Provide opportunities at all grade levels for students to apply math concepts in “real world” situations, recognizing this means different things in K-2, 3-5, 6-8, and high school

• Teachers in content areas outside of math ensure that students are using grade-level-appropriate math to make meaning of and access content
Problems vs. Exercises

**Problems**

• Students learn new mathematics
• Students are asked to use what they already know to answer mathematical questions that they haven’t been taught to solve
• Opportunity for discussion and allows teachers to see what students understand

**Exercises**

• Students apply what they have already learned to build mastery
• Repetition to develop a skill
Engaging With Problem Solving

My pool is a rectangular shaped pool. It has a width of 10 feet and a length of 18 feet. The whole pool has a depth of 5.5 feet. The delivery trucks can carry 3,000 gallons of water and there are approximately 7.5 cubic feet in a gallon. They charge $0.04 per gallon and an extra delivery fee of $15 per truck load.

How much water is needed to fill the pool?

How many trucks are needed?

How much will the delivery cost?

7. A plastic rectangular container measures 5 inches wide, 8 inches long and 3 inches high. How much water will fit in the container?

8. In order to ship an item, the company requires a box that is 48 cubic feet. The box they currently have is 3 feet wide, 4 feet high and 2 feet long. Will this satisfy the companies requirements?

How much water is needed to fill the pool?

How many trucks are needed?

How much will the delivery cost?

www.CommonCoreSheets.com
Rigor in the Standards

The language of the standards assist in determining the appropriate use of rigor in instruction.

• Conceptual Understanding
• Procedural Skill and Fluency
• Application
Conceptual Understanding

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

1a. Which of these tables of values satisfy the equation $y = 2x + 3$? Explain how you checked.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

1b. By completing the table of values, draw the lines $y = 2x + 3$ and $x = 1 - 2y$ on the grid.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

1c. Do the equations $y = 2x + 3$ and $x = 1 - 2y$ have one common solution, no common solutions, or infinitely many common solutions? Explain how you know.
Procedural Skill and Fluency

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

Solve each of the following systems:

-4x – 2y = -12
4x + 8y = -24

x – y = 11
2x + y = 19

8x + y = -1
-3x + y = -5

5x + y = 9
10x – 7y = -18

Source: Leinwand, S., Brahier, D., and Huinker, D., Principles to Action, pg. 20
A.REI.C.6
You are trying to decide which two smartphone plans would be better. Plan A charges a basic fee of $30 per month and 10 cents per text message. Plan B charges a basic fee of $50 per month and 5 cents per text message.

a) How many text messages would you need to send per month for Plan B to be the better option? Explain your decision?

b) If the cell phone company decided to offer unlimited texts for $80 per month, do you think that you would change your smartphone plan? Use mathematical reasoning to support your decision.

Adapted from Illustrative Mathematics and Leinwand, S., Brahier, D., and Huinker, D., Principles to Action, 2014, pg. 20

A.REI.C.6
What does student learning look like if the teacher is balancing rigor?
Look Fors

Students are either working toward or using understanding.

**Conceptual Development**
- Students work on tasks that elicit ideas, discussions, and connections that lead to new understanding
- Students ask and answer questions about understanding

**Fluency and Procedural Skills**
- Students engage in meaningful practice
- Students connect procedural skills with representations

**Application**
- Students apply knowledge to new, but accessible situations that extend their understanding
Rigor in Tasks

Task selection in critical in order to effectively address rigor in instruction.

- Conceptual Understanding
- Procedural Skill and Fluency
- Application
Summary of Rigor

- Balance of conceptual understanding, procedural skill and fluency, and application
- Targeted aspect of rigor in a lesson should be determined by the standard being addressed
- Equal intensity throughout the course of a year, not necessarily in each lesson or unit
Finding Evidence in the Lessons
Planning for Standards-Aligned Instruction

• Analyze and interpret the lesson plan to collect and discuss evidence of the shifts

• Review the guiding questions related to each shift in the Lesson Plan Analysis Template

• Respond to the questions using evidence from the lesson plan provided.
Instructional Practice Guide

• **Core Action 1:** Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.

• **Core Action 2:** Employ instructional practices that allow all students to learn the content of the lesson.

• **Core Action 3:** Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.
Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.

- The lesson focuses on the depth of grade-level cluster(s), grade-level content standard(s) or part(s) thereof.
- The lesson intentionally relates new concepts to students’ prior skills and knowledge.
- The lesson intentionally targets the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.
Core Action 2: *Employ instructional practices that allow all students to learn the content of the lesson.*

Indicators:

• The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples.

• The teacher strengthens all students’ understanding of the content by strategically sharing students’ representations and/or solution methods.

• The teacher deliberately checks for understanding throughout the lesson to surface misconceptions and opportunities for growth, and adapts the lesson according to student understanding.

• The teacher facilitates the summary of the mathematics with references to student work and discussion in order to reinforce the purpose of the lesson.
Watch the Lesson Video
Debrief

• What core actions were clearly evident during the lesson?
• What evidence did you gather to support your assertions?
• What core actions were not clearly evident?
• How did you know?
Conclusion

*How do I know if the lesson reflects the Shifts?*

- Is the lesson addressing on grade-level content?
- What is the full intent of the standard(s) being addressed?
- Is the aspect of rigor required by the standard(s) the same as the aspect(s) being addressed in the lesson?
- How does the lesson connect to and build on students’ prior skills and knowledge?
Beyond the Lesson Guide

“After discussing the observed lesson, use the ‘Beyond the Lesson’ questions to help clearly delineate what practices are in place, what already occurred, and what opportunities might exist in another lesson, further in the unit, or over the course of the year to incorporate the Shifts into the classroom.”

• Prioritize the question for the observed lesson.

• Utilize the resources provided in the guide to support teacher’s understanding of standards-aligned instruction.
Resources to Support Teachers

- **Self-Paced Learning Modules**
  - Math Module 1: Focus on Practice Standards
  - Math Module 2: Focus on Content Standard
  - Supporting Success for ALL Students with the CT Core Standards
  - CCS Mathematics Success for Students with Disabilities
  - CCS Mathematics Success for English Learners

- **Illustrative Mathematics**
  - Course Blueprints
  - Tasks and other resources for each content standard

- **Student Achievement Partners**
  - Major work of the grade
  - Textbook alignment and adaptations (enVisionmath 2.0, GO Math!)
  - Instructional Materials Evaluation Tool
  - Most Misunderstood Middle School Mathematics Standards

- **YouCubed**
  - Tasks
  - Research evidence on best ways to learn math facts
Thank You

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