**Unit 3: Investigation 3 (2 Days)**

**Parallel and Perpendicular Lines**

**Common Core State Standards**

* ***G-CO-9*** Prove theorems about lines and angles.
* ***G-CO-12*** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)... *constructing a line parallel to a given line through a point not on the line.*

**Overview**

Beginning with the need to demonstrate that all rectangles are parallelograms, students will conjecture that two lines perpendicular to the same line are parallel. This can be proven as a consequence of a more general theorem, that if a line intersects two other lines making corresponding angles congruent, then the other two lines are parallel. We emphasize that this theorem is the converse of the Parallel Lines Corresponding Angles Theorem proved in Unit 2. The proof is indirect and depends upon the Exterior Angle of a Triangle Corollary. Finally we use this theorem to construct parallel lines with compass and straight edge.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Understand the distinction between a theorem and its converse.
* Apply theorems to prove that two lines are parallel.
* Construct a line through a point parallel to a given line with compass and straightedge.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 3.3** asks students to write a short proof that two lines are parallel given certain conditions.
* **Journal Entry** asks students to explain to another student why the Parallel Lines Corresponding Angles Theorem and its converse do not say the same thing.

**Launch Notes**

Ask students to define rectangle. Many students will typically describe a rectangle using all the properties of which they are aware. Some may define it as a special type of parallelogram. Most will give more conditions than are absolutely necessary. At the end of this discussion you should get the class to agree on one of the following definitions:

1. A rectangle is a quadrilateral with four right angles
2. A rectangle is a quadrilateral with at least three right angles.

If the class accepts definition (1), they can then prove that three right angles are sufficient to guarantee that a quadrilateral is a rectangle. Or if they accept definition (2) they can prove that every rectangle has four right angles. In both cases the proof rests on the Quadrilateral Sum Theorem.

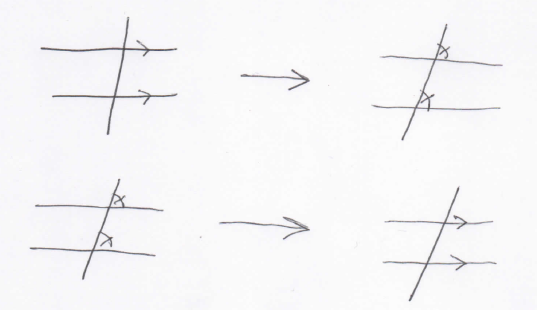
Now turn your attention to the relationship between rectangles and parallelograms. Most students should already understand that a rectangle is a special type of parallelogram. Ask them to prove this. Their response is likely to be that each pair of opposite sides is perpendicular to the same side and therefore must be parallel. Point out that they are assuming this conjecture: If two lines are perpendicular to the same line, then they are parallel to each other. That conjecture will be proved in the course of this investigation.

**Teaching Strategies**

In **Activity 3.3.1 Parallel and Perpendicular Lines** students review properties of slopes learned in Algebra 1: (1) if two lines have the same slope, then the lines are parallel, and (2) if the slopes of two lines are opposite reciprocals, then the lines are perpendicular. They apply these properties to a line with a given slope, *m*, and two lines that are perpendicular to it. They see that both lines have the same slope, –1/*m,* and are therefore parallel to each other. They also consider the special cases where the lines are vertical or horizontal.

**Differentiated Instruction (Enrichment)** Extend the coordinate properties of perpendicular lines to show that two vectors <*e, f*> and <*g, h*> are perpendicular if and only if *eh + fg* = 0. The quantity *ef + fg* is called the “dot product” of the two vectors.

In **Activity 3.3.2 Parallel Lines Corresponding Angles Converse** students prove that if a transversal intersects two lines so that a pair of corresponding angles are congruent, then the lines are parallel. It is an indirect proof based on the idea that if the two lines intersect, then a triangle is formed in which an exterior angle is equal to a non-adjacent interior angle. From this theorem, it follows that if two lines are perpendicular to the same line, they must be parallel.

**Differentiated Instruction (For Learners Needing More Help)** Help students clarify the distinction between a theorem and its converse. For some students a visual representation may help. In this figure the Parallel Lines Corresponding Angles Theorem is followed by its converse.

Most of Activity 3.3.2 should be completed by the end of the first day. Question 2 from Activity 3.3.2 and questions 1 and 2 from Activity 3.3.3 may be assigned for homework. You may also want to assign the journal prompt at the end of the first day. If you did not use **Activity 2.3.5 Converses of Conditional Statements** in Unit 2, you may want to introduce it now.

**Journal Entry**Suppose another student says to you, “I don’t understand why we have two theorems about parallel lines and congruent corresponding angles. Don’t they mean exactly the same thing?” How would you respond? Look for students to explain the difference between a conditional statement and its converse.

**In Activity 3.3.3 Proving that Lines are Parallel** the Parallel Lines Corresponding Angles Converse is used to prove the Parallel Lines Alternate Interior Angles Converse and the Parallel Lines Same Side Interior Angles Converse. In addition, students apply these theorems to a variety of exercises involving parallel and perpendicular lines. These will include examples where a pair of exterior angles is given to be congruent or a pair of same side interior angles is given to be supplementary. Also some exercises will involve several pairs of parallel lines.

In **Activity 3.3.4 Constructing Parallel lines** students use the Parallel Lines Corresponding Angles Converse to construct a line through a point parallel to a given line with compass and straightedge alone. This activity may be done with physical tools or with software. Point out to students that both Geogebra and GSP have built in short-cuts for this construction.   
  
As an alternative some students may want to use a different approach: first construct a line through *P* perpendicular to line *m*. Then construct a line perpendicular to the constructed line at *P*. Use the Perpendicular and Parallel Lines Theorem to prove that the construction works.

**Activity 3.3.5 Proofs with Parallel Lines** is designed as a group activity. See below.

**Group Activity**See Activity 3.3.5. Put students in groups of 3. The activity sheet gives a figure with several intersecting lines and several known angle measures. In addition there is a set of statements to be proved or disproved. Student A picks one of the statements. Student B attempts to prove or disprove the statement to everyone’s satisfaction. Student C helps B if necessary. Once the task has been completed for one statement, rotate roles A to B, B to C, and C to A, and repeat the process.

**Exit Slip 3.3** may be used at the end of the second day.

**Closure Notes**

Ask the students to describe how two lines may be proven parallel. They should be able to recall the three theorems introduced in this investigation as well as the property of slopes used in the first activity.

**Vocabulary**

Converse

Parallelogram

Rectangle

**Theorems and Constructions**

**Corresponding Angles Congruent 🡪 Parallel Lines (Parallel Lines, Corresponding Angles Converse):** If two lines are cut by a transversal so that a pair of corresponding angles are congruent, then the lines are parallel.

**Alternate Interior Angles Congruent 🡪 Parallel lines (Parallel Lines, Alternate Interior Angles Converse):** If two lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the lines are parallel.

**Same Side Interior Angles Supplementary 🡪 Parallel lines (Parallel Lines, Same Side Interior Angles Convers):** If two lines are cut by a transversal so that a pair of same side interior angles are supplementary, then the lines are parallel.

**Perpendicular and Parallel Lines Theorem:** If two lines are perpendicular to the same line, then they are parallel.

**Line through a given point parallel to a given line (Construction)**

**Resources and Materials**

Activity 3.3.1 Parallel and Perpendicular Lines

Activity 3.3.2 Parallel Lines Corresponding Angles Converse

Activity 3.3.3 Proving That Lines Are Parallel

Activity 3.3.4 Constructing Parallel Lines

Activity 3.3.5 Proofs with Parallel Lines

Compass and Straightedge for **Activity 3.3.4.** Alternatively use the compass and straightedge tools with Geogebra or Geometer’s Sketchpad.