**Unit 8: Investigation 1 (3 - 4 Days)**

**Operations with Matrices**

**Common Core State Standards**

N-VM 6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM 7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

N-VM 8 Add, subtract, and multiply matrices of appropriate dimensions.

N-VM 9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM 10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.

The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

**Overview**

This investigation uses the context of a community engagement project to introduce matrix structure and operations with matrices. Students will demonstrate facility with the matrix operations of addition, subtraction, multiplication by a scalar and multiplication of a matrix by another matrix (where possible) in and out of context and with and without technology. There will also be a focus on promoting a deeper understanding of a commutative operation, an associate operation and a distributive property of one operation over another.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able To Do?**

* Given a table of contextual mathematical data, students will be able to choose an appropriate *m* × *n* matrix and organize the data into the matrix.
* Students will be able to perform addition of matrices and be able to multiply a matrix by a scalar.
* Students will use these operations with contextual problems.
* Students will use matrix addition to discover the zero matrix and will be able to find the additive inverse of a matrix.
* Students will then be able to subtract matrices by adding the additive inverse of a matrix and do this with contextual data.
* Students will know the requirements for two matrices *A* and *B* in order that the product matrix *AB* will be defined and be able to multiply two matrices.
* Students will perform matrix multiplication in context and understand the product matrix in context.
* Students will discover the identity matrix *I* as a matrix with the property that $AI=IA=A for all A$ and be able to determine that the identity matrix must be a square matrix.
* Students will explore the associate, commutative and distributive laws as they apply to matrix multiplication using technology.

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 8.1.1** assesses a student’s ability to add and subtract matrices and to use examples to decide which of these operations is commutative. They will also see that matrix multiplication by a scalar is distributive.
* **Exit Slip 8.1.2** assesses a student’s ability to perform matrix addition, subtraction, and multiplication by a scalar.
* **Exit Slip 8.1.3** assesses a student’s ability to use technology to multiply matrices and to identify the properties of the identity matrix.
* **Journal Prompt 1** asks students to write about carbon dioxide as a greenhouse gas and how various recycling programs reduce carbon dioxide emissions. This could be a group activity with each person in the group assigned to a different recycling activity. (For example: clothing and textiles, paper and cardboard, aluminum cans, plastic bottles and glass bottles)
* **Journal Prompt 2** asks students to do an internet search about available technology to perform matrix multiplication. It also asks students to see if they can find a cell phone app that will make matrix multiplication easier and to write at least three sentences about using technology for matrix multiplication.
* **Journal Prompt 3** asks students to compare and contrast properties of matrices with the properties of real numbers.
* **Activity 8.1.1** **Introduction into Matrices** introduces students to matrices as a way to organize data.
* **Activity 8.1.2** **Matrix Terminology and Laws for Matrix Operations** has students continue using the data from activity 8.1.1 to introduce the more general notation for matrices and identifying elements of a matrix by row and column.
* **Activity 8.1.3 Using Technology with Matrices** has student use technology to facilitate working with matrices. They could validate their calculations in the previous two activities.
* **Activity 8.1.4 Multiplying Matrices and the Identity Matrix** has students learn the basic steps for multiplying two matrices in a contextual framework.
* **Activity 8.1.5** **Laws of Matrices** has students first review all of the properties of matrices that have been studied thus far and reviews the properties of Real Numbers.

**Launch Notes**

**Activity 8.1.1 Introduction into Matrices** can begin with a discussion about everyday situations that involve organizing things in “containers.” The teacher can suggest several examples like clothes in a dresser, game pieces in a game box, beads in a storage box, etc. The teacher can have students make a list of some different categories of items that they organize into containers. For each item, they can write down how they organize items into containers and explain how organization makes working with the items easier. As they share their lists, ask if there is something in common about all of them. Is there something that containers for organizing have in common that makes the organizing easier?

Next, ask for examples of situations that involves organizing data that makes it easier to work with. What “containers” do we use when working with data? Hopefully, students will suggest lists, tables, spreadsheets, and other ways of creating arrays and labeling arrays. Ask students to compare and contrast the various methods. The term matrix may not emerge among the suggestions, as frequently students have not been exposed to this term before.

Next introduce the theme for the first investigation, a school community engagement project related to the CT DEEP initiative to decrease the amount of waste that ends up in landfills. The initiative is called Reduce    Reuse     Recycle. This website is the homepage for this initiative: <http://www.ct.gov/deep/cwp/view.asp?a=2714&q=324884&deepNav_GID=1645>

Everyone is familiar with these terms, however they have very specific meanings in this context. Reduce waste means throwing away less. Reuse means extending the life of an object. Recycle means to make new items from material that would otherwise be thrown away.

**Teaching Strategies**

1. In **Activity 8.1.1**, students are introduced to matrices as a way to organize data. Students are introduced to the context of the CT DEEP reduce/reuse/recycle initiative and a hypothetical school community engagement project. Students organize data from the project into a matrix format and learn how to create and perform addition with matrices and multiplying a matrix by a scalar.

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| **Group Activity** **Activity 8.1.1 Introduction into Matrices** Form groups of two to four students. Students are given data from the following scenario: Your school is planning a community engagement project by sponsoring community support for the CT Department of Energy and Environmental Protection (CT DEEP) Reduce  Chasing Arrows  Reuse   Chasing Arrows  Recycle initiative. Reduce means consume less and throw away less, Reuse is lengthening the life of an object, and Recycle means to create new products from used materials. The project is endorsed by your town and school officials because it will help reduce costs, promote community awareness and show leadership in an important effort that benefits everyone. Students should if possible help plan a real project in various classes including math, science, and language arts. There will be a lot of data to collect, analyze, and synthesize. In this activity, students will learn the terminology for describing matrix size and matrix elements and construct appropriate matrices to organize data collected from a pretend project. They will explore adding and subtracting matrices and multiplying a matrix by a scalar. |
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You can assign **Exit Slip 8.1.1** and **Journal Prompt 1**

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| **Journal Prompt 1**The greenhouse gas carbon dioxide can be reduced by reusing items. Read information from various websites about the benefits of reusing textile products. Write a paragraph about why reusing items results in less carbon dioxide emissions into Earth’s atmosphere. Examples:* <http://www.energysavings.com/blog/what-to-do-with-old-athletic-shoes/>
* <http://www.motherearthnews.com/diy/braided-rug-from-recycled-wool-zmaz08djzgoe.aspx>
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1. **Activity 8.1.2** **Matrix Terminology and Laws for Matrix Operations** can be started in class and any part of the activity that has not been completed can be assigned for homework. It can also be used as a group activity in any class but especially for learners needing more help. This activity introduces the more general notation for matrices and identifying elements of a matrix by row and column. This leads to generalizing the commutative and associative laws for matrix addition. It is also a good opportunity for asking about matrix subtraction and whether one counterexample suffices for deciding that matrix subtraction is not commutative. Following this, the zero matrix is defined and students look at the properties of the zero matrix. They see what happens when you multiply a matrix by -1 and then add that matrix to the original matrix.

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| **Group Activity** **Activity 8.1.2** **Matrix Terminology and Laws for Matrix Operations**You can use groups of two to four students. Students continue using data from activity 8.1.1 after first discussing their results from exit slip 8.1.1.  |

**Differentiated Instruction (For Learners Needing More Help).** Groups of two to four students. Students continue usingdata from activity8.1.1after first discussingtheir results fromexit slip 8.1.1 and then proceed to work on Activity 8.1.2.

You can assign **Exit Slip 8.1.2**

1. **Activity 8.1.3 Using Technology with Matrices** can be started in whole class mode and the technology of choice can be demonstrated. Students can then be paired or arranged in larger groups. They should validate their calculations in the previous two activities with a graphing calculator or apps for ipad or smart phone. For example, Matrix Calculator for Students 2.0.6 APK is an easy to use app for android phones. Step by step instructions for working with matrices on the TI graphing calculator family are included in a hyperlink under resources below. In these activities, the TI graphing calculator screenshots are used as examples.

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| **Group Activity -----Activity 8.1.3 Using Technology with Matrices**Groups of two to four student should use technology to facilitate working with matrices. Students will first enter the collection data for the first two weeks into their graphing calculators. They will practice matrix operations with their calculators by redoing some of the activities in 8.1.1 and 8.1.2 and comparing with previous results. They will then explore the basic steps for multiplying two matrices in a contextual framework. It is essential that they understand matrix multiplication as related to this contextual framework. Finally, they will use their graphing calculators to see the results of multiplying a matrix by the identity matrix. Activity 8.3.2 will explore the identity matrix further.  |

1. **Activity 8.1.4 Activity 8.1.4 Multiplying Matrices and the Identity Matrix** should be started in whole class mode to be sure students understand the application and then students could be encouraged to complete it in pairs and after a few problems are done by hand, technology can come to the rescue.

You can assign **Exit Slip 8.1.3** and **Journal Prompt 2**

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| **Journal Prompt 2**Do an internet search about available technology to perform matrix multiplication. See if you can find a cell phone app that will make matrix multiplication easier. Write at least three sentences about using technology for matrix multiplication. |

1. Before distributing **Activity 8.1.5** **Laws of Matrices Laws** begin the class by reviewing the properties of real numbers. For each property, ask the class for several examples. What do we do if a property applies some, but not all of the time. Discuss when a property may not apply to a certain operation and how much evidence is needed before we reject a potential property, for example a commutative law for subtraction. Note that we have not yet discussed inverse matrices and have just mentioned the identity matrix in passing.

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| G**roup Activity****Activity 8.1.5** **Laws of Matrices**It is recommended that you pair students for this activity. It might be best to choose pairs with a stronger student as one member of the pair. In this activity, students first review all of the properties of matrices that have been studied thus far. The list is not complete yet for students still need to explore inverse matrices. Investigations 3 and 4 will complete the list of properties. Students should make their own list and then the list from Activity 8.1.5 can be examined. You may want to do 1 – 5 in whole class and then for 6 – 8 you may want each pair of students to select their own 3 matrices. When all find the property has worked for their concrete example matrices the question can then be posed, “Because we now have 15 or 20 true examples where the distribution works have we proven it always will work? How many example would we need to do before we can say it always works?” Of course, no amount of examples will do the trick. Extension: You may want to design an activity that looks at the distributive property for matrix multiplication over addition for the case of general 2 × 2 matrices: $C\left(A+B\right)=CA+CB$This activity requires attention to precision. |

You can assign **Journal Prompt 3.**

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| **Journal Prompt 3** Write a paragraph contrasting the properties of real numbers with the properties of matrices.  |

**Differentiated Instruction (For Enrichment)** Students can use Activity 8.1.5 as a model and prove the other form of the distributive property for matrices:

 $\left(A+B\right)C=AC+BC$and you could ask why we need to **prove** two forms.

**Closure Notes**

The investigation culminates with students successfully demonstrating their ability to compare and contrast the properties of real numbers and the properties of matrices. This is an important cognitive leap since many students take properties of real numbers for granted and are surprised when other mathematical structures are useful structures with elements and operations, but where operations fail to be commutative and may not even be defined for all pairs of elements. This is an important reason for introducing matrices and vectors. Students learn that care must be taken with each new mathematical structure to ascertain which properties hold and which are violated. They learn new respect for the real number system and learn to be careful with other structures, particularly vectors and matrices.

**Vocabulary**

Associative property of addition

Carbon dioxide (CO2)

Column of a matrix

Commutative property for addition

Greenhouse gas

Identity matrix

Row of a matrix

Scalar

Square matrix

Zero matrix

**Resources and Materials**

**All activities should be completed.**

Activity 8.1.1Introduction into Matrices

Activity 8.1.2 Matrix Terminology and Laws for Matrix Operations.

Activity 8.1.3 Using Technology with Matrices.

[education.ti.com/media/.../ti84plus\_guidebook\_en](file:///C%3A%5CUsers%5Cany%5CDocuments%5CGeometry%20and%20Algebra%20II%5CInvestigations%5CInvestigation%201%5Ceducation.ti.com%5Cmedia%5C...%5Cti84plus_guidebook_en)

Activity 8.1.4 Multiplying Matrices and the Inverse of a Matrix.

Activity 8.1.5 Laws of Matrices.

Bulletin board for key concepts

Example storage containers from crafts store

Graphing Calculators

Student Journals

Projector

Computers

Rulers

**Web sites**

* [http://www.ct.gov/deep/cwp/view.asp?a=2714&q=324884&deepNav\_GID=1645](http://www.ct.gov/deep/cwp/view.asp?a=2714&q=324884&deepNav_GID=1645http://www.ct.gov/deep/cwp/view.asp?a=2714&q=324884&deepNav_GID=1645)
* [http://www.ct.gov/deep/cwp/view.asp?a=2718&q=325464&deepNav\_GID=1646%20](http://www.ct.gov/deep/cwp/view.asp?a=2718&q=325464&deepNav_GID=1646%20%20%20)
* <http://www.gaia-movement.org/Article.asp?TxtID=534&SubMenuItemID=103&MenuItemID=47>
* <http://www.epa.gov/epawaste/nonhaz/municipal/index.htm>
* <http://www2.epa.gov/recycle>
* <http://www.epa.gov/epawaste/conserve/materials/textiles.htm>
* <http://www.energysavings.com/blog/what-to-do-with-old-athletic-shoes/>
* <http://www.smartasn.org/>
* <http://www.motherearthnews.com/diy/braided-rug-from-recycled-wool-zmaz08djzgoe.aspx>