**Unit 5 Circles and Other Conics**

**(4-5 Weeks)**

**UNIT PLAN**

In this unit a variety of methods, including formal synthetic proof, compass and straightedge constructions, and coordinate methods are applied to the study of circles and other conic sections.

In **Investigation 1** the circle is defined as the locus of points that are at a given distance to a given point. From the locus definition the equation of a circle in the coordinate plane with center (*h*, *k*) and radius *r* is derived. We also observe that all circles with equation may be considered images of the unit circle under translation and dilation. Also some equations of the form may be rewritten in the above form by completing the square. Once the square is completed, as long as the right side is greater than 0, the equation represents a circle.

In **Investigation 2** the concept of locus is applied to the perpendicular bisector of a line segment. We return to the locus definition of circle to show that the center of a circle lies on the perpendicular bisector of any chord. This allows us to locate the center of any circle given two chords and to circumscribe a circle about any triangle.

**Investigation 3** examines central angles and arcs. Given the measure of a central angle in degrees, the ratio of the intercepted arc to the length of the radius is constant regardless of the size of the circle. This leads to the definition of radian as another way to measure angles. (This concept is introduced here and developed more fully in Algebra 2). The length of an arc is therefore given by where is measured in radians. Similarly the area of a sector is given by . Notice that when we have the circumference and area of the entire circle.

In **Investigation 4** we see an informal proof of the fact that a tangent to a circle is perpendicular to a radius drawn to the point of tangency. Two right triangles are congruent if their hypotenuses and one pair of legs are congruent, a theorem that will be used to prove that tangent segments drawn from an external point to a circle are congruent. One activity will revisit the equation of a circle in the coordinate plane and use the slopes of perpendicular lines to find the equation of a line tangent to a circle at a given point.

**Investigation 5** leads to the construction of the center of the inscribed circle in a triangle, using the property that the bisector of an angle is the locus of points equidistant from the sides.

In **Investigation 6** we show that an inscribed angle in a circle is equal in measure to ½ the measure of its intercepted arc. A special case is Thales’ Theorem that the angle inscribed in a semicircle is a right angle. We will also show that the opposite angles of a cyclic quadrilateral are supplementary. An optional activity (addressing standard C-4+) will have students construct tangents to a circle from an external point.

**Investigation 7** introduces the locus definition of parabola and uses it to find the equation of a parabola with focus at (0, *p*) and directrix y = –*p*. Students who have studied Unit 8 in Algebra 1 will see the connection to the parabola as the shape of the graph of a quadratic function.

**Investigation 8 (+)** extends the study of conic sections to ellipses and hyperbolas. The Common Core Standards include this for STEM intending students, so this is an optional investigation.

**Essential Questions**

* How are the equations of conic sections related to their locus definitions?
* What relations among angles, chords, and tangents to circles can be proved?
* How are the lengths of arcs, areas of sectors, and radian measure related to central angles in circles?
* What are the properties of inscribed and circumscribed triangles and inscribed quadrilaterals?

**Enduring Understandings**

Most properties of circles may be derived from the definition of a circle as the locus of points at a given distance from a given point.

**Unit Contents**

Investigation 1 Circles in the Coordinate Plane (2 days)

Investigation 2 Radii and Chords (3 days)

Investigation 3 Central Angles and Arcs (2 days)

Investigation 4 Tangents to Circles (3 days)

Mid-unit test (1 day)

Investigation 5 Angle Bisectors (2 days)

Investigation 6 Inscribed Angles and Cyclic Quadrilaterals (2 days)

Investigation 7 Parabolas (2 days)

Investigation 8 Ellipses and Hyperbolas (+ optional, 2-3 days)

Performance Task (1-2 days)

End of Unit Test\* (1 day review, 1 day test)

\* Covers up through Investigation 7, since Investigation 8 is optional.

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. **Construct viable arguments and critique the reasoning of others**.

4. Model with mathematics.

5. **Use appropriate tools strategically.**

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Standards Overview**

* Understand and apply theorems about circles
* Find arc lengths and areas of sectors of circles
* Translate between the geometric description and the equation for a conic section
* Use coordinates to prove simple geometric theorems algebraically

**Standards**

G-CO.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.*

G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G-C 4. (+) Construct a tangent line from a point outside a given circle to the circle.

G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.2. Derive the equation of a parabola given a focus and directrix.

G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant; or given foci and directrices.

G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

**Vocabulary**

arc

arc length

asymptote (+)

center

central angle

chord

circle

circumscribed circle (of a triangle)

completing the square

conic section

cyclic quadrilateral

diameter

directrix (of conic section)

ellipse

equidistant

focus (of conic section)

hyperbola

inscribed angle (of a circle)

inscribed quadrilateral (of a circle)

inscribed circle (of a triangle)

intercepted arc

locus of points

major arc

major axis (of ellipse +)

minor arc

minor axis (of ellipse +)

parabola

point of tangency

radian measure

radius

secant

sector

semicircle

tangent

tangent segment

transverse axis (of hyperbola +)

vertex (of parabola)

**Theorems**

**Perpendicular Bisector Theorem:** the locus of points that are equidistant from the endpoints of a segment is the perpendicular bisector of the segment.

**Perpendicular Bisector of a Chord Theorem**: The perpendicular bisector of any chord of a circle passes through the center.

**Radius Chord Midpoint Theorem:** If a radius of a circle bisects a chord (that is not a diameter), then it is perpendicular to the chord.

**Radius Chord Midpoint Converse:** If a radius of a circle is perpendicular to a chord then it bisects the chord.

**Construction:** To circumscribe a circle about a triangle.

**Arc Length Theorem:** The length of an arc of a circle is equal to the product of the radius and the radian measure of the central angle that intercepts the arc.

**Area of a Sector Theorem:** The area the sector of a circle is equal to the half product of the square of the radius and the radian measure of its central angle.

**Shortest Distance Theorem:** The shortest segment joining a point to a line is the perpendicular segment.

**Radius-Tangent Theorem:** A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

**Radius-Tangent Converse:** If a line is perpendicular to the radius of a circle at a point on the circle, then it is tangent to the circle.

**HL Congruence Theorem:** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

**Tangent Segments Theorem**: If two tangents are drawn to a circle from the same point outside the circle, the segments from that point to the circle are congruent.

**Angle Bisector Theorem:** The locus of points equidistant from the sides of an angle is the bisector of the angle.

**Inscribed Angle Theorem**: The measure of an angle inscribed in a circle is one-half the measure of the central angle that intercepts the same arc.

**Construction:** To inscribe a circle in a triangle.

**Thales’ Theorem:** An Angle inscribed in a semi-circle is a right angle

**Cyclic Quadrilateral Theorem:** The opposite angles of a cyclic quadrilateral are supplementary.

**Construction:** To construct the tangent segments to a circle from a point outside the circle (+)

**Coordinate Geometry Equations**

**Circle with Center at the Origin and Radius *r*:**

**Circle with Center (*h*, *k*) and Radius *r*:**

**General Equation of a circle:**

**Parabola with Focus (0, *p*) and Directrix *y* = – *p*:**

**Ellipse in Standard Position(+):**

**Hyperbola in Standard Position(+):**

**Focus-Directrix Definition of Conic Section** **(+)** (with directrix *y = –*1, focus (0,*e*)**:**

**Assessment Strategies**

**Performance Task**

Students will apply the properties of circles to determine the most efficient way to pack cylindrical cans, given the diameter of the can and the dimensions of the box.

You will need rolls of pennies for the introduction to the performance task. Give students page 1 only to begin. After they have discovered square and hexagonal packing, you may give them the remaining page. Note that there is a group component to this project.

From previous study, students should have sufficient experience with volume and surface area to be able to do this task. However, if you would prefer to review these concepts you may delay implementing this task until after Investigation 1 of Unit 6.

**Other Evidence (Formative and Summative Assessments)**

* Mid-unit test
* End-of-unit test
* Exit slips
* Class work
* Homework assignments
* Math journals