**Activity 2.3.2 Imaginary Numbers**

In 50 A.D., Heron of Alexandria ran into a problem when he tried to take the square root of a negative number. He quit when he could not figure it out! In the 1500’s Girolamo Cardano solved an equation and came up with  as one of the solutions. He found no pleasure in working with these numbers and considered them useless. Many other mathematicians agreed with him. In 1637 Rene Descartes called these numbers “imaginary”, meaning it in a derogatory way. Other mathematical greats such as Isaac Newton and Albert Girard called these “solutions impossible.” In the 18th century, Euler introduced the notation

More and more mathematicians found uses for these numbers and helped the world understand them.

1. Discover the interesting pattern occurs when you raise *i* to a power:

a. Fill in the table:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  | \_\_\_\_ |
| 1 | \_\_\_\_ |
|  | \_\_\_\_ |
|  | \_\_\_\_ |
|  | \_\_\_\_ |
|  | \_\_\_\_ |
|  | \_\_\_\_ |
| = | \_\_\_\_ |

b. What is the pattern that occurs when *i* is raised to the exponents 1, 2, 3, 4, 5 …in order?

c. How many terms does it take till the pattern is repeated?

2. a. Fill in the blank:

\_\_\_\_

\_\_\_\_

\_\_\_\_

\_\_\_\_

b. Notice that 16 = 4·4, 20 = 4·5, 44 = 4·11, and 100 = 4·25 . We say that given any natural number ‘n’ 4n is a multiple of 4. Also 4*n* is divisible by 4.

Conclusion: \_\_\_\_

3. a. Consider

\_\_\_\_

\_\_\_\_

\_\_\_\_

\_\_\_\_

b. If we divide the exponent of *i* into groups of 4, and the remainder is 1 then we have

4. Try these, then write a procedure for simplifying *i* is raised to a natural number.

a. \_\_\_\_ b. \_\_\_\_ c. \_\_\_\_ d. \_\_\_\_ e. \_\_\_\_ f. \_\_\_\_

g. \_\_\_\_ h. \_\_\_\_ j. \_\_\_\_ k. \_\_\_\_\_

5. In full sentences, explain how to simplify *i* when it is raised to a natural number e.g., .

6. Remember the multiplication rules for a square roots? Does this rule hold if a square root is an imaginary number?

Recall that , because the left side is 3·5 and the right side is that equals 15, also. In general, if a and b are positive real numbers, then

Let’s look at this ‘proof’ that -1 = 1. Either -1 really does equal 1, or you have to find a flaw in the argument:

and also equals . Since is equivalent to , which we then write as . Multiplying -1 by -1 gives 1, so .

So , from the beginning: , and . In other words , and -1 =1

What is the flaw in this argument?

7. When multiplying or dividing by the square root of a negative number, you must rewrite each radical that has a negative radicand so that it is a real number multiplied by .

Example:

a. b.

c. d. e.

\

f. g. h.

i. j. k.

l. m. n.