**Unit 1: Investigation 5 (3 Days)**

**Composition of Functions +**

**(This Investigation is intended for STEM-Intending Students)**

**Common Core State Standards**

F.BF.1c (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

**Overview**

This Investigation, intended for STEM-intending students, concerns composition of functions. The primary metaphor for composite functions should be that an output from one function becomes the input for another function. Students will learn how to find the composition of two functions from a table of values and from graphs of the functions prior to finding composite functions symbolically. It is hoped that the emphasis on functions in tabular and graphical form will help students understand that the domain of a composite function cannot always be determined directly from its final symbolic representation; the domain of each function individually must be taken into account. Several problems on the decomposition of functions into two functions help to set the stage for the use of the Chain Rule and other topics that STEM-intending students will see in later precalculus and calculus courses. The final activity in this Investigation demonstrates the usefulness of composite functions in modeling realistic situations. Students who complete Investigation 5 will use composite functions again in Investigation 6 in the study of inverse functions, when they verify that two functions are inverses of each other by composing the functions. They will also return to it Unit 5 where they will demonstrate that the logarithmic and exponential functions base b are inverses of each other.

**Assessment Activities**

**Evidence of success: What will students be able to do?**

* Given a table of values or a graph of two functions, find a table of values or a graph for the composition of those functions.
* Given two functions, find a formula for the composite function.
* Given a composite function, find the domain of the composite function.
* Given a function, be able to decompose the function into two simpler functions.
* Use composite functions as mathematical models in realistic problems.

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 1.5** asks students to take two functions and find their composition symbolically, graphically, and by use of a table of values.
* **Journal Prompt 1** asks students to explain the main ideas of composite functions to a friend who skips Investigation 5 of Algebra 2.
* **Activity 1.5.1 Composing Composite Functions** asks students to start with a table of values for two functions and find a table of values of the composite function, and to start with the graphs of two functions and find the graph of the composite function.
* **Activity 1.5.2 Domains and Graphs of Composite FUNctions** gives students two functions and asks them to find the composite function and its domain, and to decompose one function into two functions.
* **Activity 1.5.3 Functions Inside Functions** asks students to use composite functions as mathematical models.

**Launch Notes**

Because Investigation 5 is written for STEM-intending students, it is expected that at least some of the students will have some familiarity with cars and how they are manufactured. Prepare a set of colored index cards, or slips of colored paper, with the following words written on them, one on each card:

* Blue cards: iron, chromium, nickel, crude oil, silica sand, limestone
* Red cards: Steel, plastic, glass, aluminum, rubber
* Yellow cards: (Group 1: car chassis, door beams, car roof, car body panels, exhaust system), (Group 2: dashboard, door handles, floor mats, seat belts, airbags), (Group 3: body panels, engine block, wheels), (Group 4: tires, wiper blades, seals, belts, hoses), (Group 5: windshield, mirrors, navigation screen)
* White card: car

Distribute these to the class at random. It is important to distribute all of the blue, red, and white cards, and at least one yellow card from each of the groups listed above, if it is possible. Once each student has a card, ask them to form groups as follows: How many of you have a card listing a raw material? How many of you have a card listing a type of material made from other raw materials? How many of you have a card listing a car part? How many of you have the card listing “car”? Note that they should be grouped according to the color of the card they have.

Next, ask students in the “raw material” or “blue” group to try to group themselves according to which of them are needed to make the materials in the “red” group. They should recognize the following (or you can guide them to recognize the following):

* Iron, chromium, and nickel are used to make steel.
* Crude oil is used to make plastic.
* Silica sand and limestone are used to make glass.
* Aluminum and rubber are themselves raw materials, not made from other raw materials.

Now ask students in the “red” group to try to group themselves according to which material is used to make the car parts in the “yellow” group. This should work as follows:

* Steel is used to make the car chassis, door beams, car roof, car body panels, and exhaust systems.
* Plastic is used to make the dashboard, door handles, floor mats, seat belts, and airbags.
* Aluminum is used to make the body panels, engine block, and wheels.
* Rubber is used to make the tires, wiper blades, seals, belts, and hoses.
* Glass is used to make the windshield, mirrors, and navigation screen.

Finally, all of these parts are used to make a car, which has about 30,000 individual parts.

Now, what does all of this have to do with Algebra 2? Think of it this way:

* If you own a business that mines or harvests the raw materials (the “blue” cards), you would have functions that describe how much of those materials you make (your output) depending on the conditions of your business (your inputs).
* If you own a business that manufactures other raw materials (the “red” cards), you would have functions that describe how much of your materials you make (your output) depending on the quantity of the “blue” raw materials (your inputs).
* If you own a business that manufactures car parts (the “yellow” cards), you would have functions that describe how many parts you make (your output) depending on the quantity of the “red” raw materials (your inputs).
* If you own a business that manufactures cars (the “white” card), you would have functions that describe how many cars you make (your output) depending on the quantity of the “yellow” car parts you have (your inputs).

See what happens? The output from one part of the chain in making cars becomes the input to the next part, which creates a new output that becomes the input to the next part, etc. Mathematically, when the output from one function becomes the input for the next function, we have what is called a composition of functions.

(Sources used for this Launch include the following websites: <http://auto.howstuffworks.com/under-the-hood/auto-manufacturing/5-materials-used-in-auto-manufacturing.htm>, <http://visual.ly/materials-used-make-car-0>, <http://en.wikipedia.org/wiki/>.)

**Teaching Strategies**

Build on the idea of the Launch by stressing that a composition of functions means that the output from one function is the input for another function—we are no longer thinking of the input to a function as a number or variable, but as another function. As you introduce the notation for composite functions to your students, you should strongly consider color-coding the symbols you use. For example, if f(x) = x2 – 3x + 5 and g(x) = $\sqrt{x-2}$, then f ◦ g (x) = f(g(x)) = f($\sqrt{x-2}$) = ($\sqrt{x-2}$)2 – 3($\sqrt{x-2})$ + 5.

**Activity 1.5.1 Composing Composite Functions** looks at composite functions using both tables and graphs. Depending on the exposure students had in Algebra 1 to quadratic functions, they may not be able to determine what kind of a function f ◦ g or g ◦ f is. You might remind students about Activity 1.3.1 and the growth of functions demonstrated there. Since students have tables of values for these functions, you can ask them to find the first differences, the ratios, and the second differences between successive values in the tables as was described in Activity 1.3.1. This would help to reinforce the concept of function growth and how this can be used to describe a family of functions.

**Activity 1.5.2 Domains and Graphs of Composite FUNctions** extends the first Activity by considering the domain of composite functions. Be sure to make students aware that they need to consider the domain of both functions in order to determine the domain of the composite function, and not just the formula of the composite function. By definition, in order for x to be in the domain of the composite function f ◦ g, x must be both in the domain of the function g and the domain of the resulting composite function f ◦ g. For example, if f(x) = x2 and g(x) = $\sqrt{x-1}$, then (f ◦ g)( x) = x – 1, but the domain of f ◦ g is x ≥ 1, since the domain of g is x ≥ 1. Several of the problems in Activity 1.5.2 have similar outcomes.

Activity 1.5.2 also asks students to find two functions f and g such that a given function F is equal to f ◦ g. Decomposing functions in this way anticipates problems that STEM-intending students may see later in precalculus and calculus (for example, problems where finding a derivative in calculus using the Chain Rule). Encourage students to think of one function as being “inside” another function when attempting to find a suitable decomposition. It is important for students to understand that such a decomposition is not unique, that there is usually more than one correct way to decompose functions. For example, one of the questions in Activity 1.5.2 asks students to find a decomposition for F(x) = $\frac{3}{\sqrt{7x-4}}$. Two possible correct solutions are f(x) = $\frac{3}{x}$, g(x) = $\sqrt{7x-4}$; and f(x) = $\frac{3}{\sqrt{x}}$, g(x) = 7x – 4. On the completion of this Activity, it would be beneficial to share with members of your class all the different correct answers from your students.

**Differentiated Instruction (Enrichment)**

Consider asking students to try to find as many ways to decompose the functions in Activity 1.5.2 as they can find. Also consider asking students if it would be possible to decompose a function into more than two functions. If they think this would be possible, challenge them to find a decomposition of some of the functions in Activity 1.5.2 into three or more functions.

**Differentiated Instruction (Enrichment)**

One of the questions in Activity 1.5.2 asks students to decompose the function $F\left(x\right)=\frac{x^{3}}{x^{3}+6}$. It is a common mistake to decompose this by saying that $f\left(x\right)=x^{3}$ and $g\left(x\right)=x^{3}+6$. However, this decomposes the function into $\frac{f(x)}{g(x)}$, not $f(g\left(x\right))$. Can it ever be true that $\frac{f(x)}{g(x)}=f(g\left(x\right))$ for two functions f and g? Why or why not?

Finally, Activity 1.5.2 has one pair of functions (question #2) that are inverses of each other. After students complete this Activity, ask students if they observed anything interesting about the solutions they found to the Activity. If no one volunteers it, be sure to point out that most of the time, f ◦ g and g ◦ f are two different functions, but for question #2 they were the same; in fact, both (f ◦ g)(x) and (g ◦ f)(x) are equal to x itself. Ask students to think about what happens with these two functions: we start with a value of x, apply one function to x, then apply the other function to the result, and we get back to x itself. Such pairs of functions will play an important role in Investigation 6 (Inverse Functions).

Students can complete **Exit Slip 1.5** after completing Activity 1.5.2.

**Activity 1.5.3 Functions Inside Functions** provides a number of situations that can be modeled using composite functions, including percentage discounts and markups, unit conversions, and currency conversions. Students can complete **Journal Prompt 1** after completing Activity 1.5.3.

**Journal** **Prompt 1** Not all students are asked to complete Investigation 5 of Algebra 2, the one you are working on now. Suppose you have a friend who is taking Algebra 2 but they skip this Investigation on composite functions. How would you explain the main ideas of composite functions to your friend? Be sure to give examples using tables and graphs, and give an example of at least one realistic problem, different from the examples studied in the Activities, where you would use composite functions.

[Possible partial response: Suppose we have two functions f(x) and g(x). When you make the composite function (f ◦ g)(x), the output of one function becomes the input to the other function. For example, if f(x) = 2x + 4 and g(x) = x2 – 1, then (f ◦ g)(x) = f(g(x)) = f(x2 – 1) = 2(x2 – 1) + 4 = 2x2 + 2. You can find the graph or a table of values of (f ◦ g)(x) by remembering that the output of g becomes the input for f.]

**Closure Notes**

There are many realistic problems that can be modeled by composite functions, where the output of one function becomes the input of another function, and this is one of the main reasons to study them in Algebra 2. Composite functions are also immediately useful in the next Investigation in Unit 1, since students who have completed Investigation 5 will use composite functions to verify that a given pair of functions are inverses of each other. In addition, composite functions are used in a number of ways in more advanced areas of mathematics. For example, you have just studied functions of the form f ◦ f. What happens if you form the function f ◦ f ◦ f, or f ◦ f ◦ f ◦ f, or … ? This is called *iterating* the function f, and is the idea behind the generation of fractal images that you might have studied in Geometry. Consider showing the class some fractal images (a good places to start is with Google images, [www.google.com](http://www.google.com)). All of the images shown rely in some way on the idea of iteration, which is one of the more important tools used by mathematicians.

**Vocabulary**

Composite functions

Coordinate axes/coordinate plane

Decomposing a function

Domain

Family of functions

Function Growth

Input

Output

**Resources and Materials**

**All STEM-intending students should complete all Activities in this Investigation.**

Activity 1.5.1 Composing Composite Functions

Activity 1.5.2 Domains and Graphs of Composite FUNctions

Activity 1.5.3 Functions Inside Functions

Graphing calculator/computer software with a graphing utility for all activities

Graph paper for all activities

Colored index cards/slips of paper for Launch

Colored pens/pencils to be able to “color code” graphs

<http://auto.howstuffworks.com/under-the-hood/auto-manufacturing/5-materials-used-in-auto-manufacturing.htm>

<http://visual.ly/materials-used-make-car-0>

<http://en.wikipedia.org/wiki/>

[www.google.com](http://www.google.com)