**Unit 6: Three-Dimensional Geometry**

**(4 Weeks)**

**UNIT PLAN**

This unit provides an in-depth study of the vocabulary, properties and formulas for the surface area and volumes of polyhedra and other solid figures. Students make informal arguments for the formulas for surface area and volume. Later in the unit students study Cavalieri’s principle and encounter more formal derivations of the formulas for volume of the pyramid, cone and sphere. The unit explores several real world applications of surface area and volume as well as the geometry of the sphere.

The first three investigations may be treated as a mega-investigation culminating in a mid-unit performance task. The performance tasks motivates the need to learn about surface area and volume of polyhedra. Although these topics should have been introduced in the middle grades, we assume this will be new material for many students.

**Investigation 1- Polygons and Polyhedra (3 days):** Students classify solids by their properties. They identify solids as prisms, pyramids, regular and semi-regular polyhedra as well as cones, cylinders and spheres. This activity develops terminology such as: polyhedron, faces, edges, vertices, convex, non-convex, regular, semi-regular, irregular. Students use isometric dot paper to create perspective drawings of polyhedra. Students prove informally that there are exactly five Platonic solids and they begin to explore semi-regular polyhedra Students investigate the relationship of faces, edges and vertices and discover Euler’s formula in the case of convex polyhedra. They make real world connections through viewing illustrations of crystal formations and dice from the Ptolemaic dynasty.

In **Investigation 2 – Nets and Surface Area (2 days) s**tudents build models by creating nets for specific solids. They determine if a plane configuration of polygons is a net of a polyhedron. Students apply the formulas for area of plane figures (learned in earlier grades and proved earlier in this course) to compute surface areas of prisms, pyramids, cylinders and cones. Students relate the real world applications of surface area to problems of packaging products.

In **Investigations 3 – Volume (3 days) s**tudents do informal explorations using sand, water or beans to fill shapes with measuring cups and compare and contrast similar dimensional shapes with their volume. They explore Cavalieri’s Principle informally by stacking coins, paper etc. They explore the relationship between prisms and pyramids that have common bases and heights. They extend the work with prisms and pyramids to cones and cylinders. The final activity introduces frustums of cones and pyramids.

The **Mid-Unit Performance Task (2 days)** provides an opportunity for students to use the skills and concepts from Investigations 1-3 to solve a concrete task.Students work in groups to choose five different solid figures and make a mobile of figures sized to have the same surface area. The volumes of the figures are calculated and another mobile is constructed so that the figures, when filled with a uniform substance, can balance using the volumes of the shapes.

In **Investigation 4 - Cross Sections and Solids of Rotation (2 days)** Students use modeling clay and dental floss to explore cross sections of various solid figures. They use geometry software to explore solids formed by rotating triangles, rectangles, isosceles trapezoids, and semicircles. They are introduced to three-dimensional rectangular coordinates via geometric software and use the software to generate solids of revolution.

**Investigation 5 – Spheres (2 days)** extends the work begun in investigations 2 and 3 to justify formulas for the surface area and volume of a sphere. Using Cavalieri’s principle it anticipates ideas that some students will explore more deeply in a calculus course and provides some beginning experience with applications of the formulas to real world problems.

**Investigation 6 – Geometry on the Sphere (3 days)** introduces students to a geometry different from the Euclidean model used so far. They learn that the shortest path through two points lies on a great circle and that the sum of the interior angles of a spherical triangles is greater than 180° but less than 540°. Students work with coordinate systems on a sphere which are related to latitude and longitude on the Earth’s surface. They are introduced to applications of spherical geometry to cartography and Global Positioning Systems (GPS).

**Investigation 7 Size and Shape in the Real World (2-3 days):** Students explore a real world problem of making a three-dimensional scale model. Reference is made throughout the investigation to the application of a 3-D printer. Students calculate volume of a shape and its weight given a 10% fill. In the final, optional, activity students design their own object. This may be used as a mini-performance task.

**Essential Questions**

* What kinds of solid geometric figures are there?
* What are some of the properties of these solids?
* How can we find surface areas and volumes for these solids?
* Where do the formulas come from?
* How can we use these formulas to solve problems we might face in the real world?
* Is there only one kind of geometry, the Euclidean Geometry with which we are familiar?

**Enduring Understandings**

Formulas for solid figures relate to each other in meaningful ways, just as area formulas for plane figures connect to each other.

**End of Unit Test (2 days including review)**

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning.*

1. **Make sense of problems and persevere in solving them.**

2. **Reason abstractly and quantitatively.**

3. Construct viable arguments and critique the reasoning of others.

4. **Model with mathematics.**

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Standards Overview**

* Explain volume formulas and use them to solve problems.
* Visualize relationships between two-dimensional and three-dimensional objects.
* Apply geometric concepts in modeling situations

**Standards**

***6G4*** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

***7G6*** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

***8G9*** Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.

**G-GMD.1**. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri’s principle, and informal limit arguments.*

**G-GMD.2.** (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.

**G-GMD.3.** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

**G-GMD.4**. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects

**G-MG.1**. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

**G-MG.2.** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).\*

**G-MG.3.** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios

**Postulates and Theorems**

**Regular Polyhedron Theorem:** There are exactly five regular polyhedra. (Informal proof)

**Euler’s Formula**: For any convex polyhedron, *V* + *F* = *E* + 2, where *V* is the number of vertices, *F* is the number of faces, and *E* is the number of edges. (not proved)

**Rectangular Prism Volume Postulate:** If *B* is the area of the base and *h* is the height of a right rectangular prism, then Volume = *Bh*

**Cavalieri’s Postulate (Principle):** If two solid figures lie between two parallel planes and every plane parallel to these two planes intersects both solids in cross-sections of equal area, then the two solid figures have equal volumes.

**Prism Volume Theorem:** The volume of any prism is found by multiplying the area of its base by its height.

**Pyramid Volume Theorem:** If the bases of two pyramids have the same area and if the heights of the pyramids are equal, then the pyramids have equal volumes.

**Pyramid-Prism Theorem:** The volume of a pyramid is one-third the volume of a prism with the same base and height.

**Hemisphere-Cylinder Theorem:** The volume of a hemisphere is equal to $\frac{2}{3}$ the volume of a cylinder with the same radius and height equal to its radius.

**Vocabulary**

altitude

Archimedean solid

axis of rotation
CAD (computer aided design)

circumscribed (prism or pyramid)

concave

cone

convex

cross-section

cube

cylinder
density

dodecahedron

edge (of polyhedron)

Equator

face (of polyhedron)

frustum (of cone or pyramid)

geodesic

great circle

height (of prism, pyramid, cylinder or cone)

hemisphere

hexahedron

inscribed (prism or pyramid)

icosahedron

isometric drawing

lateral surface

latitude

longitude

meridian

net

non-convex

octahedron

Platonic solid

polyhedron

Prime Meridian

prism

pyramid

projection

regular polyhedron

semi-regular polyhedron

### Schläfli symbols

Schlegel diagram

semi-regular polyhedron

slant height

solids generated by rotations of plane figures

sphere

spherical coordinates

spherical excess

spherical triangle

surface area

surface of revolution

tesseract

tetrahedron

three-dimensional figure

trilateration

vertex (of polyhedron)

volume

 **Assessments**

**Mid-Unit Performance Task –** Students construct to mobiles, one of equal surface area polyhedra and one that uses the same polyhedra and are filled with materials.

**Other Evidence (Formative and Summative Assessments)**

**•** Exit slips

• Class work

• Homework assignments

• Math journals

• Mid-unit Performance Task

• End-of-unit test