**Activity 2.4.3 Complex Zeros:**

**Do They Have the Same Properties as Real Zeros?**

We saw that for any real zeros *m* and *n* of a quadratic function, we can work backwards to show that *x=m* and *x=n* and hence *(x – m)* and *(x – n)* are factors of a quadratic function of the form *y=x2 –(m+n)x+m•n.* Vice-versa, whenever we see a quadratic function of the form

*y = x2 + bx + c*, we know that the sum of the zeros equals *–b* and the product of the zeros equals *c*.

However, can we find out if that property still holds for quadratic functions that have complex zeros? To do that let’s review how to add and multiply complex numbers.

**Addition of Complex Numbers:**

Adding or subtracting complex numbers is like adding like terms, the imaginary number *i* acts like the variable *x*. Therefore, when we add or subtract complex numbers, we can apply the commutative and associative properties to add or subtract the real parts and add or subtract the imaginary parts, separately. For example:

(1 – 2*i*) + (3 + 4*i*)

= 1 – 2*i* + 3 + 4*i*

= 1 + 3 – 2*i* + 4*i*

= 4 + 2*i*

Similarly, if we subtracted the two complex numbers, we would obtain:

(1 – 2*i*) – (3 + 4*i*)

= 1 – 2*i* – 3 – 4*i*

= 1 – 3 – 2*i* – 4*i*

= –2 – 6*i*

**Multiplying Complex Numbers:**

In like manner, multiplying complex numbers is like multiplying binomials with the exception that whenever we see *i2*, we can replace it by –1. For example:

(1 – 2*i*) • (3 + 4*i*)

= 1•3 – 2*i*•3 + 1•4*i* –2*i•*4*i*

= 3 – 6*i* + 4*i* – 8*i2*

= 3 – 6*i* + 4*i* – 8•(–1)

= 3 + 8 – 6*i* + 4*i*

= 11 – 2*i*

Before testing to see if the property holds for complex number solutions of a quadratic, practice adding, subtracting and multiplying the complex numbers, 2 – *i* and –1 + 3*i*.

**Testing to See if the Property Holds:**

For each of the following quadratic functions, 1) find the zeros of the function, 2) test to see whether the sum of the zeros is the opposite of the x coefficient, and 3) test to see if the product of the roots is the independent term. Fill in the table with your findings.

|  |  |  |  |
| --- | --- | --- | --- |
| **Function** | **Zeros** | **Sum of the Zeros****–b** | **Product of the Zeros****c** |
| *f(x)=x2–4x+5* |  |  |  |
| *g(x)=x2+6x+13* |  |  |  |
| *h(x)=x2+2x+6* |  |  |  |
| *k(x)=x2+4x+1* |  |  |  |

Use the space below for your computations.

**Property:** For any quadratic function of the form *f(x) = x2 – bx + c*, the quadratic can be expressed as the product of two binomials, *(x–r1)(x–r2),* where the sum of the zeros

*r1+r2 = b* and the product of the zeros, *r1•r2 = c* for all roots r1 and r2, real or complex.

**Consequence of the Property**

For any quadratic function, we can apply the converse of the property to find an equation given its two zeros.

1. **Given Two Zeros:**

Given the two zeros of a quadratic function, we can work backwards from the zeros to find a corresponding quadratic function that has those two zeros by finding the sum and product of the roots.

Find a quadratic function with roots, 2 and –3.

For quadratic functions with **rational coefficients**, we can apply the converse of the property to find an equation given one zero in the form $m\pm \sqrt{n}$.

2. **Given One Irrational Zero of the Form** $m\pm \sqrt{n}$

 Given one irrational zero of the form $m\pm \sqrt{n}$, its conjugate will be the second zero and the sum of the roots will equal 2*m* and the product will equal *m2 – n*. therefore, an equation for a quadratic function with roots $m\pm \sqrt{n}$, is *f(x)=x2 – 2mx +( m2 – n).*

Find a quadratic function, one of whose roots is $2-\sqrt{3}$.

For quadratic functions with **real coefficients**, we can apply the converse of the property to find an equation given one zero in the form $m\pm i\sqrt{n}$.

3. **Given One Complex Zero of the Form** $m\pm i\sqrt{n}$

 Given one complex zero of a quadratic function with real coefficients of the form $m\pm i\sqrt{n}$, its conjugate will be the second zero. The sum of the roots will equal 2*m* and the product will equal *m2 + n*. therefore, an equation of a quadratic function with roots $m\pm i\sqrt{n}$, is *f(x)=x2 – 2mx +( m2 + n).*

Find a quadratic function, one of whose roots is $1+i\sqrt{5}$.