**Unit 5: Dimension and Measurement (4-5 Weeks)**

**UNIT PLAN**

***This unit incorporates material from Units 1, 3, and 6 of the Connecticut Core Geometry Curriculum***

This unit introduces geometric concepts for the first time in the Integrated Mathematics sequence. The concept of dimension is the key, unifying theme as students move from the study of one-dimensional objects (in Investigation 1) to two dimensions Investigation 2 and to three dimensions in the remainder of the unit. The final investigation Size and Shape in the Real World gives students the opportunity to apply what they have learned to make a scale model with a 3-D printer.

**Essential Questions**

* How is distance measured between two points in the coordinate plane?
* How are formulas for areas of different plane figures related to each other?
* How can nets help us find the surface are of a three-dimensional figure?
* How can we find the volume of three dimensional figures?
* How can we use these formulas to solve problems we might face in the real world?

**Enduring Understandings**

The squares and cubes of linear measurements are used to measure areas of two dimensional figures and the volumes of three dimensional figures.

**Unit Contents**

**Investigation 1 The Pythagorean Theorem and the Distance Formula (3 days)**  presents an informal proof of the Pythagorean Theorem which is then used to proved the Distance Formula. A formal proof of the Pythagorean Theorem appears in Integrated Mathematics Unit 19. ***Use Geometry Unit 1 Investigation 1.***

**Investigation 2: Areas of Plane Figures (2 days)** reviews material original introduced in the middle grades. The difference between area and perimeter is emphasized. Students then explore the rationale behind area formulas for rectangles, parallelograms, triangle, trapezoids, kites, and circles. These will be needed in Investigation 4 when students find the surface area of polyhedra.

***Use the Overview and Activities written for this investigation.***

**Investigation 3- Polygons and Polyhedra (3 days):** Students classify solids by their properties. They identify solids as prisms, pyramids, regular and semi-regular polyhedra as well as cones, cylinders and spheres. This activity develops terminology such as: polyhedron, faces, edges, vertices, convex, non-convex, regular, semi-regular, irregular. Students use isometric dot paper to create perspective drawings of polyhedra. Students prove informally that there are exactly five Platonic solids and they begin to explore semi-regular polyhedra Students investigate the relationship of faces, edges and vertices and discover Euler’s formula in the case of convex polyhedra. They make real world connections through viewing illustrations of crystal formations and dice from the Ptolemaic dynasty. ***Use Geometry Unit 6 Investigation 1.***

In **Investigation 4 – Nets and Surface Area (2 days) s**tudents build models by creating nets for specific solids. They determine if a plane configuration of polygons is a net of a polyhedron. Students apply the formulas for area of plane figures (learned in earlier grades and proved earlier in this course) to compute surface areas of prisms, pyramids, cylinders and cones. Students relate the real world applications of surface area to problems of packaging products. ***Use Geometry Unit 6 Investigation 2.***

**Mid Unit Test**

In **Investigations 5 – Volume (3 days) s**tudents do informal explorations using sand, water or beans to fill shapes with measuring cups and compare and contrast similar dimensional shapes with their volume. They explore Cavalieri’s Principle informally by stacking coins, paper etc. They explore the relationship between prisms and pyramids that have common bases and heights. They extend the work with prisms and pyramids to cones and cylinders. The final activity introduces frustums of cones and pyramids. ***Use Geometry Unit 6 Investigation 3.***

**Performance Task (see below)**

In **Investigation 6 - Cross Sections and Solids of Rotation (2 days)** Students use modeling clay and dental floss to explore cross sections of various solid figures. They use geometry software to explore solids formed by rotating triangles, rectangles, isosceles trapezoids, and semicircles. They are introduced to three-dimensional rectangular coordinates via geometric software and use the software to generate solids of revolution. ***Use Geometry Unit 6 Investigation 4.***

**Investigation 7 Size and Shape in the Real World (2-3 days):** Students explore a real world problem of making a three-dimensional scale model. Reference is made throughout the investigation to the application of a 3-D printer. Students calculate volume of a shape and its weight given a 10% fill. In the final, optional, activity students design their own object. This may be used as a mini-performance task. ***Use Geometry Unit 6 Investigation 7.***

**End-of-Unit Test**

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

**1. Make sense of problems and persevere in solving them.**

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

**4.** Model with mathematics.

5. Use appropriate tools strategically**.**

**6. Attend to precision.**

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Standards Overview**

Use Coordinates to Prove Simple Geometric Theorems Algebraically

Explain Volume Formulas and Use Them to Solve Problems

Visualize Relations between Two-Dimensional and Three Dimensional Objects

Apply Geometric Concepts in Modeling Situations

**Content Standards**

**6 G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context solving real-world and mathematical problems.

**6 G.4**Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

**7 G.1** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**7 G. 6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

**8.G.6**. Explain a proof of the Pythagorean Theorem...

**8.G.7.** Apply the Pythagorean Theorem to determine the unknown side lengths in right triangles in real-world mathematical problems in two and three dimensions.

**8.G.8**. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**8 G.9** Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.

**G-GPE.7.** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

**G-GMD.1**. Give an informal argument for the formulas for the circumference of a circle, area of a circle, ...

**G-GMD.3.** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

**G-GMD.4**. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects

**G-MG.1**. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

**G-MG.2.** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

**G-MG.3.** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios

**Assessment Strategies**

**Performance Task: Building a Mobile** provides an opportunity for students to use the skills and concepts from Investigations 2–6 to solve a concrete task.Students work in groups to choose five different solid figures and make a mobile of figures sized to have the same surface area. The volumes of the figures are calculated and another mobile is constructed so that the figures, when filled with a uniform substance, can balance using the volumes of the shapes.
This task may be assigned at any time following Investigation 5. ***Use the Mid Unit Performance Task from Geometry Unit 6.***

**Other Evidence (Formative and Summative Assessments)**

**•** Exit slips

• Class work

• Homework assignments

• Math journals

• Mid-unit test (after Investigation 4)

• End-of-unit test

**Vocabulary**

altitude

Archimedean solid

area

axis of rotation

base
CAD (computer aided design)

circle

circumference

circumscribed (prism or pyramid)

concave

cone

convex

cross-section

cube

cylinder

diameter
density

distance formula

dodecahedron

edge (of polyhedron)

Euler’s formula

face (of polyhedron)

frustum (of cone or pyramid)

height (of prism, pyramid, cylinder or cone)

hexahedron

kite

inscribed (prism or pyramid)

icosahedron

isometric drawing

lateral surface

midpoint

midpoint formula

net

non-convex

octahedron

parallelogram

pi

Platonic solid

polyhedron

prism

projection

pyramid

PythagoreanTheorem

radius

rectangle

regular polyhedron

right triangle

rhombus

semi-regular polyhedron

### Schläfli symbols

Schlegel diagram

semi-regular polyhedron

slant height

solids generated by rotations of plane figures

sphere

surface area

surface of revolution

tetrahedron

three-dimensional figure

trapezoid

triangle

vertex (of polyhedron)

volume

**Resources**

Flores, Alfinio. Area Formulas with Hinged Figures. In *Understanding Geometry for a Changing World*, NCTM, 2009.