**Activity 5.7.1 Saving for a Down Payment (Part 1)**

Earlier in the unit, you saw the compound interest formula.

P = principal (initial deposit)

r = annual interest rate (written as a decimal)

n = number of compounding periods per year

t = number of years

F = amount in the account after t years

$$F=P\left(1+\frac{r}{n}\right)^{nt}$$

Keep in mind some of the assumptions you need when you plan to use that formula: you make an initial deposit, you make no other deposits or withdrawals, and interest is based on both the initial deposit and the accumulated interest.

What happens if you can’t deposit one big sum of money at the beginning and let it sit in the bank? For most people, savings happen gradually over time. For each compounding period, a regular deposit is made into the account. Let’s see how this works.

**Situation:** Aaron has six months to save money for a down payment on a car. He can put $100 per month in the bank on the last day of each month. His bank offers a savings account with **6% annual** **interest compounded monthly**. (This is not a realistic interest rate for a savings account right now for rates are very low, but it will allow you to focus on the ideas behind the process rather than worrying about decimals that get too messy. Don’t worry, we will encounter more realistic values soon!)

Right now we don’t have a formula to figure out how much will be in his bank account after 6 months. We will first do a problem concretely and then return to the process to see if we can develop a formula. We know he will deposit $600 over the 6 months starting on July 31, but we need to also figure out how much interest is earned each month. Let’s make a table and investigate. **Make sure you calculate the interest before you add the new deposit for the month.** If a deposit of $100 is made on July 31 and interest is compounded on July 31, the bank will not give you interest on that $100. Why? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ That means for the first month of Aaron’s savings, he doesn’t earn any interest because that deposit was made at the end of the month. But for month two he will have had $100 in the bank long enough to get 0.5% interest which is\_\_\_\_\_\_\_\_. So on August 31, $\_\_\_\_\_\_ will post to his account and he will then also make his 100 dollar deposit for the month of August. Fill in the last cell in row two for his new amount as of August 31. The complete the rows for the remaining 4 months.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Interest Earned** | **Amount in Account from the Previous Month plus Accrued Interest** | **Deposit** | **Total Amount in the Account at the End of the Month** |
| **Month 1** | $0 | $0 | $100 | $0 + $100 = $100 |
| **Month 2** |  |  | $100 |  |
| **Month 3** |  |  |  |  |
| **Month 4** |  |  |  |  |
| **Month 5** |  |  |  |  |
| **Month 6** |  |  |  |  |

**Questions:** Hopefully the process that you just went through to figure out the amount in the account after 6 months reminded you of some of your earlier work with the compound interest formula.

1. How much did Aaron have in the bank after 6 months?
2. Why do you think you were only asked to make a table for a 6 month savings plan rather than a 2 years or ten years?
3. Without doing the calculations, in which scenario would Aaron earn more interest: the given scenario or a situation where he could deposit $600 right away and let it sit for 6 months. Explain your reasoning and include the amount of interest he would have earned if he could have put the $600 in the bank on July 1 instead of putting in 100 dollars a month for the 6 months.

Now let us look at the problem in one other way.

The first deposit of $100 will earn 5 months’ interest 100(1+.005)5 = 102.53

The second deposit will earn 4 months’ interest 100(1+.005)4 = 102.02

The third deposit will earn 3 months’ interest 100(1+.005)3 = 101.51

The fourth deposit will earn 2 months’ interest 100(1+.005)2 = 101.00

The fifth deposit will earn 1 months’ interest 100(1+.005)1 = 100.50

The sixth deposit will earn no interest 100.00

Total: $607.56

Note then:

 F the future value or amount = C + C(1 + i) + C(1 + i)2 + C(1 + i)3 + C(1 + i)4+ C(1 + i)5

where *i = r/n* and *r* is the annual rate expressed as a decimal and *n* is the number of compounding periods.

For the total after *nt* periods where n is the number of compounding periods per year and t is the number of years, you should recognize that you have a \_\_\_\_\_\_\_\_\_\_\_\_\_\_ whose sum in general is given by

Equation (1) $S=\frac{a\_{1}\left(r^{n}-1\right)}{r-1}$

We can replace$a\_{1}$ with C, *r* with 1 + *i* and our exponent will be *nt.* But remember *i = r/n.*

Make the substitutions now in equation 1 so you can have a formula for the Future value of an annuity.

**Concept:** The situation you just looked at is a form of annuity. An annuity is a sum of money payable at regular intervals ([www.merriam-webster.com](http://www.merriam-webster.com)). Just adding that regular payment per period changes the equation significantly.

C = amount of regular deposit

r = annual interest rate (written as a decimal)

n = number of compounding periods per year

t = number of years

F = amount in the account after t years

Future Value of an Annuity (Savings)

$$F=C\left[\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right]$$

Keep in mind some of the requirements for using this formula:

* the deposit is made at the end of the period,
* interest is compounded at the end of the period before the deposit, and
* the deposit is made for a set amount and must be made every period.

**Try These:**

1. Use the Future Value of an Annuity formula to determine how much Aaron has in the bank after 6 months. Just as a reminder, Aaron has six months to save money for a down payment on a car. He can put $100 per month in the bank at the end of the month. His bank offers a savings account with 6% annual interest compounded monthly.

$F=C\left[\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right]$

Why do you think we obtained $607.56 in our second table?

1. Hilary is planning to open an account at her local bank to save for a vacation. She is saving to take a cruise. She wants to have $3000 saved in two years. Her local bank is offering a special 1.05% annual interest rate on a savings account compounded monthly. How much should she save per month in order to have $3000 at the end of two years?

$$F=C\left[\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right]$$

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1. Finbar has looked at his monthly budget and decided that he can deposit $35 per month into a savings account with 0.97% annual interest rate (APR) compounded monthly. His goal is to save $950 for a security deposit on a new apartment. At this rate, how long will it take him to save $950?

$F=C\left[\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right]$