**Unit 6: Investigation 5 (2 Days)**

**Models of Periodic Behavior**

**Common Core State Standards**

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales

F.TF.5Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

**Overview**

Students will model periodic behavior for the amount of daylight over the course of a year, ocean tides and phases of the moon, temperature, slinky (harmonic motion), car pistons, pendulums, musical notes, biorhythms, and predator-prey population interactions.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Model periodic behavior from real-world data
* Explain what the mathematical model tells us about the real world phenomena and/or make predictions based on the model
* Think critically and communicate in writing about their modeling work.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 6.5.1** Students will draw a sketch that illustrates the following: the distance of the bottom of a spring to the floor if the spring starts from its lowest point 6 cm off the floor, and one second later is at its highest point that is 26 cm off the floor. Then they will state the amplitude, period, and midline, and write a sinusoidal model for the distance of the bottom of the slinky to the floor as a function of time. (Assume that there is no damping.)
* **Exit Slip 6.5.2** State three real world situations that can be modeled by a trigonometric function. Sketch the graph of each, with labels on the independent and dependent axes.
* **Journal Prompt 1** asks students to explain a part of an activity that they found challenging. Why did they find it challenging? How did they overcome the difficulty? What were they thinking and feeling from the start until the resolution of the challenge?
* **Activity 6.5.1 Temperatures** includes a variety of situations with simple data that can be modeled with a sine or cosine function .
* **Activity 6.5.2 Daylight Hours** tells students the number of hours of daylight from Jan. 1 to Dec. 31 at various location around the globe. Students compare the parameters of the sinusoidal model for each location and the rates of change at the equinox and solstice.
* **Activity 6.5.3 Ocean Waves** provides a look at standard ocean waves, as well as tsunamis.
* **Activity 6.5.4 Modeling Automobile Pistons,** has students approximating the up and down motion with at sinusoidal function.
* **Activity 6.5.5 Constructive and Destructive Interference** investigates the sum of two sinusoidal functions.
* **Activity 6.5.6 Just the pendulum. No Pit** has students use a sinusoidal function to approximate the horizontal distance of the pendulum bob from the center as a function of time.

**Launch Notes**

As students enter the class room, try to display the work of computer animation art by Daniel Sierra called “Oscillate”. <http://dbsierra.com/Work/Oscillate/> . Remind students that they have seen works of art modeled by polynomials in unit 3, and frieze patterns in geometry that can be understood in terms of translations, reflections and rotations. Here is a dynamic work of art based on the sine function. The goal is for students to see trigonometric models or approximately trigonometric models in a variety of contexts.

You may revisit the examples from the Unit 6 Investigation 1 Launch, or you can try new examples that demonstrate functions that are sinusoidal or can be approximated by sinusoidal models. Now that students have nearly completed Unit 6, they can find a trigonometric equation to approximate data. In addition to the Ferris wheel shown previously, you can try harmonic motion, the pendulum and sound.

1. Illustrate harmonic motion by having a slinky bob either above or below a motion detector. If one assumes there is no damping, fit a sinusoidal model to the height of the slinky as a function of time using one period or one up and down motion. To model the damped sinusoidal function, multiply the model for one period by an exponential decay function fit to the relative maximum points for each period. Note that when we graphed y = a∙sin(t) we had stretched the sine function vertically by the constant function f(t) = a. The relative maximums were y-coordinates of turning points that were located on the line y = a, and the relative minimums were y-coordinates of turning points that were located on the line y = -a. For damped harmonic motion, the “stretch factor” is now a variable function with the relative maximums the y-coordinates of points falling on the decay curve y = e-kt, and the relative minimums the y-coordinates of points falling on the curve y = -e-kt .
2. Devise a pendulum by attaching a ball to a string and holding the motion detector to the side of the pendulum as the ball swings toward and away from the motion detector. Observe the ball’s horizontal distance from the motion detector as a function of time. Better results are obtained if the pendulum swings through a relatively small angle no more than 30º from the vertical. Though not exactly a sine function, the pendulum’s horizontal distance from the motion detector as a function of time can be approximated by a sinusoidal model provided the angle the string makes with the vertical is not too great. Damping can be taken into account as in the slinky example.
3. To demonstrate sound, explain that sound is the compression and rarefication of air, and have a student walk back and forth in front of the motion detector to imitate this compression and rarefication. Then either use a CBL microphone or sound probe, or open any number of free computer programs that will record sound and show the wave form. If you want a simple sine wave, using a tuning fork borrowed from the music or physics department will give a pure tone (no overtones). Have students sing a high pitched and then a low pitched tone to compare the periods (note: frequency (vibrations per minute) of a sound is the reciprocal of the period (minutes per vibration). A loud tone will have a higher amplitude than a quieter tone – ask students what other words they know that have to do with sound and volume (e.g. amplifier). A percussive sound like a clap or a yell will not look like a wave at all, but will have spikes and look very irregular.
4. Can students name other examples of periodic behavior that might be modeled by a trigonometric function?

You can use **Exit Slip 6.5.1** after the launch to confirm whether or not students can find a sinusoidal model for a situation.

**Teaching Strategies**

This investigation contains several activities for you to choose from. Have students start with **Activity 6.5.1 Temperatures,** the most straightforward of the activities, because it includes a variety of situations with simple data that can be modeled with a sine or cosine function. Do at least one other activity in this investigation, either individually or with a group.

The activities provide practice for the Performance Task. If possible, match activities with student interests, having different students doing different activities. Try to have a few students share their work with the class, not only to expose others to the various activities, but also because public showings tend to generate a better product. Expect students to give thoughtful explanations about why they used the model they did, what assumptions were made, and how they handled anomalies, if any. If students think of these activities as lab reports, they will understand that they will be graded on spelling, grammar, punctuation, neatness and good writing. You may allow students to complete the assignment by making a power point to show to the class, you may require an oral presentation for one activity, or you may ask for students to present their work in the form of an academic trifold poster. (They will need guidelines for how to make a good power point presentation or what a good academic poster is.) Note: Some activities are much easier and shorter than others.

You may want to use **Exit Slip 6.5.2** after the students have done 1 or 2 of the activities. Then consider letting students write their own activity using a topic or situation they suggested and that is approved by you. You will want to be sure that their suggested topic can be modeled by a trigonometric function. Also, check in with the students early to be sure they are not spending too much time researching data. If finding data turns out to be too time-consuming, tell the students to change their topic, re-focus it, or simply try one of the activities included in this investigation.

**Activity 6.5.1 Temperatures** looks at a variety of situations where temperature is measured, and is a good starter activity.

**Activity 6.5.2 Daylight Hours** tells students the number of hours of daylight from Jan. 1 to Dec. 31 at various location around the globe. Students compare the parameters of the sinusoidal model for each location and the rates of change at the equinox and solstice.

If you are interested in more information about the hours of daylight, investigate these websites and videos:

1. To find the number of hours of daylight for any major city in the world, go to the United States Naval Observatory website and to obtain the data. A search with “daylight US Navy” brings you to their website: <http://aa.usno.navy.mil/data/docs/Dur_OneYear.php> .
2. A You Tube video explaining why daylight increases and decreases throughout the year can be found at <https://www.youtube.com/watch?v=rcquRMaVSKU> , or search: “What Causes the Seasons” by Treetest63.
3. A very detailed explanation of the rotation earth is given by the video <https://www.youtube.com/watch?v=82p-DYgGFjI> “Earth’s Motion Around the Sun, not as simple as I thought”, by Aryan Navabi, as part of the Cassiopeia Project at <http://www.cassiopeiaproject.com> .

**Activity 6.5.3 Ocean Waves** gives a look at standard ocean waves, as well as tsunamis. Included is an idea for building a wave machine.

**Activity 6.5.4 Modeling Automobile Pistons,** has students approximating the up and down motion with at sinusoidal function.

**Activity 6.5.5 Constructive and Destructive Interference** investigates the sum of two sinusoidal functions.

**Activity 6.5.6 Just the pendulum. No Pit**. Students use a sinusoidal function to approximate the horizontal distance of the pendulum bob from the center as a function of time. Since students did a pendulum experiment from Algebra 1 in which they found that a square root function was a good model for the pendulum’s period as a function of its length, they may be confused as to how a sinusoidal function and a square root function can both model a pendulum. By collecting two sets of data from the same virtual pendulum, students will see that there can be two functions about pendulums, depending on what data is being collected. Students will understand the need to clearly identify the independent and dependent variables of a function.

**Group Activity with differentiated instruction:** Use the jigsaw model for group work: form homogeneous groups of 5 or 6 by ability, and assign the problem of the appropriate level of difficulty to each group. (You may allow these large groups of 5 or 6 to subdivide if they want). After each group is done with their problem, have the students count off in each group. Form 5 or 6 new heterogeneous groups by pulling together all the ones to form a group, all the twos to form a group, etc. Each person in the group must explain the problem they did to their new group members.

**Differentiated Instruction (For Learners Needing More Help)**

Work with student through one problem, and as you do, have them articulate the steps and strategies you used. Have students make their own check list for attacking a problem on a note card or on a paper in a special sleeve in their notebook. See that they refer to their check list regularly. Consider allowing them to use the list on tests.

Here are some ideas for a student check list for sinusoidal modeling problems.

1. Understand the problem:

a. Draw a picture

b. Underline key words

c. Try out some numbers that you make up

2. Determine what is being asked and what the answer will look like. Circle the essential part of the question.

a. Is the answer going to be an equation of a function that models this behavior?

b. Is the answer going to be a number or phrase that answers a direct question?

3. Plan a strategy:

a. For modeling with a sinusoidal function: write: y = a(sinbx) + c

i. Determine the amplitude. This is a.

ii. Determine the period, then divide the 2π or 360° by the period. This is b.

iii. Determine the midline or the vertical shift. This is c.

4. Is your answer reasonable?

Check your model by graphing it and use trace on a value to double check that a few of the points you want to be on the graph are on the graph.

**Differentiated Instruction (Enrichment)**

Some activities lend themselves well to encouraging students to find their own data, explore a situation more deeply, or by adding another layer of realism. For example, have students find an equation for damped harmonic motion by explaining that the vertical stretch or amplitude need not be constant. For damped harmonic motion the sinusoidal curve stays within an envelope given by an exponential decay function: +/-1(e-kx) is the coefficient of the sine function.

**Journal Prompt 1**

Ask students to explain a part of an activity that they found challenging. Why did they find it challenging? How did they overcome the difficulty? What were they thinking and feeling from the start until the resolution of the challenge? Answers of course will vary.

**Closure Notes**

Find some way to display student work. Encourage students to report on other phenomena that might be modeled by trigonometric functions. Ask whether all periodic behavior is modeled by trigonometric functions. Compare a slinky (yes) with a yo-yo (no), or the hours of daylight in Boston (yes) compared to Antarctica (no).

**Vocabulary**

Constructive and destructive interference

Longitudinal wave

Mathematical model

Sinusoidal model

Transverse wave

**Resources and Materials**

**Have each student do Activity 6.5.1 and 6.5.2 or have them do any 3 activities (recommend that one be 6.5.1) from this investigation either individually or in groups.**

Activity 6.5.1 Temperatures

Activity 6.5.2 Daylight Hours

Activity 6.5.3 Ocean Waves

Activity 6.5.4 Modeling Automobile Pistons

Activity 6.5.5 Constructive and Destructive Interference

Activity 6.5.6 Just the pendulum. No pit.

Graphing Calculator

Computer

Motion Detector

A computer/tablet with a microphone that will display an image of the sound wave or an oscilloscope or microphone probe.