**Unit 8: Investigation 4 (3 - 4 Days)**

**Applications with** *2* × *2* **Matrices**

**Common Core State Standards**

N-VM 6Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM 8Add, subtract, and multiply matrices of appropriate dimensions.

N-VM 9Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM 12Work with 2 X 2 matrices as a transformations of the plane.

**Overview**

This investigation reviews solving a system of two linear equations in two variables and then uses the inverse and the determinant of *2* × *2* matrices to solve systems of two linear equations arising in context. It also addresses transformations of vectors by a matrix.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Students will be able to convert a system of two linear equations to a coefficient matrix multiplied by a variable matrix to create a matrix equation.
* Students will find the general formula for the inverse of a 2 × 2 matrix algebraically
* Students will determine the characteristics of 2 × 2 matrices that determine if its inverse is defined.
* Students will find the inverse of the coefficient matrix and solve the system of equations using matrix operations, interpreting their answers in terms of the context when solving a contextual problem.
* Students will interpret systems without solutions as they relate to the determinant of the coefficient matrix.
* Students will work with 2 × 2 matrices as transformations of the plane

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 8.4.1** assesses a student’s ability to develop contextual systems of equations, write them in standard form, and then convert the system to a matrix equation.
* **Exit Slip 8.4.2** assesses a student’s ability to find the inverse of a 2 × 2 matrix (if it exists), and solve contextual problems using matrix algebra.
* **Exit Slip 8.4.3** assesses a student’s ability to use technology to find the inverse of a matrix when the matrix involves scientific notation and the results are challenging physical phenomena.
* **Journal Prompt 1** Look up Seki Kowa whohas been called the Japanese Newton. He investigated determinants in 1683. Or look up Gottfried Leibniz or Theophile Vandermonde and their role in the investigation of determinants.
* **Activity 8.4.1** **Solving Systems of Equations with Matrix Algebra** hasstudents solve a system of equations developed in a context by any method. They are then asked to put the equations in standard form and note how a matrix of coefficients can simply all the writing and how the product of the matrix of coefficients with a vector can express the left side of the system. They are then guided through a solution using the inverse matrix. The big question that still looms is, how do we find an inverse matrix?
* **Activity 8.4.2** **Finding the General Formula for the Inverse of a 2×2 Matrix** has students algebraically solve a general system of linear equations in two variables. Students are shown how to use the elimination method to solve for one of the variables in the inverse matrix. They then use the same strategy to solve for the remaining three variables. They use their result to state what condition will insure that a two by two matrix has an inverse.
* **Activity 8.4.3 More Contextual Applications with Matrix Algebra** informs students that the quantity has a specific name, the determinant of matrix and that the formula for the inverse matrix is provided that . Students use their new ability to find the inverse of a 2 by 2 matrix, assuming it exists, to solve systems of equations that result from contextual situations.
* **Activity 8.4.4 Transforming Vectors with Matrices** has students explore the effect of multiplying a vector by a matrix to give a new vector. They also discover that when a matrix has an inverse, multiplying the new vector by the inverse gives the original vector.

**Launch Notes**

Begin this investigation with an example of two linear equations written in standard form developed from a context. Activity 8.4.1 uses a movie context so if you want to keep the same context here is an example you could use. The cost of five theater tickets and four soft drinks is $53.50. The cost of three theater tickets and six soft drinks is $37.50 What is the cost of each theater ticket and soft drink?

Have students write the two equations in standard form but don’t solve the system.

We want to first convert the system into a matrix equation.

After writing the system in standard form have them make up a 2×2 matrix with the coefficients of the equations like this:

Now have them use matrix multiplication to multiply the following two matrices to see that the result of multiplying the matrices returns the original system.

**Teaching Strategies**

1. In **Activity 8.4.1**, students are asked to solve a familiar problem that uses two equations in two linear variables. Students may use substitution or elimination. But after they have solved it, if no one used elimination have them write the equations in standard form so they can see how matrix notation can be used as a shorthand notation for writing the system and the advantages of using the inverse matrix. Hopefully they will want to develop a method for finding the inverse of a matrix in addition to the button on their calculators. Activity 8.4.1 can then be distributed.

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| **Group Activity 8.4.1 Solving Systems of Equations with Matrix Algebra**  Students can be grouped in pairs or by threes. After the launch distribute this activity, and have students do numbers 1 – 5 in pairs or groups. Check to be sure that after solving algebraically all groups have their system of equations in standard form and give them the task of following the activity sheet until they obtain the matrix equation in number 8 with the solution:  In problem 9 they use technology to find that the inverse matrix for is  They can then proceed to solve the matrix equation. The procedure works as long as we can find the inverse of the coefficient matrix because and so .  Students can them complete the activity and note that the second problem posed shares the same coefficient matrix so it is easy to solve it as well. Return to your opener in the launch. Students should note the coefficient matrix is different so to solve it, they will have to find another inverse matrix. So you might try to guide the discussion to note that essential to this process is finding A-1. How do we or our calculators internally find the inverse? |

**Exit Slip 8.4.1** can be used. This exit slip can be used in one of two ways. For classes where you believe students are ready to translate a situation and solve the system, all questions can be used as the exit slip. For classes still struggling to take a context and determine the equations feel free to do the introduction in whole class mode and then assign the second part as the exit slip which only assesses the ability to take a system, write it as a matrix equation and given the inverse matrix solve the problem.

**Activity 8.4.2** **Finding the general formula for the inverse of a 2×2 matrix** if it has an inverse. This activity involves algebraically solving a general system of linear equations in two variables.

You might want to have a few 2 by 2 matrices and their inverses on a sheet for students to examine and see if students can determine any patterns between the entries in a matrix A and the entries in A-1, for example b and c now have their opposites in the inverse matrix and a and d are reversed. For students needing challenge, see the box below. For students needing support see the box below. Students are shown how to use the elimination method to solve for one of the variables in the inverse matrix. They then use the same strategy to solve for the remaining three variables. They use their result to state what condition will insure that a two by two matrix has an inverse.

**Differentiated Instruction (For Enrichment)** In activity 8.4.2 after *e* has been solved for in whole class mode have students try to solve for *g* on their own. Then have them see if they can find two equations that can be used to solve for *f* and *h*. They can then solve that system as individuals or in pairs and a few could present to the class.

**Differentiated Instruction (For students Needing Support)** In activity 8.4.2 have students seek patterns with many concrete examples of predetermined inverse matrices. After solving for *e*, have them return to the concrete examples and see if the factor can be found in each inverse matrix. Instead of solving the other 3 systems, you may want to now finds inverses using the new formula and then check via multiplication of matrices that the new matrix is indeed an inverse.

**Exit Slip 8.4.2 can be used.** This exit slip can be used in one of two ways. For classes where you believe students are ready to translate a situation and solve the system, all questions can be used as the exit slip. For classes still struggling to take a context and determine the equations feel free to do the entire introduction in whole class mode but hopefully students may be able to do some of the questions alone and then assign the second part as the exit slip which only assesses the ability to take a system, write it as a matrix equation and find the inverse matrix for a 2 by 2, and solve the problem.

In **Activity 8.4.3 More Contextual Applications with Matrix Algebra** students learn that the quantity has a specific name, the determinant of matrix and that the formula for the inverse matrix is provided that . Having a formula allows us to solve more difficult problems using technology especially those with values that are noninteger and possibly even complex. Contextual problems developed by students are more engaging to them and offer more classroom dialogue. To get started, it is easy to assign fractional or decimal values to a common setting, for example the theater ticket context. The activity contains a two series/parallel electrical circuit problems as well as a theatre problem. The activity can be assigned inclass or for homework and can be done in groups or as an individual assignment. You may certainly want to use other applications or encourage students to each make up a contextual problem and have students solve those instead.

Students may need to be reminded once more to:

* Write the system of equations and define the variables
* Write the matrix equation
* Find the inverse of the matrix
* Multiply the matrix equation by the inverse matrix

**Exit Slip 8.4.3** can be assigned.If you have examined other applications, feel free to replace with an application studied by your students.

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| **Journal Prompt 1**  Look up Seki Kowa whohas been called the Japanese Newton. He investigated determinants in 1683. Or look up Gottfried Leibniz or Theophile Vandermonde and their role in the investigation of determinants. You may want to split the class in thirds asking just for a paragraph from each student and then have them share their research.) |

**Activity 8.4.4** **Transforming Vectors with Matrices.**  In this activity, students explore the effect of multiplying a vector by a matrix to give a new vector. They also discover that when a matrix has an inverse, multiplying the new vector by the inverse gives the original vector.

In the last activity, students multiplied matrices by vectors to solve equations using the inverse of a matrix. They know that when a matrix multiplies a vector, the result is vector because when we multiply an *m* × *n* matrix by an *n* × 1 vector the result is a *m* × 1 vector.

If *m* and *n* are equal then we are multiplying a square *m* × *m* matrix by an *m* × 1 vector and this gives a vector of the same dimension. So in that case, we transform one vector to another by multiplying by a matrix.

**Closure Notes**

The investigation culminates with students successfully demonstrating their ability to compare and contrast entries in a two by two matrix with an effect on a vector that is reversible using the inverse matrix, when of course it exists. Transformations using matrix multiplication of a vector is a way to view the process of spatial transformations and demonstrates relationships between coordinate systems. It is also reversible if the matrix has an inverse.

**Vocabulary**

determinant

**Resources and Materials**

**All activities should be completed but all 4 entries of the inverse matrix in activity 8.4.2 do not have to be completed. All students should experience the solution of one entry.**

Activity 8.4.1 Solving systems of equations with matrix algebra

Activity 8.4.2 Finding the general formula for the inverse of a 2×2 matrix

Activity 8.4.3More Contextual Applications with Matrix Algebra

Activity 8.4.4Transforming Vectors with Matrices

* Bulletin board for key concepts
* Graphing Calculators
* Student Journals
* Projector
* Computers
* Rulers