**Unit 3: Investigation 4 (3 Days)**

**Regular Polygons**

**Common Core State Standards**

* G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

**Overview**

Having established in Investigation 1 that the sum of the interior angles of a convex polygon is (*n*–2)180°, students will now be able to find the measure of an interior angle and an exterior angle in a regular polygon. With compass and straight edge they will construct equilateral triangles, squares, and regular hexagons. Using a protractor or rotations with GeoGebra or Geometer’s Sketchpad they will construct regular polygons with any number of sides. They will also discover that a regular *n-*gon has *n-*lines of symmetry.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Given a regular polygon with *n* sides, find the measures of the interior and exterior angles.
* Use compass and straightedge tools to construct equilateral triangles, squares, and regular hexagons
* Use a variety of tools to construct regular polygons with any number of sides
* Find lines of symmetry in a regular polygon

**Assessment Strategies: How Will They Show What They Know?**

* + **Exit Slip 3.4.1** asks students to calculate the measures of interior and exterior angles.
	+ **Exit Slip 3.4.2** asks students to draw lines of symmetry for a regular pentagon
	+ **Journal Entry** asks students to describe how to construct a regular hexagon given a circumscribed circle.

**Launch Notes**

Ask students what an equilateral triangle and a square have in common. They should observe that both figures have all their sides congruent and all their interior angles congruent. Then introduce the definition of “regular polygon.” An equilateral triangle can be described as a regular triangle and a square as a regular quadrilateral. In this investigation we will extend the concept of regular polygon to pentagons, hexagons, and to polygons with any number of sides.

Alternatively, introduce students to the Cherokee people who originally inhabited the southern Appalachian mountain region of North Carolina, Tennessee, and Georgia. Their society was organized into 7 clans. Their council houses had seven sections, one for each clan to sit in. Consequently a council house was built in the shape of a regular heptagon. Pose the question: What is the size of the angle formed where two sides of the council house meet?



You may want to use this video:[www.youtube.com/watch?v=Ax5s9bTfGog&list=PLDFmYxZHDXBmlewn3TloxvFiR0GFpPT0w&index=10](http://www.youtube.com/watch?v=Ax5s9bTfGog&list=PLDFmYxZHDXBmlewn3TloxvFiR0GFpPT0w&index=10). Start at 5:00 and run for about 2 or 3 minutes.

(Figure at right from Sharpe, 1970)

**Teaching Strategies**

In **Activity 3.4.1** **Exterior Angles of a Polygon** students discover through measurement that the sum of the exterior angles (one at each vertex) for any convex polygon, regardless of the number of sides, is 360°. This rather surprising result can be proved algebraically since the sum of the linear pairs at each vertex is 180°*n.*

**Differentiated Instruction (For Learners Needing More Help)**

Remind students that the exterior angle of a polygon forms a linear pair with the adjacent interior angle. Some students have a tendency to extend both sides at the same vertex and identify as “exterior” the angle that is vertical to the interior angle.

In **Activity 3.4.2 Angles in Regular Polygons** students will discover and then prove the **Angles in a Regular Polygon Theorem:** In a regular polygon the measure of each interior angle is $\frac{\left(n-2\right)180°}{n}$ and the measure of each exterior angle is $\frac{360°}{n}$.

Following this activity you may give students **Exit Slip 3.4.1**.

**Activity 3.4.3 Applying Angle Properties in Regular Polygons** consists of relatively straightforward exercises based on the Angles in a Regular Polygon Theorem. It is suitable for a homework assignment.

In **Activity 3.4.4** **Constructing Regular Polygons with Compass and Straightedge** students construct equilateral triangles, squares, and regular hexagons, given the length of one side. (Equilateral triangles and squares have already been constructed in Unit 2 Investigation 6. The proof for the equilateral triangle construction was shown in Unit 2 Investigation 7.) They then construct these regular polygons given the radius of a circumscribed circle. They observe that regular polygons have rotational symmetry. As an extension students are challenged to construct a regular octagon with these tools.

**Note to teachers:** It is also possible to construct a regular pentagon with compass and straightedge. That construction is related to the Golden Ratio and is introduced as a special topic in Unit 8. However, there are many regular polygons that cannot be constructed with compass and straightedge. These include polygons with 7, 11, 13, and 14 sides. It has been proved that the only values of *n* for which a regular *n-*gon is “constructible” are of the form 2*k* or *p*2*k* where *p* is a “Fermat prime” of the form $2^{2^{m}}+1$, or the product of two or more distinct Fermat primes. The first four Fermat primes are 3, 5, 17, and 257.

**Differentiated Instruction (Enrichment)**

Have students research which *n*-gons are constructible. See note to teachers above.

**Journal Entry**

Describe the steps necessary to construct with compass and straightedge a regular hexagon inscribed in a circle. Look for students to construct six equilateral triangles.

In **Activity 3.4.5 Constructing Regular Polygons with Other Tools** students will start with a circle and a point on the circle. Using software they will rotate this point by 360°/*n* to locate points on the circle that are the vertices of a regular *n­-*gon. An alternative is to use a compass to draw the circle and a protractor to draw angles measuring 360°/*n* at the center. There are two versions of this activity, one for GeoGebra, and one for compass and protractor. A second approach starts with a side of the regular *n-*gon. Interior angles are drawn either using a protractor or the rotation feature of GeoGebra. These methods enable us to draw regular polygons with any number of sides, regardless of whether it is possible to construct the polygon with compass and straightedge alone.

**Group Activity**

For Activity 3.4.5 place students in groups and have each group member assigned a particular value of *n*. Each student constructs an *n*-gon inscribed in a circle and an *n*-gon given one side using geometric software or protractors. When students have completed their task, they check each other’s work.

In **Activity 3.4.6 Lines of Symmetry in Regular Polygons** students draw lines of symmetries in regular polygons. From a template they cut out the polygons and demonstrate the symmetry through folding. They record their results on the activity sheet. If the polygon has an odd number of sides, each symmetry line contains a vertex and the midpoint of the opposite side. If the number of sides is even, half of the symmetry lines connect vertex to vertex, and the other half midpoint to midpoint. In all cases the number of lines of symmetry is equal to the number of sides.

**Differentiated Instruction (Enrichment)** Ask students to prove that if a regular polygon has a line of symmetry it must either contain at least one vertex or pass through the midpoints of two sides.

Following this activity you may give students **Exit Slip 3.4.2**.

**Closure Notes**

Make a table (or have students make tables) showing the relationship between the number of sides of a regular polygon and the measures of its interior and exterior angles. Discuss the patterns observed: as the number of sides increases the size of the interior angle increases, but never exceeds 180° and the size of the exterior angle deceases, but is always greater than zero. If you have time, use a graphing calculator to displays these data on a graph.

|  |  |  |
| --- | --- | --- |
| *n* | Interior angle | Exterior angle |
| 3 | 60° | 120° |
| 4 | 90° | 90° |
| 5 | 108° | 72° |
| 6 | 120° | 60° |
| 7 | 128.57° | 51.43° |
| 8 | 135° | 45° |
| 9 | 140° | 40° |
| 10 | 144° | 36° |
| 12 | 150° | 30° |
| 15 | 160° | 20° |
| 20 | 165° | 15° |
| 30 | 168° | 12° |

**Vocabulary**

Exterior angle

Circumscribed circle

Inscribed polygon

Pentagon

Hexagon

Heptagon

**Theorems and Constructions**

**Exterior Angles of Polygon Theorem:** The sum of the exterior angles of a [convex] quadrilateral is 360°.

**Angles in Regular Polygon Theorem:** In a regular polygon the measure of each interior angle is $\frac{\left(n-2\right)180°}{n}$ and the measure of each exterior angle is $\frac{360°}{n}$.

**Equilateral Triangle Constructions**

**Square Constructions**

**Regular Hexagon Constructions**

**Resources and Materials**

Activity 3.4.1 Exterior Angles of a Polygon

Activity 3.4.2 Angles in Regular Polygons

Activity 3.4.3 Applying Angle Properties in Regular Polygons

Activity 3.4.4 Constructing Regular Polygons with Compass and Straightedge

Activity 3.4.5 Constructing Regular Polygons with Other Tools

Activity 3.4.6 Lines of Symmetry in Regular Polygons

Tools:

 Compass and Straightedge

 Protractors

 Geogebra and GSP

Information on the Cherokee council house:

<https://www.southern.edu/outdoor/facilities/Pages/cherokeecouncilhouse.aspx>

<http://www.hmdb.org/Marker.asp?Marker=36033>

[www.youtube.com/watch?v=Ax5s9bTfGog&list=PLDFmYxZHDXBmlewn3TloxvFiR0GFpPT0w&index=10](http://www.youtube.com/watch?v=Ax5s9bTfGog&list=PLDFmYxZHDXBmlewn3TloxvFiR0GFpPT0w&index=10).

Sharpe, J. Ed. *The Cherokees: Past and Present*. Cherokee, NC: Cherokee Publications, 1970.