**Unit 8: Investigation 4 (2 Days)**

**The History of Pi**

**Common Core State Standards (extended)**

* 7-G4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**Overview**

In ancient times people had many conjectures about the value of the ratio of the circumference of a circle to its diameter, a ratio they observed to be independent of the size of the circle. Today we call this constant value π (pi, the Greek letter that begins perimeter, the meaning of circumference for a circle). We take for granted approximate values of π such as 3.14159 but people in the past worked very hard to establish their accuracy. Today we have a number of methods and procedures to calculate π with any degree of precision limited only by the computing capabilities of the computers and the efficiency of the algorithms. Since 1761 it has been known that π is an irrational number, so no computer can give us an exact value for π.

If we use the letters *A*, *c*, *d*, and *r* to represent the area, circumference, diameter, and radius of a circle respectively then π represents the constant ratio $\frac{c}{d}$. It is remarkable that the same number π is the constant of proportionality between the area of circle and the area of a square of side *r*, a fact that we represent by $A=πr^{2}$. This was established when Archimedes proved that the area of a circle is equal to the area of a right triangle with legs equal to the radius and circumference of the circle.

In this investigation students will explore the history of π, find how close early estimates were, and use inscribed and circumscribed regular polygons, as did Archimdedes, to obtain estimates of π. Implicit is the notion of convergence: as the number of sides of a regular polygon, inscribed in or circumscribed about a unit circle, increases without bound, the perimeter converges to 2π units and the area to π square units.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Rewrite expressions involving ancient estimates of π.
* Recognize that inscribed polygons give under-estimates for the area of a circle and circumscribed polygons give over-estimates for the area of a circle. Together they “sandwich” an interval where π must fall.
* Use trigonometry to find areas of regular polygons.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 8.2** asks students to analyze area relationships among inscribed and circumscribed regular polygons.
* **Journal Entry** asks students to explain Archimedes’ overall method.

**Launch Notes**

Ask students why π is special. Do they just know its estimated value? Ask them how we use it. Do they have any idea where it comes from? In this Investigation they will learn something about its history and some of the work required to find its value.

**Teaching Strategies**

**Activity 8.4.1 Historical Values for Pi** introduces students to estimates of pi from ancient cultures. They are asked to rewrite some formulas and to find percent errors for some values.

**Activity 8.4.2 Areas of a Circle from Regular Polygons** guides students to generate formulas for the area of a regular *n-*gon. Students are introduced to the Archimedes’ idea of estimating the area of a circle by sandwiching it between areas of inscribed and circumscribed regular *n*-gons. The students use trigonometry, a tool not available to Archimedes.

Take time with students to be sure they see what is meant by polygons inscribed within a circle and circumscribed around a circle. It may help to ask them to use the roots *in-* (inside) and *circum*- (around). [Magellan was the first to circumnavigate the earth.]

Take time, too, to have students share their reasoning for *A* =$\frac{1}{2}$ *ap,* the area of a regular polygon in terms of its perimeter and apothem.

The trigonometry will also require work. What makes these problems accessible is using a circle with a radius of 1 to simplify the problem.

**Differentiated Instruction (For Learners Needing More Help):**

1. Provide enlarged versions of the inscribed and circumscribed polygons. Let students measure angles and then say how they could have predicted those angles. If necessary have them revisit Activity 3.4.2 where they earlier derived the angles in regular polygons.
2. Have students review finding the sine, cosine, and tangent of an acute angle in a right triangle (Activity 4.6.1).

**Group Activity:** Have students work in groups to complete the table in question 11. Have them re-explain to one another how they arrived at the formulas.

Following **Activity 8.4.2** you may give **Exit Slip 8.4.**

**Activity 8.4.3** **Archimedes’ Method** guides students through a few cases of finding perimeters and areas of inscribed and circumscribed regular polygons. The goal is to give students the flavor of successive approximation by continuing to double the number of sides of a regular polygon inscribed or circumscribed around a circle of radius 1.

You may want to point out that Archimedes did not have decimal notation and did all of his work with fractions.

Students need to recognize that they can use the Pythagorean Theorem repeatedly. They also need to take maximum advantage of everything they had coming into the problem: knowing the radius is one, knowing the previous side length, and knowing where there are perpendiculars. After computing the perimeter of the inscribed regular 24-gon, students may use technology (TI-84) to extend the doubling process to 48 sides, 96 sides, and beyond. Students without programming experience may still copy the program and use it to carry out this exploration.

Point out that although the calculator gives us a much more precise approximation of pi than Archimedes was able to find, its precision is still limited. For hundreds of digits visit <http://www.wolframalpha.com/input/?i=pi>.

**Journal Entry** What was the major idea Archimedes used to get better and better approximations or pi? Look for students to explain the importance of doubling the number of sides to get a more accurate estimate of the area or circumference of the circle.

**Activity 8.4.4** **Extensions for Pi** provides a number of questions that students may be interested in exploring either independently or with partners.

**Closure Notes**

Close the investigation by reminding students that the decimal expansion of pi is infinite and non-repeating. The results obtained in this investigation, including those using the calculator are only approximations of the exact value of pi.

**Vocabulary**

Apothem

circumscribe

inscribe

**Resources and Materials**

Activity 8.4.1 Historical Values for Pi
Activity 8.4.2 Area of a Circle from Regular Polygons
Activity 8.4.3 Archimedes’ Method
Activity 8.4.4 Extensions for Pi
Exit Slip 8.4
TI-84 calculator for Activity 8.4.3 (optional)

Bruni, James. (1977). *Experiencing Geometry*. Belmont, CA: Wadsworth Publishing, Co. [Based on work of Emma Castelnuovo.]

Bunt, Lucas N. H., Phillip S. Jones, and Jack D. Bedient. (1976). *The Historical Roots of Elementary Mathematics.* New York: Dover Publications.

Dunham, William. (1990). *Journey through Genius: The Great Theorems of Mathematics.* New York: Penguin Books.

Gillings, Richard J. (1972). *Mathematics in the Time of the Pharaohs*. New York: Dover Publications.

Jacobs, Harold R. (1974). *Geometry*. San Francisco: W. H. Freeman and Co.

Joseph, George Gheverghese. (1991) *The Crest of the Peacock: Non-European Roots of Mathematics.* Hardmondsworth, England: Penguin Books.

Kasner, Edward & James Newman. (1940). *Mathematics and the Imagination.* New York: Simon and Schuster.

National Council of Teachers of Mathematics. (1969). *Historical Topics for the Mathematics Classroom.*Thirty-first Yearbook of the National Council of Teachers of Mathematics. Washington, D. C.: NCTM.

Rubenstein, R. (1998). Historical Algorithms: Sources for Student Projects. In Morrow, Lorna J. and Margaret J. Kenney (Eds.) *The Teaching and Learning of Algorithms in School Mathematics*  (1998 yearbook). Reston, Virginia: National Council of Teachers of Mathematics.

Stein, Sherman (1999). Archimedes Traps Pi. In *Archimedes: What Did He Do Besides Cry Eureka.* Washington, D.C.: Mathematical Association of America.

[http://en.wikipedia.org/wiki/File:Pi-unrolled-720.gif](http://en.wikipedia.org/wiki/File%3APi-unrolled-720.gif)

(Just shows circle rolling on number line to yield pi.)

<http://illuminations.nctm.org/Lesson.aspx?id=2477>

(This was basis for Activity 8.4.2 – If Archimedes knew trig)

<http://bulldog2.redlands.edu/fac/beery/math115/m115_activ_est_pi.htm>

(This was basis for Activity 8.4.3 – Archimedes’ doubling numbers of sides.)

<http://www.math.illinois.edu/igl/> (Illinois Geometry Lab ).