**Activity 6.1.2 What Is a Radian?**

In the previous activity 6.1.1, we learned how to measure angles by the amount of rotation from the initial side to the terminal side. We measured angles that are larger than 90° and angles that are negative. This will allow us to take the sine, cosine and tangent of an angle that measures any number of degrees, not just between 0 and 90 degrees.

Since our goal is to be able to use trigonometric functions to model real world periodic phenomena, there is problem with using angles measured in degrees. We will want to be able to add, subtract, multiply a trigonometric function and a non-trigonometric function so we need a common domain. Degrees will not work for “x” if we try to interact with trigonometric functions and non-trigonometric functions. Recall the work in unit 1 when you found the sum, difference and product of two functions. What was the domain of the new sum, difference and product function? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

For example, if we describe the vertical height of a bungie jumper as a function of time, the independent variable x can’t represent both the time in seconds for the exponential function factor and the angle in degrees for the cosine function in the function f(*x*) = .75*x*(cos *x*) + 30.

**The solution to this problem is to measure angles in ‘radians’ that was conceptualized by Roger Cotes (1682-1716), a co-worker of Sir Isaac Newton.**

Follow the steps below, in order to learn about radians. You will need a piece of tracing paper or other thin or transparent paper and some string about 15 or 20 cm long.

1. Draw a circle centered at (0, 0) on a coordinate axis with a compass. Be sure to mark the center. Draw the initial ray of an angle in standard position.

2. Next create your own unique tape measure for your particular circle. The tape measure will be marked off in lengths the same size as the radius of your circle. Lay a piece of string on one radius of your circle, make a mark the length of 1 radius on the string; and then mark off another radius’s length on your string, and so on.

The unit of measure for your special tape measure is “1 radius long”. Hopefully, your tape measure has at least 6 or so radii marked along its length.

We say that the radius of your circle is 1 unit in length. We use the word “unit,” because we may or may not use any standard unit of measure such as centimeters or inches. You might want to give your unit a name. You might name your unit ‘*your first name*’ and pretend to be queen/king for the day since so many units came from kings and queens. Then it is more like inches or centimeters.

3. On the circumference of the circle, mark off an arc that is one radius long. Do this by placing one endpoint of your tape measure at the point that the initial ray intersects the circle. We call this the initial point of the arc in standard position. It is the point that is at 3 o’clock on a circular clock face.



Wrap your tape measure counterclockwise

around the circumference of the circle, so you can

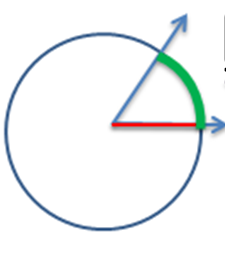
measure the length of any distance around the circle.

With a pen or pencil, mark off an arc on the circle

that is the same length as the radius of the circle.

4. Use a straight edge to draw the central angle that is subtended by your arc, the same arc that is the same length as the radius. Note: the central angle subtended by an arc is the angle that is made by drawing the initial ray from the center of the circle to the initial point of the arc and the terminal ray from the center of the circle to the endpoint of the arc.

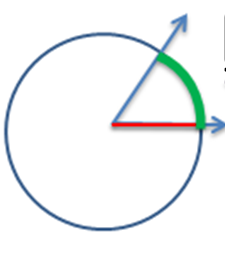
Terminal ray of the central angle



Initial ray of the central angle.

(The word “subtend” means “stretches under”. The arc on the circle stretches under the central angle from the initial ray to the terminal ray)

5. The length of the arc is 1 “*your name*”. The length of the radius is 1 “*your name*”. The measure of the central angle is 1 radian. Label the angle in your picture “1 radian”.



This central angle measures 1 radian.

6. Lay your angle on top of those of your neighbor. Each of you has an angle that measures 1 radian. Note that the length of the radius does not affect the size of the angle. Even though the circles are different sizes, the central angles are congruent.

Sketch 3 concentric circles and the angle that

measures 1 radian in standard position here:

**BIG IDEAS**

**An arc that is 1 radius long subtends an angle that measures 1 radian.**

**The arc subtended by a central angle with measure 1 radian has the same length as the radius of the circle.**

7. Go back to your original sketch, where you drew an angle that subtended an arc that was the size of 1 radius. Use a protractor that is marked in degrees to find the degree measure of the angle that is 1 radian. An angle that measures one radian is approximately equal to \_\_\_\_\_\_\_\_ degrees.

8. Compare the measure of your angle with that of other students. Fill in the blank with “greater than, less than or equal to”: An angle that measures 1 radian in a bigger circle will have a measure \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ an angle of 1 radian in a smaller circle.

9. a. To the right, sketch a circle. Indicate an arc in standard

position that is 2 radii long. Draw the initial ray and the terminal

ray of the central angle that subtends the arc.

What is the radian measure of the central angle subtended by

an arc that is 2 radii long?\_\_\_\_\_\_

b. If an angle measures 3 radians, how many radii long

is the intercepted arc? \_\_\_\_\_\_\_\_\_\_\_\_\_

c. What would be the radian measure of a central angle

that subtends an arc that is 3.1 radii long?\_\_\_\_\_\_\_

10. a. To the right, sketch a circle. Indicate an arc

with initial point (0, 1) that is 2π radii long (approx. 6.28).

Indicate the angle that is subtended by the arc.

What is the radian measure of the central angle subtended by an arc

that is 2π radii long?\_\_\_\_\_\_

b. One revolution is\_\_\_\_\_\_\_ ° and that equals \_\_\_\_\_ radians.

11. An arc that is ‘r’ radii long is subtended by and angle that measures \_\_\_\_\_ radians.