**Unit 8: Investigation 2 (2 Days)**

***Flatland* and the Fourth Dimension**

**Common Core State Standards (extended)**

* G-GPE 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
* G-GMD 4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

**Overview**

Students will explore the concept of dimension, first by reading the book or seeing the movie *Flatland*, and then extending their understanding to the fourth dimension and beyond.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Describe what life would be like if we lived in a space of only two dimensions.
* Explain how segments, squares, cubes, and hypercubes are related.
* Extend by analogy our knowledge of three-dimensional objects to four dimensions.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 8.2** asks students to find the number of vertices, edges, faces, and cubical spaces in a four-dimensional hypercube.
* **Journal Entry** asks students to explain why it is difficult to imagine a fourth dimension.

**Launch Notes**

Begin by showing all or part of the movie Flatland. (Running time: 36 minutes) At this point or later in the investigation you may also want to show the segment in which Prof. Thomas Banchoff discusses the fourth dimension. The book *Flatland* written by Edwin Abbott in 1884 is freely available on Prof. Banchoff’s web site: <http://www.geom.uiuc.edu/~banchoff/Flatland/>. Before assigning the book you may want to discuss the social context in which it was written and that besides introducing mathematical ideas, Abbott was commenting on the class structure and role of women in England during the Victorian era. You do not need to assign the entire book. The main ideas are presented in sections 1-4 and 11-17.

**Teaching Strategies**

**Activity 8.2.1 Exploring *Flatland*** has students respond to questions about the book and/or movie Flatland.

**Differentiated Instruction (For Learners Needing More Help):** Encourage students to act out scenes from the book to get a fuller understanding of what the author is describing. For example, in Section 1 the author writes,

“Place a penny on one of your tables in Space; and leaning over it, look down upon it. It will appear a circle. But now, drawing back to the edge of the table, gradually lower your eye (thus bringing yourself more and more into the condition of the inhabitants of Flatland), and you will find the penny becoming more and more oval to your view; and at last when you have placed your eye exactly on the edge of the table (so that you are, as it were, actually a Flatlander) the penny will then have ceased to appear oval at all, and will have become, so far as you can see, a straight line.”

Similarly with the explanation in Section 4 about how a needle, representing a woman of Flatland, can become “practically invisible.”

Finally you may want to blow up a balloon to demonstrate as Thomas Banchoff does in his video how a four-dimensional hypersphere would appear to us as she enters and leaves our space.

In **Activity 8.2.2 Moving to the Next Dimension** students explore one of the key ideas in *Flatland:*  how a figure in a higher dimension is generated from a figure in a lower dimension. Thus a zero-dimension point moves to form a one-dimensional line segment. The segment moves in another direction to form a two-dimensional square. The square moves through another dimension to form a three-dimensional cube, which in turn moves through yet another dimension (which we are unable to perceive) to form a four-dimensional hypercube.

In **Activity 8.2.3** **Properties of a Hypercube** students discover how many vertices, edges, faces, and spaces are in a hypercube and look at ways to represent a hypercube in two or three dimensions.

**Group Activity:** Have students work in groups to construct a three-dimensional representation of a four-dimensional hypercube. There are a variety of materials they may use. See suggestions under the list of resources.

**Exit Slip 8.2** may be given after **Activity 8.2.3.**

**Activity 8.2.4** **Coordinates in Higher Dimensions** extends what students already know about the coordinate plane and three-dimensional coordinates (introduced in Unit 6 Investigation 5) to the fourth dimension. While visualizing the fourth dimension is difficult for us, extending mathematical representations into the fourth dimension is more straightforward. Thus we can represent points in four-dimensional hyperspace, compute distances between points and find equations of hyperspheres and hyperplanes based on analogies with their lower dimensional counterparts.

**Differentiated Instruction (Enrichment):** A natural extension to this investigation is to consider five-dimensional space and beyond. Ask students to extend their findings from Activities 2, 3, and 4 to higher dimensions. Ask them to generalize to *n* dimensions.

**Activity 8.2.5** **Topics for Independent Investigation** raises a number of questions that students may be interested in exploring. The first few may be assigned to the entire class. Items 3–8 are suitable for independent investigation depending upon student interest.

**Journal Entry:** Why is it difficult for inhabitants of a three-dimensional space, such as ourselves, to comprehend a fourth dimension? Look for students to use the difficulty Flatlanders had in comprehending a third dimension in their explanations.

**Closure Notes**

Close the investigation by having students who have worked on Activity 5 present their findings.

**Vocabulary**

dimension

hypercube

hyperplane

hypersphere

hyperspace

**Resources and Materials**

Activity 8.2.1 Exploring Flatland

Activity 8.2.2 Moving to the Next Dimension

Activity 8.2.3 Properties of a Hypercube

Activity 8.2.4 Coordinates in Higher Dimensions

Activity 8.2.5 Topics for Independent Investigation

Exit Slip 8.2

<http://flatlandthemovie.com/>

<http://www.geom.uiuc.edu/~banchoff/Flatland/> for a free download of the book

For building hypercubes, a good commercial product is D-Stix <http://www.enasco.com/product/TB19388T>. This is also helpful in constructing models of three-dimensional figures studied in Unit 6. For a “homemade” approach, consider using pipe cleaners or glue and popsicle sticks.