**Unit 5: Investigation 6 (3 Days)**

**Geometric Series**

**Common Core State Standards**

A-SSE-4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

**Overview**

This unit ends with two investigations whose focus is on applications of the exponential family.

The logarithmic family will be needed to solve some of the equations. Investigation 6 begins with geometric series and the folding paper activity from Investigation 1. The first activity defines a finite geometric series, in general, and computes a finite sum. Sierpinski’s Carpet is in the fourth activity. Students investigated Sierpinski’s Triangle as part of the unit 1 on patterns in Algebra 1. Activities on calculating mortgages and investments follow in investigation 7. This unit includes the derivation of the formula for the sum of a finite series.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Recognize a geometric sum .
* Be able to find a closed form expression for a finite sum.
* Use a geometric sum in an application problem.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 5.6** The exit slip is one word problem.
* **Journal Prompt 1** Consider the geometric sum

 Find this sum two different ways by first using the difference of two formulas and

second, by factoring out from each term and then using the formula once. Which way did you prefer and why?

* **Activity Sheet 5.6.1 Finite Geometric Series** has students find the sum of some concrete finite geometric series and then based on the observations they make in the process of doing the two concrete examples , derive the formula for the sum of a finite geometric series and use it to solve some applied problems including the launch question of how many rectangles were made in folding the paper 50 times.
* **Activity Sheet 5.6.2 Sam’s Story** has students return to Sam from Investigation 1 of this unit and calculate how much money his uncle will have given him by January 14, February 2 and other related problems whose answers need a finite sum.
* **Activity Sheet 5.6.3 Rumors, Worms, and Viruses** has student solve problems related to rumors, computer worms and viruses.
* **Activity Sheet 5.6.4 Return to Sierpinski** has students return to Sierpinski and recursive rules from Algebra One Unit 1 and they have to determine whether they should be using geometric sequences/exponential models or geometric series .

**Launch Notes**

This investigation begins by recalling the paper folding activity in Investigation 1. In that first investigation, students counted the number of rectangles generated by each fold. Remember: Do not count rectangles you can make by putting several rectangles together. In this investigation, students are counting the total number of rectangles they generated from the start of that process. Begin with one rectangle. Fold the paper in half, and students see two rectangles. If the folding stopped here there would be a total of 1 + 2 = 3 rectangles, one from the starting point before the paper was folded and then 2 rectangles after the one fold. Do another step. When the paper is folded in half again, they should count four rectangles. Now the total number of rectangles is 1 + 2 + 4 = 7. If this folding were to continue for 10 folds, what would be the total number of rectangles generated? What about 50 folds?

The students should see that to count up to 10 folds is not unreasonable as it is easy to do.

1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 = 2047

But now asking for the total up to 50 folds is more challenging, so finding a general formula is in order!

In the first activity, students will continue with the launch activity, and generate the closed form expression for the solution to the problem for n folds.

**Teaching Strategies**

A finite numerical sequence is a set of (ordered) numbers, denoted A geometric sequence is defined when the elements of the sequence, given as the starting number, are defined recursively,

Students studied geometric sequences recursively in Unit 1 of the Algebra 1. Another way to describe a geometric sequence is that the ratio of consecutive elements is the same constant r, r ≠ 1. Students examined this approach in Algebra 1, units 1 and 7.

A geometric series is the sum of the elements of a geometric sequence. To find a formula for the sum of the terms, students are guided though some concrete numerical examples and then follow those steps they used to find the general sum S.

To find a closed form expression for , let

Then

Now notice that and have most terms in common. Then

Hence

 and r ≠ 1.

In **Activity 5.6.1**, students discover the formula, given above, for the sum of a finite number of geometric terms. Then they are in a position to answer the question posed in the Launch activity as well as to answer other geometric series question.

After working through **Activity Sheet 5.6.1** students can be grouped to solve the following problem or for students needing more support, the Differentiated Instruction Problem below could be used instead for the group activity or can be done first before the group activity.

**Group Activity**

A ball is dropped from a height of 20 feet. Every time it falls down, the ball then bounces up 80% of the height that it was dropped. How far has the ball travelled in 10 bounces? In 20 bounces?

**Differentiated Instruction (For Learners Needing More Help)**

Suppose your parents put $1 in a savings account for you on the day you were born, and doubled the amount of their deposit on each of your birthdays. How much would you have in your account on your 5th birthday? Your 10th birthday? When you graduate from high school at the age of 18, will you have enough money to buy a new car?

Given the closed formula for the sum of a finite geometric series, **Activity 5.6.2** returns to Sam introduced in Investigation 1 and uses the formula to determine various sums of money his uncle would have given him.

**Differentiated Instruction (Enrichment)**Students can research a Ponzi scheme,

a fraudulent investing scam promising high rates of return by generating returns for older investors by acquiring new investors. It continues to return to earlier investors as long as there are enough new investors. The following model can be examined by students and series can be used to explain why this scheme is doomed to collapse. Suppose the person generating the scheme, Mr. Bad, recruits 2 investors in his nonexistent venture and asks them to contribute $200 each. He then needs 4 new ones and with his new $800, pays his first two $200 each a 100% return and keeps $400 for himself. The he seeks out 8 new investors and with his $1600 gives $1200 to each of his first 6 investors and keeps $400 for himself. Now he needs 16 new investors so he has $3200 so the 14 old investors can each have $200 and he has $400 for himself. Students should research Ponsi schemes in general, study one that was caught, and for the made up problem here examine the sequences formed by the number of investors and the money collected and using their knowledge of series and sums of finite series explain why it will not be able to continue .

After the discussion of Activity 5.6.2 problems the **Journal Prompt 1** can be assigned.

**Journal Prompt 1**

Consider the geometric sum

Find this sum two different ways by first using the difference and the secondly by factoring out from each term and then using the formula once. Which way did you prefer and why? Students preferences will vary. Both yield .88887700825.

Applications of a geometric series are explored in **Activity 5.6.3** where students examine the spread of rumors, computer worms and computer viruses. **Exit Slip 5.6** can be assigned after Activity 5.6.3 has been discussed.

**In Activity 5.6.4** students have to determine whether they should be using geometric sequences/exponential models or geometric series as they  return to Sierpinski and recursive rules from Algebra One Unit 1 and then apply their knowledge of series and sequences .

**Closure Notes**

Have students answer the question posed in the well known children’s rhyme and show that the answer to the question is the sum of a finite geometric series or maybe it is not!!!!!.

Assume everyone is going to Saint Ives.

As I was going to St. Ives

I met a man with seven wives

Every wife had seven sacks

Every sack had seven cats

Every cat had seven kits

Kits, cats, sacks and wives

How many were going to St Ives?

Answer is 2801+ narrator = 2802 but students need to show the series and the use of the sum formula.

Now the actual nursery rhyme does not say assume everyone is going to St. Eves. It just starts with “As I was going to St Ives” so the man with his 7 wives etc. was returning and hence only the narrator of the rhyme was going with perhaps others that are not mentioned.

But the man with wives, kits and sacks and cats totals 2801.

A similar but not a trick question is problem 7 of the Rhind Papyrus written about 1650 B.C. There were 7 houses each with 7 cats, each with 7 mice, each with 7 spelt, each with 7 hekat. How many are there in all? Sum is 19,607.

**Vocabulary**

Closed form expression

Finite

Finite geometric series

Geometric series

Infinite

Infinite geometric series

Mortgage

Sequence

Series

Sierpinski

**Resources and Materials**

**Activities 5.6.1 – 5.6.3 should be completed. If time permits do Activity 5.6.4.**

Activity Sheet 5.6.1 Finite Geometric Series

Activity Sheet 5.6.2 Sam’s Story

Activity sheet 5.6.3 Rumors, Worms, and Viruses

Activity sheet 5.6.4 Return to Sierpinski

Graphers or equivalent technology