**Unit 11: Quadratic Functions and Relations**

**UNIT PLAN**

***This unit incorporates material from Unit 2 of the Connecticut Core Algebra 2 Curriculum and Unit 5 of the Connecticut Core Geometry Curriculum***

In IM Unit 10 students were introduced to quadratic functions and quadratic equations. They learned to write quadratic functions in standard, vertex, and factored form and to solve quadratic equations in one variable by a variety of methods. This unit builds on the foundation laid in IM Unit 10. Students study complex numbers and learn to solve quadratic equations with complex solutions. They learn the fundamental theorem of algebra and solve equations containing radicals. They model real world data that can be fitted with a quadratic model. They then look at quadratic relations in two-variables (not all of which are functions), i.e. the conic sections. The locus approach is used to define and derive equations for circles, parabolas, ellipses, and hyperbolas.

**Investigation 1 Complex Numbers (Algebra 2 Unit 2 Investigation 3)** fosters deep mathematical understanding of complex numbers. It develops the complex numbers as a mathematical structure that retains the field properties of the real numbers but also brings closure to the operation of taking the square root of a negative number. Activities in this investigation develop the definition of the imaginary number *i*; the powers of *i*; the basic operations of addition, subtraction, and multiplication of complex numbers; and the solution of quadratic equations that have complex solutions in the form of complex conjugates, $a+bi$ and $a-bi$.

**Investigation 2** **Fundamental Theorem of Algebra** (**Algebra 2 Unit 2 Investigation 4)** extends the findings about the connection between the zeros of a quadratic function and its factors to illustrate that the Fundamental Theorem of Algebra is true for quadratic functions and lays the foundation for the statement of the theorem for all polynomials. In order to do this, students will verify that the roots of the quadratic equation $x^{2}+bx+c=0$ satisfy the relationship that the sum of the roots equals the opposite of *b,* and the product of the roots equals *c,* for complex roots as well as real roots. This property, along with the fact that complex roots of quadratic equations come in complex pairs will enable students to create quadratic functions given two real roots or and a third point on the graph or one complex root and one point on the graph.

**Investigation 3** **Modeling with Quadratic Functions (Algebra 2 Unit 2 Investigation 5)** provides students opportunities to model real-world phenomena using quadratic functions. Students are presented with a variety of tasks (open-ended, prescriptive and non-prescriptive) and algebraically develop quadratic functions and use quadratic regression to model data to explore contextual situations. Student complete activities that enable them to completely experience multiple steps in the mathematical modeling process: understanding the real-world problem, identifying model assumptions, developing a mathematical model, applying the model, and interpreting and validating the mathematical results. The activities in this investigation center on real-life applications and involve the use of graphing and statistical technology.

**Investigation 4** **Radical Equations** (**Algebra 2 Unit 2 Investigation 6)** introduces students to real-life applications that can be described by radical functions and radical equations. Students explore radical functions using a variety of representations, solve a variety of radical equations, explore the concept of extraneous solutions, and examine contextual scenarios that can be described by square root functions.

In **Investigation 5** **Circles in the Coordinate Plane** **(Geometry Unit 5 Investigation 1)** the circle is defined as the locus of points that are at a given distance to a given point. From the locus definition the equation of a circle in the coordinate plane with center (*h*, *k*) and radius *r* is derived. We also observe that all circles with equation $(x-h)^{2}+(y-k)^{2}=r^{2} $may be considered images of the unit circle under translation and dilation. Also some equations of the form $x^{2}+ax+$ $y^{2}+by+c=0$ may be rewritten in the above form by completing the square. Once the square is completed, as long as the right side is greater than 0, the equation represents a circle.

**Investigation 6** **Parabolas (Geometry Unit 5 Investigation 7)** introduces the locus definition of parabola and uses it to find the equation of a parabola with focus at (0, *p*) and directrix y = –*p*. Students will see the connection to the parabola as the shape of the graph of a quadratic function.

**Investigation 7 (+) Ellipses and Hyperbolas (Geometry Unit 5 Investigation 8)** extends the study of conic sections to ellipses and hyperbolas. The Common Core Standards include this for STEM intending students, so this is an optional investigation.

 **Essential Questions**

* How can I determine the nature of the roots of a quadratic equation based on the value of the discriminant?
* What is the definition of the imaginary number *i*?
* What is the structure of the complex numbers?
* What is the Fundamental Theorem of Algebra, and what is its relation to quadratic functions?
* How do I use quadratic functions as mathematical models?
* How do I solve radical equations?
* How is the locus concept used to define the conic sections?

**Enduring Understandings**

* Quadratic functions can be used to model real world relationships and the key points in quadratic functions have meaning in the real world context.
* Dynamic software, graphing calculators, and other technology can be used to explore and deepen our understanding of mathematics.

**Unit Understandings**

* Transform quadratic functions through vertical shifts, horizontal shifts and stretches relate these transformations to the standard and vertex forms of a quadratic.
* Solve quadratic equations by graphing, factoring, completing the square, and the quadratic formula.
* Add, subtract, and multiply complex numbers.
* Use the discriminant to determine the nature of the roots of a quadratic equation.
* Solve radical equations and identify extraneous solutions.
* Model real-world situations with quadratic functions and use the functions to solve problems.

**Unit Contents**

Investigation 1: Complex Numbers (3 days)

Investigation 2: Fundamental Theorem of Algebra (3-4 days)

Investigation 3: Modeling with Quadratic Functions (3 days)

Mid-unit test (1 day)

Investigation 4: Radical Equations (3 days)

Investigation 5: Circles in the Coordinate Plane (2 days)

Investigation 6: Parabolas (3 days)

Investigation 7 (+): Ellipses and Hyperbolas (3-4 daysP

Performance Task: (1 day)

Review for End-of-Unit Test (1 day)

End-of-Unit Test (1 day)

Teacher Testing Note: At your discretion you may provide the quadratic formula to some or all students on some or all assessents.

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. **Construct viable arguments and critique the reasoning of others.**

4. **Model with mathematics.**

5. Use appropriate tools strategically.

6. Attend to precision.

7. **Look for and make use of structure**.

8. Look for and express regularity in repeated reasoning.

**Common Core State Standards**

A.SSE.3 Choose and produce an equivalent from of an expression to reveal and explain

properties of the quantity represented by the expression.

A.SSE.3a Factor a quadratic expression to reveal zeros of the function it defines.

A.SSE.3b Complete the square in a quadratic expression to reveal the maximum/minimum

value of the function it defines.

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples

showing how extraneous solutions may arise.

A.REI.4 Solve quadratic equations in one variable.

A.REI.4b Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots,

completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a\pm bi$ for real numbers *a* and *b*.

BF.A.1 Write a function that describes a relationship between two quantities.

CED.A.1 Create equations and inequalities in one variable and use them to solve problems.

*Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.

CED.A.2 Create equations in two or more variables to represent relationships between

quantities; graph equations on coordinate axes with labels and scales.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key

features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by

hand in simple cases and using technology for more complicated cases.

F.IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step

functions and absolute value functions**.**

N.CN.1 Know there is a complex number *i* such that $i^{2}=\sqrt{-1}$, and every complex

number has the form *a* + *bi* with *a* and *b* real.

N.CN.2 Use the relation i2 = -1 and the commutative, associative, and distributive

properties to add, subtract, and multiply complex numbers.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic

polynomials. (Note that only functions with real coefficients are considered in this investigation.)

N.RN.3 Explain why the sum or product of two rational numbers is rational and the sum of a rational and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**Assessment Strategies:**

**Performance Task**

In the Algebra 2 Unit 2 Performance Task: Modeling Basketball Shots, students develop and explore linear and quadratic models of the trajectories of basketball shots. Students are prompted to develop six models for shots taken by six different players at various positions on a court.  Students are provided information about each shot and a framework for developing models. It is the student’s task to use the information to model the shot, determine characteristics about the ball’s movement, and in some cases, determine whether the shot was successful.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* Unit 11 Mid-unit test
* Unit 11 End-of-unit test

**Vocabulary**

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| Absolute value |
| Absolute value |
| Axis of symmetry |
| Binomial |
| Binomial expansion |
| Closed setsClosure of sets under an operation |
| Completing the square |
| Complex Conjugate |
| Complex Number |
| Concave up/down |
| Cost Function |
| Decreasing function |
| Distributive property |
| End behavior |
| Extraneous root |
| Extraneous solution |
| Factor |
| Factor a quadratic over the integers |
| Family of functions |
| Imaginary Number |
| Increasing function |
| Infinity |
| Inside change |
| Mathematical model |
| Mathematical modeling |
| Modeling diagram |
| Monomial |
| Outside change |

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| Parabola |
| Parabolic |
| Parametric equations |
| Parent function |
| Piecewise defined function |
| Profit function |
| Quadratic formula |
| Quadratic Function |
| Quadratic regression |
| Radical equation |
| Radicand |
| Revenue function |
| Root |
| Sets of numbers {Natural numbers (counting numbers), Whole numbers, Integers, Rationals, Irrationals, Real numbers, Complex numbers) |
| Slope |
| Solution |
| Square root equation |
| Standard form of a quadratic function |
| Transformation |
| Vertex |
| Vertex |
| Vertex form of a quadratic function |
| *x*-intercept |
| *y*-intercept |
| Zeros of a function |

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