**Unit 4: Investigation 2 (2 Days)**

**Similar Figures**

**Common Core State Standards**

* G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
* G-C.1 Prove that all circles are similar.

**Overview**

In this investigation students will begin by learning the definition of similarity as it applies to all 2-dimensional figures. Students will then define similarity transformations as the composition of isometries and dilations for polygons and circles. Once students have mastered similarity of all 2-dimensional figures, the focus of similarity transformations will remain on triangles,setting up for investigation 3.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Understand that figures are similar if their corresponding angles are congruent and all pairs of corresponding sides are in proportion.
* Use similarity transformations to prove that two polygons or circles are similar.
* Understand that congruence is a special case of similarity.
* Determine if two figures are similar or not using similarity transformations.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 4.2** asks students to determine if two quadrilaterals are similar. Then given that two pentagons are similar they determine missing side lengths.
* **Journal Entry** asks students to explain why congruence is a special case of similarity. In addition, students are asked to write about specific examples they have seen in the public that show how both congruency and similarity are used in marketing.

**Launch Notes**

Similar figures are used everywhere in people’s lives. In fact, one profession that uses similar figures in much of their work is that of a graphic designer. Graphic designers are often presented with a small design drawn on paper and are asked to construct *similar* versions for use on trucks, letterhead, billboard, etc. The process used to create the design is based on dilations and the definition of similar figures. Using this connection to introduce students to similar figures can be a great way to not only help students understand the definition of similar figures but to see that math is used in everyday life and in many different professions.

You might begin by telling students that they work for a design company and they have been presented with a design for a company logo. The customer wants to see what the logo will look like when it is small on their letterhead, when it is large and posted on the back of a trailer or billboard, and when it is placed on company shirts. Show a picture of the design [find one on the internet or make your own] to students and ask them how can they enlarge or shrink the design so that it looks exactly like the original. Try to get students to think about how the parts of the pre-image and image should compare.

Has to look good in all sizes. Figure out the aspect ratio. If enlarged will all parts still fit on the paper. How will the figure look on wrapping paper? A sphere?

How is it going to look on a truck? The corner of the paper on a letterhead.

Could be a mural, tattoo

**Teaching Strategies**

In **Activity 4.2.1 Similarity Transformations** students use dynamic geometry software to explore similarity transformations. Students begin with a pre-constructed Geogebra file that contains similar figures. They measure angles and sides and compare corresponding measures. They discover how dilations along with other transformations are used to map one figure onto another. In this activity they are introduced to formal definitions for *similarity transformation* and *similar figures*:

* A **similarity transformation** is defined as a translation, rotation, reflection, or dilation or the composition of two or more of these transformations.
* **Similar figures** are defined as figures where one is the image of the other under a similarity transformation.

**Differentiated Instruction (For Learners Needing More Help)**

For some students, the instantaneous actions that take place when using technology can be too quick for them to realize the transformation. In this case, you can place an image on an overhead projector. Then move the projector closer or further to/from the board. This will help students to see a dilation in “action”. The use of the overhead in this way can also help with visualizing translations and rotations.

In **Activity 4.2.2 Similar Figures**, students explore the properties of corresponding sides and corresponding angles of two similar figures. Using a protractor and ruler, students measure all of the parts of both images while recording measurements at the same time. Students then compare the measures of the corresponding angles and the ratios of the pairs of corresponding sides. In the first example, the figures are similar. For the next example, students should complete this process again but for a different type of polygon. Students will then look at a figure where the pairs of corresponding angles are in proportion but the angles are not congruent. This is very important for students to see as they need to understand that in order for the figures to be similar BOTH the corresponding angles are congruent and the measures of the corresponding sides are in proportion.

In **Activity 4.2.3 Corresponding Parts of Similar Polygons s**tudents are work with various types of polygons to establish the properties of corresponding sides and angles of similar polygons. When we state that two polygons are similar then it means that all of the pairs of corresponding angles are congruent and the pairs of corresponding sides are in proportion. This understanding is very important to have before moving to the similarity “shortcuts”. This activity concludes with an informal proof that all equilateral triangles are similar to each other.

**Differentiated Instruction (For Learners Needing More Help)**

For students that are struggling with identifying a mapping of one figure onto another you might first have them create their own mappings in GeoGebra. Using a blank GeoGebra page have them begin with a polygon, dilate it, rotate and then translate it. GeoGebra will leave an image after each step. Have students study the results and then do it again in a different order. After a few attempts these students might be ready for activity 4.2.3 or a simpler version.

**Activity 4.2.4 Similarity and Circles** extends the concept of similarity to circles. Students first use GeoGebra to map one circle onto another with similarity transformations. In question 2 students are then asked to prove that all circles are similar to each other. There are two versions of this Activity. In **Activity 4.2.4** the proof in question 2 is open ended. In **Activity 4.2.4b** it is scaffolded for students who need more support. Follow up questions bring out the fact that while every pair of circles are similar, this property does not apply to every pair of triangle

**Group Activity*.*** Have students work in groups of 3 or 4. Each student creates a pair of similar triangles using dilation along with at least one other transformation (translation, rotation, or reflection). The other students write the statement of similarity (e.g. ∆*ABC* ~ ∆*DEF*) and try to figure out which transformations were used.

**Differentiated Instruction (Enrichment)**

Ask students to show that given two similar triangles in Geogebra, it is always possible to map one onto the other with four or fewer transformations. (Suggested strategy: translate to map one vertex onto its corresponding vertex. Then rotate to get two sides to lie on the same ray. Reflect around one of the sides, if necessary and then dilate.

**Journal Entry:** Why are all pairs of congruent figures also similar? Give some examples of where congruent and similar figures are used in everyday life. Look for students to understand that congruence is a special case of similarity with a scale factor = 1.

**Exit Slip 4.2** may be given at the end of the second day.

**Closure Notes**

Ask students what they would need to know to be sure that two triangles are similar. Elicit from them that they would need to know that three pairs of corresponding angles are congruent and three pairs of corresponding sides have the same ratio. Remind them that for congruent triangles we found some shortcuts, so we would not need to know about all six parts to prove that two triangles are congruent. Ask if they think there may be comparable shortcuts for similar triangles. Listen to their suggestions and then tell than that this will be the subject of the next investigation.

**Vocabulary**

Dilation

Rotation (from Unit 1)

Reflection (from Unit 1)

Translation (from Unit 1)

Isometry (from Unit 1)

Scale factor

Center of dilation

Ratio

Proportion

Similar figures

Similarity transformation

**Theorems**

**Triangle Similarity Theorem:** If two triangles are similar then pairs of corresponding sides have the same ratio and pairs of corresponding angles are congruent.

**Circle Similarity Theorem:** All circles are similar to each other.

**Resources and Materials**

Activities:

Activity 4.2.1 Similarity Transformations

Activity 4.2.2 Similar Figures

Activity 4.2.3 Corresponding Parts of Similar Polygons

Activity 4.2.4 Proving all Circles are Similar

GeoGebra file: ctcoregeomACT421.ggb

School Club Logo

Protractors

Rulers

Compasses