**Activity 7.4.3 – Estimating Population Means**

In 2014 the Center for Disease Control (CDC) reported that American adults, on average, exercise for 65 minutes per week. This was alarming to national health and fitness experts because 65 minutes is significantly lower than the weekly exercise amount the CDC recommends. The report listed a margin of error of 5.8 minutes per week at a 95% confidence level. What does this mean?

The purpose of this large, national sample survey was to estimate the mean exercise time per week of all adults in the US. The sample mean (65) and margin of error (5.8) can be combined to form an *interval estimate* for the population mean. Interval estimates have the form:

The point estimate is the sample mean; the margin of error is based on the sample mean and sample size. In this example, the interval estimate is , which in interval form is (59.2, 70.8). The CDC concluded that, with 95% confidence, the mean amount of time US adults exercise each week is between 59.2 and 70.8 minutes.

**What is the Margin of Error?**

A sample mean’s margin of error represents the *likely* maximum possible distance between the sample mean and population mean. This distance is based on the sampling distribution. To see how this works, let’s examine a sampling distribution of sample means from a population with a *known* population mean. We will find the margin of error for a 95% *confidence interval.*

A recent study found that US teenagers spend an average of 7.6 hours a day consuming media. This involves watching television and using computers and mobile devices. Let’s assume that this value is the true population mean (, and that the population standard deviation is 2 hours a day (). The dot plot below shows an empirical distribution of sample means from random samples of size *n* = 30, taken from this population.

Samples = 500

Mean = 7.6

SE = 0.32



As we expect, the population mean (7.6) is the mean of the sample means. Based on the Empirical Rule, the interval that contains the *middle 95% of the sample means* is:

This interval is Thus, we can conclude that 95% of sample means are within 0.64 (in either direction) of the population mean. 0.64 is the *margin of error* for a 95% confidence interval; it is the *maximum* possible distance between the population mean and the middle 95% of sample means.

In general, the margin of error for a 95% confidence interval for a population mean is two standard errors.

**How Is the Margin of Error Used?**

When the goal is to estimate the population mean, the margin of error can be added to and subtracted from the sample mean to form an interval estimate for the population mean. Why? The key is to understand how sample means are distributed within a sampling distribution.

The dotplot below shows the middle 95% of sample means in a sampling distribution of samples means from random samples of size with *n* = 50 from a population with and .

Note that the width of this interval is 1.28.



What happens if we create intervals of the *same width* centered at sample means instead of the

population mean ? The three number lines below show intervals of the same width (1.28) centered at three sample means, , , and .







The intervals centered at and (colored green) contain the population mean . However the interval centered at (colored red) does not. These examples highlight the reality that some intervals centered at sample means will contain the population mean and others will not. If a sample mean is in the middle 95%, the interval constructed around it will contain the population mean; if, however, a sample mean lies outside of the middle 95%, the interval constructed around it will not. Since 95% of sample means generate intervals that *do* contain the population mean, it is highly likely that a randomly selected sample mean leads to an accurate interval estimate.

Of course, since the population mean is unknown, we never know whether or not our interval estimate actually does contain the population mean.

**Approximating the Standard Error**

The standard error (standard deviation of sample means) is based on the population standard deviation . Since we often don’t have this population parameter, we can calculate an *approximate standard error* by using the sample standard deviation to estimate the population standard deviation.

The following procedure outlines the process of constructing a 95% confidence interval for a population mean based solely on sample statistics.

**Process for Constructing 95% Confidence Intervals for a Population Mean**

1. Find a point estimate (sample mean), .
2. Calculate the margin of error. The margin of error is two approximate standard errors.
3. Create the interval estimate:

Confidence intervals are interval estimates developed from sample data. The confidence level is the long-run probability that an interval estimate actually contains the true value of the population parameter, assuming this estimation process is repeated a large number of times. We will use the 95% confidence level but other confidence levels could be used.

1. A student was interested in estimating the mean number of smartphone apps that students at his high school who own smartphones use on a monthly basis. He randomly surveyed 40 students and found a sample mean of 24 apps per month with a standard deviation of 4.5 apps per month. Determine a 95% confidence interval for the population mean.
2. Determine the sample mean.
3. Determine the approximate standard error.
4. Calculate the margin of error for a 95% confidence interval.
5. Construct a 95% confidence interval and sketch the interval on the number line below.



1. Interpret the confidence interval.
2. A statistician is interested in the mean amount of time teenagers who watch movies online spend watching movies online per week. She surveys 50 teenagers who watch movies online and finds a sample mean of 4.2 hours per week with a sample standard deviation 1.3 hours per week. Determine a 95% confidence interval for the mean amount of time teenagers who watch movies online spend watching movies online per week.
3. Determine the sample mean.
4. Determine the approximate standard error.

1. Calculate the margin of error for a 95% confidence interval.
2. Construct a 95% confidence interval and sketch the interval on the number line below.



1. Interpret the confidence interval.
2. How would the confidence interval change if the sample size was quadrupled and the sample mean and sample standard deviation remained the same?