**Activity 2.4.4: Putting It All Together**

Two students, Will and Beth, are having an argument. Will says that the expression $x^{2}+1$ can’t be factored but Beth says that it can. They each show their work to support their claim.

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| Will’s work:Since the middle term is 0•x and the last term is 1, I need to find two numbers that multiply up to 1 and add up to 0. The only numbers that multiply to 1 are 1•1=1 or -1•-1=1. But since 1+1=2 and -1+-1=-2, there aren’t any factors of $x^{2}+1$, i.e., it is prime. | Beth’s work:Since the factors of a quadratic can be found from the zeros of the related quadratic equation, I set $x^{2}+1=0$. To solve for *x*, I isolated *x2* = –1 and took the square root of both sides. Therefore, $x=\pm i$ and the factors are *(x+i)* and *(x–i)* so $x^{2}+1=(x+i)(x-i)$. |

1. Which student is correct? Explain your answer.

2. Is it possible that different conditions for factoring would allow Will or Beth to be correct? Explain your answer.

In fact, both Will and Beth can be considered correct if you decide what numbers you will allow the coefficients of the polynomial to equal: rational numbers only, real numbers only, or complex numbers.

When we allow the coefficient to equal complex numbers, then any quadratic can be factored as argued by Beth. Whatever the zeros of the quadratic equation $ax^{2}+bx+c$ = 0 are, then the quadratic expression can be factored into $a(x-r\_{1})(x-r\_{2})$, where *r1* and *r2* are the solutions to the quadratic equation $ax^{2}+bx+c=0$.

The general property is called the **Fundamental Theorem of Algebra** and states that every polynomial can be broken down into a product of linear factors with complex coefficients. This activity will verify this property for any quadratic of the form $ax^{2}+bx+c,$ where *a, b,* and *c* are real numbers. Note that any real zero, *m*, can be considered a complex number of the form *m + 0i*.

For each of the following, solve the quadratic equation and use the resulting zeros to factor the quadratic expression into a product of linear factors with complex or real coefficients. For each example, multiply out the linear factors to check your answer.

1. $x^{2}+2x+5$ = 0

2. $-x^{2}+2x+24=0$

3. $2x^{2}+x+3=0$

How do we know that when *m + ni* is a zero of a quadratic function with real coefficients, that *m – ni* is also a zero? Fill in the table below to look for a pattern of the values of *f(m + ni)* and *f(m –ni)* for the quadratic functions with real coefficients: *f(x)=x2 + 2x + 6*. Put your calculator in complex mode and evaluate the quadratic for each complex number and its conjugate.

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| --- | --- | --- | --- | --- | --- | --- | --- |
| *m + ni* | *f(m + ni)* | *Real Part f(m + ni)*  | *Imag.**Part**f(m + ni)* | *m – ni* | *f(m – ni)* | *Real Part f(m – ni)*  | *Imag.**Part**f(m – ni)* |
| *1+2i* |  |  |  |  |  |  |  |
| *-2+3i* |  |  |  |  |  |  |  |
| *5–i* |  |  |  |  |  |  |  |
| *-1–i* |  |  |  |  |  |  |  |
| *.5+.5i* |  |  |  |  |  |  |  |

4. What do you notice about the value of *f(m + ni)* and *f(m – ni)?*

*5.* What conclusion can you make about *f(m – ni)* if *f(m + ni)=0*?

6. Find a quadratic function if you know that one of the factors of the quadratic expression forming the function is *x – (2 + 3i).*

7. Is the answer you got for #6 unique? If not explain why not.