**Activity 8.1.4 Multiplying Matrices and the Identity Matrix**

In addition to adding matrices, subtracting matrices, and multiplying a matrix by a scalar number, we can also *multiply* matrices. The process of multiplying matrices is logical and based on familiar ideas.

Consider the data from the reduce-reuse-recycle project.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Clothing (lbs.) | Wool(lbs.) | Shoes(pairs) | Rags (lbs.) |
| Pickup Site A | 92 | 36 | 14 | 88 |
| Pickup Site B | 46 | 87 | 34 | 73 |
| Pickup Site C | 57 | 44 | 37 | 44 |

The corresponding 3×4 collection matrix *A* is:

$$A=\left[\begin{matrix}92&36&14&88\\46&87&34&73\\57&44&37&44\end{matrix}\right]$$

The collected items have a monetary (money) value that depends on the item’s category. Also, since the collection and reuse of these items results in less need for new items, the collection and reuse of the items results in a reduction of carbon dioxide (CO2) emission into the atmosphere. The *benefit* of this project can be measured in the amount of money earned and the reduction in the amount of carbon dioxide produced.

To calculate the total benefit for all sites during the first week, we need the dollar value (measured in dollars per unit) for each category and the carbon dioxide emission reduction (measured in pounds of CO2) for each category. This is shown in the table below.

|  |  |  |
| --- | --- | --- |
|  | $ per unit | lbs. of CO2 |
| Clothing (lb) | .67 | 5.9 |
| Wool (lb) | 1.20 | 14 |
| Shoes (pairs) | 2.50 | 30 |
| Rags (lb) | .50 | 1.7 |

This information can be presented in the 4×2 benefit matrix, *B*,

$$B=\left[\begin{matrix}.67&5.9\\1.2&14\\2.5&30\\0.5&1.7\end{matrix}\right]$$

To determine the monetary benefit for items in a pickup site, or the reduction in C02 benefit for items in a pickup site, we can multiply matrices.

To find the monetary value (dollar value) for Pickup Site A, we multiply entries in the first row of matrix *A* by entries in the first column of matrix *B*. The first entry in the first row of matrix *A* (92) is pounds of clothing and the first entry in the first column of matrix *B* (0.67) is the dollars per pound of reused clothing. Multiplying the two gives the dollar value of the the clothing collected at Site A in the first week. To find the total dollar value of all items at Site A, we continue this process (see yellow hightlighted entries below) and add the result. Multiply the second entry in the first row by the second entry in the first column, then the third entry in the first row times the third entry in the first column and finally the fourth entry in the first row by the fourth entry in the first column and then add the results:

$$A=\left[\begin{matrix}92&36&14&88\\46&87&34&73\\57&44&37&44\end{matrix}\right] B=\left[\begin{matrix}.67&5.9\\1.2&14\\2.5&30\\0.5&1.7\end{matrix}\right]$$

$$92\left(.67\right)+36\left(1.2\right)+14\left(2.5\right)+88\left(0.5\right)=61.64+43.20+35+44=\$183.84$$

The result of this calculation ($183.84) is placed in the first row and first column of the newly created matrix $AB$ as shown below:

$$AB=\left[\begin{matrix}183.84&\\&\\&\end{matrix}\right]$$

In a similar way, the total amount of CO2 reduction based on the collections from Site A is found by multiplying the first row of matrix *A* entry by entry with the second column of matrix *B* and adding the results:

$$92\left(5.9\right)+36\left(14\right)+14\left(30\right)+88\left(1.7\right)=542.8+504+420+149.6=1616.4 lbs of CO\_{2}$$

The result of this calculation (1616.4) is placed in the first row and second column of the newly created matrix $AB$ as shown below:

$$AB=\left[\begin{matrix}183.84&1616.4\\&\\&\end{matrix}\right]$$

Continuing this process gives the remaining entries for the product matrix *AB*. (see below)

1. Find the dollar value of all items at Pickup Site B. Show your work.

Place this result in the second row, first column of matrix *AB* below:

$$AB=\left[\begin{matrix}183.84&1616.4\\&\\&\end{matrix}\right]$$

1. Find the amount of CO2 reduction based on all items at Pickup Site B. Show your work.

Place this result in the second row, second column column of matrix *AB* below:

$$AB=\left[\begin{matrix}183.84&1616.4\\&\\&\end{matrix}\right]$$

We will use technology to obtain all of the entries for matrix *AB*

1. Using your graphing calculator, enter in data for the benefit matrix as matrix *B.*
2. Use technology to find the product of the two matrices *AB* and compare with the results you obtained above. Write your answer below.
3. Explain all of the entries in matrix *AB*.
4. Would the matrix *BA* make sense? Explain.
5. Attempt to find the product matrix *BA* using your graphing calculator. What happens?
6. Find the product *DE*.

$D=\left[\begin{matrix}5&12\\9&6\end{matrix}\right]$

$E=\left[\begin{matrix}0.65&0.15&0.35\\0.68&0.12&0.40\end{matrix}\right]$

1. Enter the following matrices in your graphing calculator:

*C* = $\left[\begin{matrix}2&3&1\\3&-2&-1\end{matrix}\right]$ *D* = $\left[\begin{matrix}-2&1\\3&-1\\0&-3\end{matrix}\right]$

Use your calculator to find $CD$ and $DC$

1. Did you expect these products to be the same? Why or why not?
2. What does this tell you about matrix multiplication? Hint: Does the order matter?
3. Explain the matrix size for matrix $CD$ and matrix $DC$ and note any differences.

1. So to multiply two matrices, the number of columns of the \_\_\_\_\_must match the number of \_\_\_\_\_ of the second.
2. For the matrices *A, B, C, D* below, match the matrices that may be multiplied, and then multiply them by hand or using your graphing calculator. In each case make sure to decide if you can multiply from the left or from the right and write the product matrix.

 $A=\left[\begin{matrix}1&3\\4&5\end{matrix}\right]$

$$B=\left[\begin{matrix}2&3&4\\3&4&5\\5&6&7\end{matrix}\right]$$

*C* = $\left[\begin{matrix}2&3&1\\3&-2&-1\end{matrix}\right]$

*D* = $\left[\begin{matrix}-2&1\\3&-1\\0&-3\end{matrix}\right]$

The following matrix products are possible:

The product matrices are as follows:

**Square Matrices and Identity Matrices**

A *square matrix* is a matrix that has the same number of rows and columns. A square matrix with 1’s entered on the main diagonal and zeros everywhere else is called an *identity* matrix. The main diagonal is the diagonal running from top left corner to bottom right corner. Identity matrices have a special property.

The following matrices are identity matrices:

$I=\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right]$ $J=\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$

1. Find the product *DJ*. (See number 8 above for matrix D) What do you notice? \_\_\_\_\_ Remember, that to multiply two matrices, the number of columns of the first must match the number of rows of the second.
2. Find $CBI.$ Use the matrices in number 14 above

$$CBI=$$

1. Find BDJ. Use the matrices in number 14 above.

$$BDJ=$$

1. What is the special property of matrices $I=\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right]$ and $J=\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$?