**Unit 8: Investigation 3 (3 Days)**

**Applications Using Vectors and Matrices**

**Common Core State Standards:**

N.VM 9*:* Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N.VM 10*:* Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N.VM 11:Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

**Overview**

This investigation uses the inverse of a matrix as a backdrop for introducing the determinant of a matrix and for solving systems of two linear equations using matrices.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Given two matrices *A* and *B*, students will determine when the product *AB* is defined based on the number of rows and columns of *A* and *B*. If the product *AB* is defined, for example *A* is *m* × *n* and *B* is *n* × *p*, then students will find the correct product *m* × *p* matrix by hand and using technology.
* Students will find product matrices including the product of a matrix and a vector viewed as an *n* × *1* matrix.
* Given contextual situations, students will find product matrices and the context can be used to solve problems and understand solutions.
* Students will use these contextual situations to analyze why matrix multiplication is not in general commutative.
* Given a vector, students will express the vector using matrix notation and explain why a vector may be considered as an *n* × *1* matrix.
* Students will be able to multiply a matrix by a vector and determine contextual circumstances where the result makes physical sense.
* Given two matrices, students determine if they are inverses of each other.

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 8.3.1** hasstudents use the contextual data from Investigation 8.3.1 to perform matrix multiplication and to explain the meaning of the cell entries of the product matrix in the product that is defined and to explain why the other possible product ca not be performed.
* **Exit Slip 8.3.2** has students use data from a column of a matrix, realizing that it is just a vector. They will multiply a matrix by a vector and interpret the meaning of the entries in the resultant vector.
* **Exit Slip 8.3.3** has students multiply square matrices and verify whether or not they are inverses. This is done by hand and using technology.
* **Journal Prompt 1** Write a paragraph about how multiplying matrices can simplify the job of keeping a chain store properly stocked with merchandise.
* **Journal Prompt 2** Look back through your notes and see if you can find the “zero product principle”. If you can’t find it, see if you can find it online. Write a sentence explaining the principle in your own words. Since some matrices can be multiplied, do you think the principle applies to matrices? See if you can find an example where the principle is violated by matrices.
* **Journal Prompt 3** Do an internet search about matrix multiplication using technology. See if you can find a cell phone app that would make this easy.
* **Activity 8.3.1 Matrix Multiplication in Context** has students explore matrix multiplication in a context so the meaning of each entry will make sense and students can understand why the definition to find each cell entry in the product matrix is sensible.
* **Activity 8.3.2** **Matrix Identity Crisis** has students examine whether or not matrix multiplication is commutative not just be computing cell entries but also by examining the context of matrices AB and BA. Students also examine the effect of multiplying by the identity matrix and examine if AI = IA.
* **Activity 8.3.3** **A Vector Is a Matrix?** has students explore the concept of viewing a vector as a matrix.
* **Activity 8.3.4** **Am I Your Inverse?** has students determine if given 2 matrices are they inverses and in the 2 by 2 case how we can determine algebraically if a matrix has in inverse. Students must use technology at this point to find A-1 is a is 3 X3 or greater, opening the door to further study, how do we find inverse?
* **Activity 8.3.5 Matrices and Cable Television** asks students,” Have you even turned on your television and flipped to a channel you don’t pay for, hoping to see a cool movie or PayPerView event? How is it that the person next door to you is able to watch HBO but you are not? It is all based on matrices.” Students will see a use of a matrix and its inverse matrix. Students will use technology but hopefully will hopefully have a desire to see what is happening behind the technology.

**Launch Notes**

You can begin this investigation with a discussion that revisits the first week collection spreadsheet and matrix from investigation one:

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| --- | --- | --- | --- | --- |
|  | Clothing | Wool | Shoes-pairs | Rags |
| Pickup Site A | 92 | 36 | 14 | 88 |
| Pickup Site B | 46 | 87 | 34 | 73 |
| Pickup Site C | 57 | 44 | 37 | 44 |

The 3×4 matrix *A*:

The benefits of reusing items in dollars and equivalent carbon dioxide emission reduction are shown in this spreadsheet:

|  |  |  |
| --- | --- | --- |
|  | $ per unit | lb CO2 |
| Clothing (lb) | .67 | 5.9 |
| Wool (lb) | 1.20 | 14 |
| Shoes (pairs) | 2.50 | 30 |
| Rags (lb) | .50 | 1.7 |

which can be organized in this 4×2 benefit matrix:

The requirements for matrices *A* and *B* to havea defined product matrix *AB* needs to be reviewed and redefined. Using a contextual meaning for the entries in the product matrix *AB* will make the definition of how each entry is obtained meaningful. Students should continue to find some products manually before using technology.

**Teaching Strategies**

**Activity 8.3.1** begins by revisiting the context for collecting textile items for reuse in order to generate income, reuse valuable resources and reduce carbon dioxide emissions into the atmosphere. The collection’s matrix and the benefit’s matrix can be multiplied to find the results of collection by collection site. This review from activity 8.1.1 is important because it reviews the process of matrix multiplication, the requirements that must be met for matrices to be multiplied, and the meaning of the product matrix.

You can use a different context that involves matrices of other sizes to illustrate the three main concerns: Can two matrices be multiplied? What is the process for finding the product matrix? What is the meaning of the product matrix? Students should see another context to insure that they can answer all three questions.

You can then distribute the **activity 8.3.1**.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Group Activity 8.3.1** Matrix Multiplication in Context  Groups of two to four students can be assigned.  Use the common context of a builder’s supply matrix context as follows:  A building contractor constructs three different house models in three different  cities. The following example table shows the number of each model being built in each city:   |  |  |  |  | | --- | --- | --- | --- | |  | Colonial | Cape | Split Level | | Newington | 9 | 6 | 2 | | New Britain | 6 | 8 | 3 | | Southington | 8 | 9 | 4 |   Each model has a specific requirement for exterior windows and doors and bundles of siding as listed in this spreadsheet:   |  |  |  |  | | --- | --- | --- | --- | |  | Windows | Doors | Siding Bundles | | Colonial | 18 | 4 | 12 | | Cape | 16 | 3 | 12 | | Split Level | 14 | 2 | 10 |   In this activity, students will create and name matrices from the spreadsheets and discuss the three questions. Since both matrices will be *3* × *3*, this will lead to an informative discussion about multiplying them and the meaning of the product matrix. They should be encouraged to find some entries of the product matrix (which product?) by hand, and also verify their results with technology. Next, each group should use the context for the collections from the collection sites and the benefit realized from the collections. They should review the requirements for two matrices to be multiplied, including labeled spreadsheets and matrices. They should explain the meaning of the product matrix and find the product matrix with technology. |

**Journal Prompt 1** can be assigned.

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| **Journal Prompt 1**  Write a paragraph about how multiplying matrices can simplify the job of keeping a chain store properly stocked with merchandise. |

**Activity 8.3.2** can be assigned using groups or individually. It can be started in class and completed for homework.

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| **Group Activity 8.3.2** Matrix Identity Crisis  Groups of two to four students can be formed. The two example contexts from activity 8.3.1 form a backdrop. Each group should test each example to see if matrix multiplication is commutative. They should also examine the meaning of the matrix product and explain why the meaning would completely change if the order of multiplication is changed. They should also explain why the number of columns of the first matrix must match the number of rows of the second matrix in order to perform the operation of matrix multiplication.  Write or display the identity matrices *I2*, *I3*, and *I4*. Have students test if multiplication by the identity matrix is commutative and describe the results. |

**Exit Slip 8.3.1** can be assigned**.**

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| **Journal Prompt 2**  Look back through your notes and see if you can find the “zero product principle”. If you can’t find it, see if you can find it online. Write a sentence explaining the principle in your own words. Since some matrices can be multiplied, do you think the principle applies to matrices? See if you can find an example where the principle is violated by matrices. (Answer: doesn’t apply: Are there more examples? |

**Activity 8.3.3** **A Vector Is a Matrix?** can be done individually, in pairs or whole class. But before students work on the questions, review vectors and the salient features of a vector. This should include the various notations that are used to denote vector quantities and that ordered pairs can denote vectors in two-space. Equally important is to remind students that the components of a vector need not be distance or velocity or anything to do with motion. They just need to be different changing quantities like prices of stocks or the population of states or inventory of items in a department store.

Again use the conservation project context of collecting textiles for reuse, and refer to the first week collection matrix *A*:

Suppose we only wished to find the amount of CO2 reduction due to the items collected in the first week at each of the collection sites. Could we multiply matrix *A* by the following value matrix (which is just the second column of matrix *B*)? Review the three requirements.

Have the students try this with their calculators. What is the meaning of the result?

Ask if the matrix *H* could be considered a vector and how the matrix representation might be compatible with the other representations already learned.

**Exit Slip 8.3.2** can be assigned**. Journal prompt 3** can also be assigned.

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| **Journal Prompt 3**  Do an internet search about using technology to multiply matrices. See if you can find a cell phone app that would make this easy. |

**Activity 8.3.4** **Am I Your inverse?** explore the general idea of an inverse and more specifically what a multiplicative inverse of a matrix would mean. Students have already seen that matrix multiplication can only be done under certain conditions and that it is not generally a commutative operation. They also explored the situation of multiplying a square matrix by the identity matrix and saw it was commutative. They also saw in exit slip 8.1.4 that some matrices have inverses and that multiplying a matrix by its inverse turns out to be commutative. Inverse matrices are very useful as we will soon see, but how can we find out if a matrix has an inverse? How can we find it? It turns out not to be easy, but one thing we can already do. Check to see if a matrix *N* is the inverse of a matrix *M.*

**Exit Slip 8.3.3** can be assigned.

**Activity 8.3.5 Matrices and Cable Television** asks students,” Have you even turned on your television and flipped to a channel you don’t pay for, hoping to see a cool movie or PayPerView event? How is it that the person next door to you is able to watch HBO but you are not? It is all based on matrices.” Students will see a use of a matrix and its inverse matrix. This activity is an exciting way to end the investigation.

**Closure Notes**

After being reminded that this investigation continues to build an understanding of matrix multiplication using context so that students will see the meaning of the resultant product matrix and why matrix multiplication isn’t generally commutative, that vectors can be reframed as an *m* × *1* matrix, and that some matrices have inverses; students can use a matrix and its inverse in the context of Cable television. Hopefully, they will desire to be able to find out how one obtains an inverse matrix besides pressing a key on a calculator and will also want to discover other uses of matrices and their inverses.

**Vocabulary**

Inverse matrix

Identity matrix

Vector

Zero Product Principle

**Resources and Materials**

**All activity sheets should be completed.**

* Activity 8.3.1Matrix Multiplication in Context
* Activity 8.3.2 Matrix Identity Crisis
* Activity 8.3.3 A Vector is a Matrix?
* Activity 8.3.4 Am I Your Inverse?
* Activity 8**.**3.5 Matrices and Cable Television
* Carbon Footprint Analysis of Textile Reuse and Recycling
* US EPA Municipal Solid Waste Generation, Recycling, and Disposal in the United States: Facts and Figures for 2012
* <http://www.epa.gov/epawaste/nonhaz/municipal/index.htm>

[http://www.ct.gov/deep/cwp/view.asp?a=2714&q=324884&deepNav\_GID=1645](http://www.ct.gov/deep/cwp/view.asp?a=2714&q=324884&deepNav_GID=1645http://www.ct.gov/deep/cwp/view.asp?a=2714&q=324884&deepNav_GID=1645)

* <http://www.usctcgateway.gov/tool/>
* <http://www.popularmechanics.com/science/environment/recycling/4291576>

[education.ti.com/media/.../ti84plus\_guidebook\_en](file:///C:\Users\any\Documents\Geometry%20and%20Algebra%20II\Investigations\Investigation%201\education.ti.com\media\...\ti84plus_guidebook_en)

* Bulletin board for key concepts
* Graphing Calculators
* Student Journals
* Projector
* Computers
* Rulers